Futures
What is a Future?

• An agreement between a buyer and seller to trade an asset at a future date.

• Typical assets traded as futures: shares, bonds, metals, oil and gas, corn, pork bellies, …

• The future date at which asset to be traded is called the settlement date. It is usually 1st day of January, April, July or October.
Futures Exchanges

• Futures are traded through an *exchange* (like shares), such as Chicago Board of Trade.

• Exchange organises standard contract conditions, and handles any payments between buyer and seller.
Examples of Futures (1)

1. On Jan 2nd 2016, Investment Bank A puts for sale a future to sell 1 million shares in Microsoft on July 1st 2016 at $28 per share.

2. On March 23rd 2016 a commodities broker buys from an agricultural co-operative a future for the purchase of 100,000 lbs of pork bellies from the co-operative on October 1st 2016 at 80c per lb.
Examples of Futures (2)

3. Today, you agree to sell 5,000 shares in AIB to me on July 1st 2017 at €13.30 per share

4. A commodities broker sells to another broker for $3.1M a future contract to sell 10,000 barrels of Brent Crude on January 1st 2017 at $30 per barrel.
A Future Can be Bought and Sold

- Futures are bought and sold, just like underlying asset.
- The price of the future changes over time, according to:
  - how close to the settlement date;
  - the price of the underlying asset.
- If you commit to sell an asset as a future then:
  - Someone will turn up on the settlement day to buy it;
  - It may not be the person you originally sold to;
  - The future may have changed hands several times.
What is a Fair Price for a Future?

• Should equal what you would earn by buying asset now.

• Suppose you have €S available.

• You have two strategies:
  1. Buy €S of an asset now;
  2. Buy a futures contract to buy the same amount of asset in \( t \) days time for €\( F \).
What is a Fair Price for a Future? (2)

Let

• $S_t =$ value of asset in $t$ days;

• $r$ be the simple *daily* interest rate available at a bank over $t$ days;

• $C_t$ be any money you earn from asset between now and $t$ days (e.g., share dividend).
Value of Strategies

Strategy 1 (buy asset now):
• In t days time, asset worth €\(S_t\) and has earned €\(C_t\). So you have €\(S_t + C_t\)

Strategy 2 (buy future):
• Put your €\(S\) in bank, so after t days it has grown to €\((1+rt)S\). Then pay €\(F\) for future, which is worth €\(S_t\). So you have €\((1+rt)S - F + S_t\).
What is a Fair Price for a Future? (3)

• It seems reasonable that these two values are equal:

\[ S_t + C_t = (1+rt)S - F + S_t. \]

• Solving for \( F \), we have the fair price for a future:

\[ F = (1+rt)S - C_t. \]
Example 1

You have €30k to invest over 100 days. Two strategies:

• Buy 1000 shares in Microsoft today at €30 per share. Expected dividend of €0.05 per share in the next 100 days.

• Agree to purchase 1000 shares in Microsoft in 100 days at €30.20 per share. Bank currently offers 0.01% per day simple interest.
Example 1 (2)

• What is fair price for future?

• Which strategy should you pursue?

• If buy future, at what price of Microsoft stock will you break even with respect to buying $30k of stock today?
The Margin

• In principle do not need any money at all *now* when purchasing future to buy an asset.

• Dangerous for a reliable market (many people buying futures in the hope of making money but without any money to cover a loss).

• Two extra properties of futures overcome this: *daily marking to market* and the *initial margin*. 
Daily Marking to Market

- Each day, the 2 entities involved in the future must post the gain/loss on the underlying asset with the exchange.

- This daily gain/loss is called the daily margin.

