Tutorial Problems

1. A software supplier sells and installs educational software to schools. It has three main products A, B, C. The installation times (minutes) for the three products are considered to be Normally distributed with the parameters shown in the table. The times vary due to variations in the hardware and the combinations of other software packages installed on the machines. Information is not available separately for product B because it has always been installed together with A, up to this time. You may assume installation times are independent of each other.

<table>
<thead>
<tr>
<th>Product</th>
<th>A</th>
<th>A+B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>15</td>
<td>35</td>
<td>12</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>2.5</td>
<td>3.91</td>
<td>1.5</td>
</tr>
</tbody>
</table>

(a) For product A, what percentage of installations will take more than 20 minutes? What value K can be quoted such that it may be claimed that 99% of all installations take less than K minutes?

(b) Calculate the mean and standard deviation for product B (round answers to one decimal place).

(c) If a technician is required to install ten type C products in one school, what is the probability that the installation time will exceed 130 minutes?

(d) Two technicians X and Y are sent to adjacent schools. Technician X will install five type A, 5 type B and two type C products. Technician Y will install five type A and ten type C products. What is the probability that X will finish first?

(e) Define the variance of a random variable. If X and Y are independent random variables with means \( u_1 \) and \( u_2 \) and standard deviations \( \sigma_1 \) and \( \sigma_2 \), respectively, derive the variance of their difference, i.e., \( \text{Var}(X-Y) \). State explicitly any results you assume in the derivation.

(T-2006)
2. A chemist regularly carries out a procedure which involves adding three solvents to a flask. The first, A, is delivered using a pipette in such a way that the volume delivered may be considered to be Normally distributed with mean 100 mL and standard deviation 1 mL. The other two are delivered using a second pipette twice; in each case the volume delivered may be considered to be Normally distributed with mean 50 mL and standard deviation 0.5 mL. The three volumes are statistically independent. The total volume, T, is the sum of the three.

(a) Derive the expectation and variance of T, clearly stating any results which are assumed in your derivation.

(b) What is the probability that the total volume will exceed 200.2 mL?

(c) Twenty five such procedures are carried out every day. What is the probability that the deviation from 200 mL of the average of the 25 total volumes exceeds 0.2 mL, in either the positive or negative direction?

(d) What is the probability that less than three of the twenty five totals will exceed 200.2 mL?

(e) A Normal probability plot might be used to verify the assumption of Normality assumed in this question. Explain the rationale underlying such plots.