1. Assuming that the probability of any one of 5 telephone lines being engaged at any instant is 1/4, calculate the probability that at any one instant (a) all 5 lines are engaged, (b) at least one of the lines is engaged.

2. You are asked to design a sampling scheme for large batches of items which are either acceptable (A) or defective (D). You decide to draw items at random from each batch until one D item is drawn. This takes k draws. You plan to fix an integer N to reject the whole batch if k ≤ N, and otherwise to accept the batch. Find the minimum value of N such that at least 95% of those batches having 10% or more defectives is rejected.

3. You are given a number of transmission switches that are unreliable in the following sense: when triggered by a sudden voltage pulse, they will close and transmit the voltage only 90% of the time. A system is to be built using a minimum number of these switches in parallel which will transmit some signal at least 99.99% of the times that a triggering voltage is put into the input. Find the minimum number required.

4. A communications channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability .2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses "majority" decoding, what is the probability that the message will be incorrectly decoded? What independence assumptions are you making? (By majority decoding we mean that the message is decoded as "0" if there are at least 3 zeroes in the message received and as "1" otherwise.)

5. Suppose that a particular trait (such as eye colour or left handedness) of a person is classified on the basis of one pair of genes and suppose that d represents a dominant gene and r a recessive gene. Thus, a person with dd genes is pure dominance, one with rr is pure recessive, and one with rd is hybrid. The pure dominance and the hybrid are alike in appearance. Children receive 1 gene from each parent. If, with respect to a particular trait, 2 hybrid parents have a total of 4 children, what is the probability that 3 of the 4 children have the outward appearance of the dominant gene?