Efficient Integral Equation-Based Propagation Modelling for Radio Coverage Prediction

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A Dissertation

Presented to the University of Dublin, Trinity College in partial fulfilment of the requirements for the degree of

Master of Science in Computer Science (Data Science)

Supervisor: Eamonn O Nuallain

September 2020

Declaration

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Mrinal Jhamb , Master of Science in Computer Science, University of Dublin, Trinity College, 2020

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This study exhaustively explores integral equation based propagation modeling techniques for electromagnetic waves and tries to improve upon the existing models by providing a method that includes the backscattering electric field as well. All the methods discussed have the computational complexity of the order of the square of the number of integral intervals (groups in FExM) as opposed to the basic algorithm (Exact method) which has a computational complexity of the order of the cube of integral intervals. It is explained that how these modeling methods can be used in conjunction with a Radio Environment Map (REM) aided Cognitive Radio Network (CRN) to implement the 5G efficiently by enabling the Dynamic Spectrum Access (DSA), to get past the limitations of traditional static spectrum allocation. The predictions are made using all the models on the most commonly used and gently undulated 'Hadsund' terrain profile and on more rugged 'German' terrain profile. All the models work equally well on the 'German' terrain profile as they did on the 'Hadsund' terrain profile. The predictions made using these models are in sync with that of the state of the art Okumura-Hata model. This study als0 addresses the problem of localization of the non-cooperative transmitter using signal strength measurements based on the Reciprocity Theorem and produces the results accurate to around 147 meters, upon a random selection of the locations of three pair of transceivers. This localization method is a huge improvement over existing signal strength measurements based methods in terms of computational complexity, with highly accurate results. This study also provides a detailed roadmap for the future scope of improvement on the limitations of the study such that sufficient accuracy and execution times can be achieved.

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Chapter 1

Introduction

1.1 Motivation

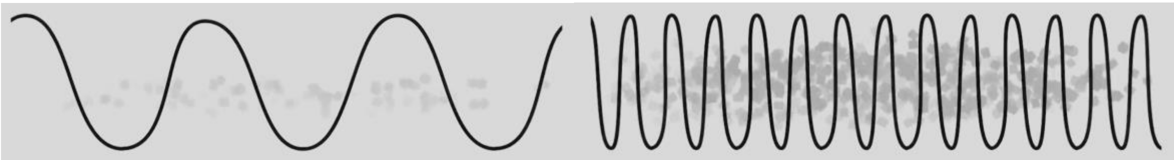
Wireless communication has come to ever good ever since its introduction in the early 1970s [1]. It has experienced exponential growth in traffic in the past decade. This is because of the growing popularity and accessibility of the internet. It has changed the way we used to live our life. The traffic is supposed to increase even further with the growing popularity of IoT and other next-gen technologies. The load on the spectrum has increased even further with the growing work from home culture, which is the need of the hour due to the pandemic.

It has become sort of a trend for the industry to advance with the introduction of new generation technologies every 10 years. First-generation wireless telecommunication services were introduced in 1982, the second in the decade following in 1992, and then third and fourth generations in 2001 and 2010 respectively. With the next decade approaching, we expect the rollout of fifth-generation wireless communication standards in the early 2020s [2]. The transfer to the next generation standards has always brought the unique and difficult challenges to tackle for the time. It is no different this time.

Humans have always tried to use the advancement of technology to their good in a manner that can increase efficiency and the quality of life. The emergence of the Internet of Things has been significant over the years. It has always been deemed to change the way we live our lives, once implemented to its full potential. The growth in the field of IoT has been immense in the recent past, which is being fuelled by the development of cheap and easily accessible small sensors. The implementation will increase people to thing and among things communication. Also, the exchange of data between computationally capable IoT process stations and sensors would be immense. Another aspect of the implementation of IoT is the wide range coverage, low latency, high speed, and high capacity. All of which are uncompromisable. Take for example self-driving cars, the high latency and sudden drop of the network is the last thing you want in such a car. These things would be a deal-breaker in such cases.

With the new breakthroughs in the field, every other day, the 4G and other old technologies are not good enough to serve the purpose because of their limitations. 5G is deemed to be the enabler of the technology. It is because of the fact that older standards 2G, 3G, and 4G were meant to handle people to people communication, and to reach its full potential IoT requires people to things and among things communication to grow as well.

The wireless telecommunication systems use radio networks to operate. A radio network is essentially a spectrum of different frequencies we can connect to, with wireless or mobile devices. There are different radio frequencies for different types of communication. Low frequencies are good for long-distance, bending and reflecting around objects and corners and penetrating buildings. High frequencies are preferred for short distance and higher capacity. But there is a finite supply of radio frequencies [3].



(a) (b)

Fig 1.1: Pictorial depiction of electromagnetic wave with (a) low frequency and (b) high frequency

5G because of its basic requirements of wide coverage and higher data speeds require the space on low mid-band frequency range for perfect operation. But unfortunately, most of the countries don't have access to this sweet spot. This range has already been used for the implementation of 4G and WIFI live [3].

The sample runs of 5G in Chicago, USA showed very high speeds, even higher than 1 gigabit per second. Even though the highest speed achieved is fascinating but what would be a more realistic measure of the performance of 5G is the median speed achieved over the range of area. The median speeds in South Korea were better than the median speeds in the United States of America. This implies that their networks are faster in more scenarios [4].

