ST1251 Second Assessment

For visiting students ONLY.

To be handed into the Mathematics Department Office by 12 noon on Friday 16\textsuperscript{th} December, 2016\textsuperscript{1}. Alternatively, answers may be handed into my office, room LB144, or sent by email to arwhite@tcd.ie

Print your name and student number on the front of your script.

Answer all questions

\textsuperscript{1}Assignments that are more than a week overdue will be subject to up to a 50\% penalty.
1. The Packaged Goods (Quality Control) Regulations, 1981 specify that the actual contents of packages shall not be less, on average, than the Nominal Quantity ($Q_n$) and that not more than 2.5% of packages may be Non-Standard (NS), i.e., deviate from $Q_n$ by more than the ‘total negative error’ (TNE.) Thus, if $Q_n = 230g$, and TNE = 9g, a package is NS if it weighs less than 221g. In what follows, assume that fill weights follow a Normal distribution and that containers are filled independently of one another.

(a) Let $Q_n = 230g$ denote the weights of TNE = 9g and suppose the manufacturer sets the average fill to be $\mu = 230g$.

i. What must the standard deviation of the filling operation be so that only 2.5% of containers are NS?

ii. Suppose that the actual standard deviation is 6g, what minimum value should $\mu$ be so that the NS regulation is met? Round your answer to whole units.

(b) Suppose $\mu$ is set at the value determined in (a) ii, and that five containers are selected at random from the production line and weighed.

i. What is the probability that two or more of them are NS?

ii. What is the probability that the combined weight of the five containers is less than 1200g?

(c) An inspector decides to sample containers from the production line until she discovers a NS container. If the process is set up as determined in (a) ii, what is the probability that she will need to weigh exactly six containers?

(d) Outline the logic underlying the construction of Normal QQ plots which are used to assess the assumption of Normality for a random sample. Give examples of behaviour which would not correspond to a normal distribution.
2. (a) If $X$ is an exponentially distributed random variable with parameter $\lambda$, derive $\mathbb{E}[X]$, the expectation of $X$.

(b) The system $S$ is composed of two components, $C_1$ and $C_2$, in parallel (see figure). In other words, $S$ works provided at least one component works. The two components have lifetimes $T_1$ and $T_2$ that are exponentially distributed with the same parameter $\lambda = 0.01$.

i. What is the probability that $T_1$ is greater than 100?

ii. What is the probability that $T_S$, the lifetime of the system $S$, is greater than 100?

iii. What is the probability density function (PDF) for $T_S$?

iv. Derive $\mathbb{E}[T_S]$, the expected lifetime of the system.

(c) A customer buys 10 such systems from the manufacturer. Write down an expression for the probability that 2 or more of the 10 will fail by the time $t = 100$. 

3
3. (a) A particular type of chocolate sweet is manufactured such that the average mass is 18.9 g and the standard deviation is 1.2 g. What fraction of chocolates will have a mass in excess of 20g? What value can be quoted such that 90% of all chocolates have a greater mass than this value? You may assume normality.

(b) A mixed box contains two types of chocolates with the following mass characteristics:

<table>
<thead>
<tr>
<th>Product</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>18.9</td>
<td>22.7</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

i. What is the probability that the mass of the contents of a mixed box, which contains 6 of type A and 5 of type B, will exceed 235 g? Note that the individual chocolates within a box are randomly selected.

ii. What is the probability that the mass of the contents of a mixed box will exceed that of a box containing 10 type B chocolates? Again, assume random selection.

(c) The number of surface blemished on any one chocolate is described by a Poisson distribution with parameter $\lambda = 0.05$. What is the probability that a box of ten chocolates will contain two or more chocolates each with one or more surface blemishes?

4. (a) A particle detector is set up to detect cosmic rays of type A. These particles are detected as a Poisson process with parameter $\lambda = 0.5$ per day.

i. What is the probability that three or more will be detected in any one day?

ii. What is the distribution of inter-detection times for these particles?

iii. What is the probability that the inter-detection time for two consecutive particles will be less than three days?

iv. The particle detector also detects type B particles. These occur with a Poisson rate of $\lambda = 0.5$ on days when no type A particles are detected, and with a Poisson rate of $\lambda = 1$ per day on days when one or more type A particles are detected. If on any one day one or more type B particles are detected, what is the probability that one or more type A particles are detected?

(b) If $X$ is a Poisson random variable with parameter $\mu$, derive the expectation and variance of $X$. 

4