- The cumulative gain/loss between future price $F$ and actual asset value at settlement $S_t$ is slowly paid over length of contract.
Initial Margin

• When entering a futures contract, you must also post an amount of money with the exchange.
• This is used to cover the daily margin if you fail to pay it.
• This amount is called the initial margin.
• It is usually up to 10% of the future price $F$.
• So, you must have the initial margin today to give to the exchange when you enter into the future.
Example 2: Daily Margin for Future on 1000 Microsoft Shares

<table>
<thead>
<tr>
<th>Day</th>
<th>Microsoft Price</th>
<th>Daily Price Change per Share</th>
<th>Daily Margin – Gain/Loss on 1000 shares</th>
<th>Cumulative Gain or Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$30.00</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>2</td>
<td>$30.20</td>
<td>+ $0.20</td>
<td>+ $200.00</td>
<td>+ $200.00</td>
</tr>
<tr>
<td>3</td>
<td>$30.10</td>
<td>− $0.10</td>
<td>− $100.00</td>
<td>+ $100.00</td>
</tr>
<tr>
<td>4</td>
<td>$30.05</td>
<td>− $0.05</td>
<td>− $50.00</td>
<td>+ $50.00</td>
</tr>
<tr>
<td>5</td>
<td>$29.60</td>
<td>− $0.45</td>
<td>− $450.00</td>
<td>− $400.00</td>
</tr>
</tbody>
</table>
Leverage

• Go back to Microsoft example (1000 shares to buy at €30.20 in 100 days, current price € 30.00)
• Suppose that $S_t = €30.50$ and for simplicity that $C_t = € 0.00$.
• Strategy 1 (buy shares). You have to have € 30,000.00 now, buy 1000 at € 30.00. This is worth in 100 days € 30,500.00 $\Rightarrow$ profit of € 500.00.
• Percentage return is $100 \times \frac{500}{30,000} = 1.67\%$
Leverage (2)

Strategy 2 (buy future at fair price).

- Recall that fair price is $F = €30,250.00$. Suppose initial margin is 10%, so you have to have $3,025.00$ now.
- In 100 days, pay €30,200 for shares worth $30,500 $\Rightarrow$ profit of $300.00$
- Percentage return on initial margin is
  \[
  100 \times \frac{300}{3025} = 9.92\%
  \]
Leverage (3)

• Extraordinary!

• By buying a future in an asset and not the same asset directly, you make a much larger return!

• This is because you only have to have the initial margin to buy the future and not the whole value of the asset

• The difference in returns is called the leverage

• There is a catch: you can lose a lot more as well!
Example 3

Let’s do Micrososft example but where price on settlement day is €29.50
Example 4

Now repeat the example again but where the initial margin is 5% and not 10%
Relationship between the Margin and Leverage

• The smaller the margin, the larger the leverage

• So if you want to play it safe, have a large %age as initial margin  
  ⇒ %age gains may not be spectacular, but neither are the losses

• If you want to be like Nick Leeson, buy futures and only have a small initial margin!!
Finally.... A model for how Share Prices Change

• This has nothing to do directly with futures, but we need it for the simulation part of this Case Study
Model for Share Price Changes

- First, we will look at a model for log10 of share price and not price directly.
- This allows us to model the occasional large changes in share price more easily.
- There are many models, but a simple one is a random walk.
Random Walk Model

• Each day, the log price changes by an amount that is:
  • Normally distributed with a mean 0 and variance $\sigma^2$.
  • Independent of price changes in previous days
• Mean 0 $\Rightarrow$ price equally likely to increase as decrease
Random Walk Model (2)

• If $S_t$ is share price at day $t$, then we have

$$\log_{10}(S_t) = \log_{10}(S_{t-1}) + \varepsilon_t,$$

where $\varepsilon_t \sim N(0, \sigma^2)$.

• We can change the mean of $\varepsilon_t$ from 0 to reflect shares whose price is generally rising or falling.
Simulating the Random Walk Model

It is easy to simulate the daily share price for $T$ days from this model:

- specify $S_1$, the price on day 1
- for $t = 2,\ldots,T$:
  - generate an $\varepsilon_t$ from a $N(0,\sigma^2)$
  - $\log_{10}(S_t) = \log_{10}(S_{t-1}) + \varepsilon_t$,
end
Simulating the Random Walk Model

• Is this a good model?
• It does not model “big” shocks (e.g. Crashes) as often as they are observed to occur;
• It is a good model for what happens under “stable” trading conditions