ST1251 Assessment 1
Michaelmas Term 2016

To be handed into the Mathematics Department Office by 12 noon on Monday 14\textsuperscript{th} November, 2016.

Print your name and student number on the front of your script.

\textbf{Answer all questions}
1. (a) Bovine tuberculosis has been a serious problem in Ireland for a number of years. Cattle are tested regularly to identify infected herds. The test used to check for the disease has the following characteristics: \( P(\text{negative test} | \text{healthy animal}) = 0.999 \) and \( P(\text{positive test} | \text{diseased animal}) = 0.90 \).

i. Assume three animals are tested, two of the three having the disease while the third is free of the disease. What is the probability that there will be two positive test results and one negative test result? All tests are independent of each other.

ii. If the disease rate is 0.4% in a particular part of the country and a randomly selected animal is tested, calculate the probability of a false positive result, i.e., the probability that the disease is absent even though the animal obtains a positive result.

iii. An administrative error contains 20 herds of which 3 include diseased animals. If 5 herds are randomly sampled for testing by the animal health officials of a country to which animals are exported, what is the probability that none of the infected herds will be included in the sample?

(b) The number of times that an individual contracts a cold in a given year is a Poisson random variable with parameter \( \lambda = 5 \). Suppose a new wonder drug (based on large quantities of vitamin C) has been marketed that reduces the Poisson parameter to \( \lambda = 3 \) for 75% of the population, while having no appreciable effect for the remaining 25%. If an individual tries the drug for a year and has 2 colds in that time, how likely is it that the drug is beneficial to him or her?

2. (a) A computer system is equipped with three hard disks, each independently having a 0.97 probability of functioning correctly. At least 2 of these are required for a working system. What is the probability that the system isn’t working?

(b) A group of individuals contain ten people with blood group O, five with group A, and five with group B. What is the probability that a random sample of size six will contain two people with each blood group?

(c) A task involving a large amount of computing time may be delayed due to a computer failure. If the probabilities are 0.6 that the computer will fail, 0.85 that the task will be completed on time if the computer doesn’t fail, and 0.35 if it does, calculate the probability that the task will be completed on time. If the task is completed on time, what is the probability that the computer failed?

(d) Suppose that the following game is played: you roll a fair six-sided die, numbered 1 to 6, and after seeing the outcome, you can choose to either keep the result, or roll again. If you roll again, then you must keep the score from your second roll. A player decides that the optimal strategy for this game is to keep the result of the first roll if it scores a 4 or higher, otherwise she rolls again. Using this strategy, what is the probability that the player scores a 5? What is the expected score of the strategy?
3. (a) In a population of women a proportion \( p \) have abnormal cells on the cervix. The Pap. test entails taking a sample of cells from the surface of the cervix and examining the sample to detect any abnormality. We will assume here that sampling is without error, i.e., that if abnormal cells are present some will be included in the sample. The following are the characteristics of the test:

- In a sample including abnormal cells, examination fails to observe them with probability \( \nu \).
- A sample free of abnormal cells is determined with probability \( \pi \) to contain abnormal cells.

If a randomly selected woman takes the test:

i. What is the probability that the result is wrong?

ii. If abnormality is reported, what is the probability that, in fact, no abnormal cells are present?

(b) A contingent of soldiers is to be screened for sexually transmitted infections (STIs) after service overseas. To reduce testing it has been decided first to group soldiers in groups of 10. The blood samples of the 10 soldiers in each group will pooled and analyzed together. The pooled test will be positive if at least one soldier in the pool has an STI. If the test is negative, one test will suffice for all 10 soldiers; otherwise, all 10 soldiers will also have to be individually tested, meaning that 11 tests in all must be conducted. Assuming that the probability that a soldier has an STI is 0.1, independently of each other, compute the expected number of tests necessary for each group.

(c) If the number of children with a particular birth disorder who are born per week in a large maternity hospital is Poisson distributed with parameter 2, what is the probability that in any given week there will be more than two such births?
4. (a) Three boxes, A, B and C each contain 10 balls, specifically 3, 6, and 8 red balls respectively, and the rest being black. A game involves two stages: i) draw a ball from A, and ii) if the first ball is red, draw a ball from B, if the first ball is black draw a ball from C.

If the second ball is red, what is the probability that the first ball is red also?

(b) There are 3 coins in a box. Coin 1 is fair, (so $P$(heads) = $P$(tails) = 0.5), Coin 2 is biased so that $P$(heads) = 0.75, and Coin 3 is two-headed.

i. A coin is picked randomly from the box and flipped. What is the probability of getting a head?

ii. The above experiment is repeated; a coin is picked from the box at random and flipped. It lands heads. What is the probability that the two-headed coin was the one that was picked from the box?

(c) The number of customers that enter a bank in a minute is random and can be described by a Poisson distribution with a mean of 1.5.

i. Write down the formula for the probability distribution for the number of customers arriving at the bank.

ii. What is the probability that no customers arrive at the bank in any given minute?

iii. What is the probability that no customers enter the bank in two consecutive minutes? You may assume that the number of customers entering the bank in different minutes are independent.