PERT Analysis

Project Scheduling
Critical Path Methodology

• Given deterministic activity times, it is possible to:
  – Describe a project as a network
  – Find a critical path through the network.
<table>
<thead>
<tr>
<th>Descript</th>
<th>Idx</th>
<th>Predece</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foundation</td>
<td>A</td>
<td>NA</td>
<td>4</td>
</tr>
<tr>
<td>Frame</td>
<td>B</td>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>Order Windows</td>
<td>C</td>
<td>NA</td>
<td>11</td>
</tr>
<tr>
<td>Walls</td>
<td>D</td>
<td>B</td>
<td>3</td>
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<tr>
<td>Wiring</td>
<td>E</td>
<td>D</td>
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<td>Plumbing</td>
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<td>Ducting</td>
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<tr>
<td>Dry Wall</td>
<td>H</td>
<td>EFG</td>
<td>3</td>
</tr>
<tr>
<td>Install Windows</td>
<td>I</td>
<td>BC</td>
<td>1</td>
</tr>
<tr>
<td>Paint &amp; Clean</td>
<td>J</td>
<td>H</td>
<td>2</td>
</tr>
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</table>
Project (NB Parallel)
To Carry out CPM

• Set up network with dummy nodes so that activities do not finish at same place.
• Do a ‘forward’ pass and a ‘backward’ pass.
• Based on this, identify the ‘critical path.’
• See if you can do this now.
Solution in Excel

• It is possible to set up the activities in Excel, and explicitly code dependencies.
• If you are careful with the choice of constraints, you can ‘solve’ the network using Solver.
• In any case it is possible to readily summarise the features of the project.
‘Realistic’ Scenario

• However, it should be clear that activities do not, in practice, take fixed time known a priori.
• CPM is appropriate for the deterministic situation.
• Another methodology, developed at the same time, is PERT.
PERT

• PERT stands for Programme Evaluation and Review Technique.
• It has its origins in the Polaris missile project.
• A key output is time to completion of project.
• Times can be uncertain.
The PERT distn is used to summarise times for parts of projects.

- It is a distn taking min \( a \), most likely \( b \) and max \( c \).
- It is ‘smoother’ than triangle.
- It is in fact a Beta distribution “stretched” to fit the interval \((a,c)\) with max at \( b \).
- \( \text{PERT}(a,b,c) = \text{Be}(\alpha, \beta)*(c - a) + a \)
Parameterisation

The $\alpha$ and $\beta$ are given by;

- Then $\alpha = (-5a + 4b + c)/(c-a)$
- And $\beta = (-a - 4b + 5c)/(c-a)$
- Mean of a beta is $\alpha/(\alpha + \beta)$
- Mean of a PERT is
  \[ \mu = a + (c-a) \frac{\alpha}{\alpha + \beta} = \frac{(a+4*b+c)/6}{6} \]
  $\mu$ is not the mode unless $\alpha = \beta$ (symmetric)
PERT(0,200,400)
<table>
<thead>
<tr>
<th>Descript</th>
<th>Idx</th>
<th>Pred</th>
<th>Param</th>
<th>Mu</th>
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<tr>
<td>Frame</td>
<td>B</td>
<td>A</td>
<td>3,4,5</td>
<td>4</td>
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<tr>
<td>Order Windows</td>
<td>C</td>
<td>NA</td>
<td>7,10,19</td>
<td>11</td>
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<tr>
<td>Walls</td>
<td>D</td>
<td>B</td>
<td>2,2.5,6</td>
<td>3</td>
</tr>
<tr>
<td>Wiring</td>
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<td>D</td>
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<td>F</td>
<td>D</td>
<td>2,2.5,6</td>
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<td>Ducting</td>
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<td>D</td>
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<td>Dry Wall</td>
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<td>EFG</td>
<td>2.5,3,3.5</td>
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<tr>
<td>Install Win</td>
<td>I</td>
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<td>0.5,1,1.5</td>
<td>1</td>
</tr>
<tr>
<td>Paint &amp; Clean</td>
<td>J</td>
<td>H</td>
<td>1.5,2,2.5</td>
<td>2</td>
</tr>
</tbody>
</table>
In Excel

- Thus, we have a table with all activities.
- Each time is considered an independent random variable.
- The total project time is also a random variable.
- It can be simulated, or approximated analytically.
Questions of Interest

• What is the expected project time?
• What is the probability the time of the project is less than 22 days (say)?
• What are the critical activities?
• Can an activity be ‘probabilistically’ critical?
Simulation Approach

• Treat activities as conditionally independent.
• Simulate a time for each.
• Examine the combination of activity times.
• Examine the effect of each on the total time if this is of interest (by, for example, adding ‘delta’ to each activity in turn.)
Summary

• PERT is another method of examining project scheduling.
• It involves the stochastic specification of activity times.
• The PERT distribution is a Beta.
• Simulation allows the answering of questions that arise in a realistic fashion.