This is because of the fact that the USA is one of those countries which has already used the low mid-band frequency range to its full extent and what they are left with is the high-frequency band, which can achieve high speeds but cannot offer wide coverage. Making 5G work properly in the higher frequency region would require a great investment in the infrastructure, for building nodes to make a blanket of coverage [4].

The fact that countries like China, South Korea, etc. are leading the race of development of 5G infrastructure is because of the fact that they still have access to this sweet spot range of 1-6 GHz on the frequency band. The trend from previous generations show that there is a great first-mover advantage when it comes to wireless telecommunication standards. As the country to enter first ends upsetting all the governing standards for the technology, this is a great amount of economic power at grasp.

Even those countries which currently have unused frequency bands in the region of 1-6 GHz, won't be able to sustain the demands of the 5G with its growth. This is because of the traditional static spectrum allotment. This traditional spectrum allocation and utilization system has every possibility of becoming the bottleneck in the 5G development and revolution is expected. Hence, it is high time to deviate from the earlier model of static spectrum allotment and shift to the dynamic spectrum allotment, more commonly known as DSA (Dynamic Spectrum Access).

Dynamic Spectrum Access has come into the picture because of the inefficient utilization of the allocated bands. As per the report of the very famous measurements made on the spectrum utilization in New York in 2004, 80% underutilization of the 30-29000 MHz spectrum was reported. This is when CRNs (Cognitive Radio Networks) come into the picture. CRNs are comprised of Cognitive Radio (CR) nodes. These nodes can sense the vacant spaces in the network. This process of sensing the vacant spaces in the network is called spectrum sensing. This network information when sensed is communicated to the Radio Environment Mapping (REM) server. This REM server decides whether to propagate the signal of the second user based upon the plethora of radio environment knowledge available at its disposal, such as geographical features, regulation, policy, radio equipment capability profile, and radio frequency emissions. This process of increasing spectrum efficiency via the real-time adjustment of radio resources, i.e. via a process of local spectrum sensing, probing, and the autonomous establishment of local wireless connections among cognitive nodes and networks is called Dynamic Spectrum Access.

The main thing here is the real-time availability of the radio emission from the primary users and the cognitive node to the REM server. This is when the efficiency of propagation modeling comes into account. Another main problem with the dynamic allocation of the secondary users after spectrum sensing, is the hidden node problem. It is when the Spectrum Analyser component of CR does not detect any primary user but the transmission from the secondary user introduces noise in the primary user's signal. This occurs because Spectrum Analyser does not detect the transmission from the primary user rather it detects only the primary user. CRNs tries to handle this with the deployment of REMs, whose working depends on efficient and accurate propagation modeling.

Apart from the low mid-band frequency range utilization, the implementation of DSA through CRNs will help utilize the other frequency bands more efficiently too. Specifically, ISM (Industrial Scientific Medical) bands that are being used by technologies like ZigBee, NFC, Bluetooth, etc. will also become overcrowded as the number of sensors will increase [5].

Proper utilization of the underutilized bands can do wonders and can help attain 5G its full potential. This will lay the basic foundation for the long-foreseen power of numerous technologies, to application. Developing upon which the human race can actually see the world unfold as it has always dreamt, like those sci-fi movies.

These factors when put together lead to this research, which focuses on the very fundamentals of what Cognitive Radio Networks and eventually Fifth Generation wireless telecommunication systems will depend upon - propagation modeling. It is the efficiency and accuracy of the propagation modeling technique which is of concern here.

This research looks at Integral Equation based models and how they can be used to address the hidden node problem. These models have been found to give numerically exact solutions and give an accurate measure of large-scale fading signals over the terrain profile in the past. It also focuses on the localization technique to locate the non-cooperative transmitter.

This research provides the reimplementation of the evolution of the research in the field. It also compares Integral Equation based models with the Path-Loss models with a focus on the accuracy of the models under consideration. The work completely focuses on the propagation modeling and tries to contribute to it with the help of integral equation-based methods.

1.2 Research Question

How well do the deterministic integral equation-based propagation models compare to the empirical models and how can they be made computationally efficient in order to put them in practice for solving hidden node problems and localization of non-cooperative transmitters?

1.3 Research Objectives

The following objectives are considered to answer the research question:

1. Understand the fundamental components and concepts of Cognitive Radio Networks, which include:
    a. Cognitive Radios
    b. Radio Environment Map
    c. Dynamic Spectrum Access
    d. Propagation modeling
2. Study and implement the Electric Field Integral Equation based method with the inverse of the Z matrix to evaluate the slow fading signal effect over semi-rural terrain profiles.
3. Implement the Electric Field Integral Equation (EFIE) based method with Forward Scattering assumption.
4. Implement the faster electric field integral equation-based method with Forward Scattering assumption, known as Field Extrapolation Method (FExM).
5. Implement the electric field integral equation-based method with intuitive iterative Forward and Backward Scattering Sweeps using:
    a. Forward Scattering EFIE.
    b. Forward Scattering FExM.
6. Compare the accuracy of results from all the electric field integral equation-based methods by plotting superimposed graphs.

7. Study and implement the Okumura-Hata's empirical path loss model.

8. Check the agreement of the predictions from the electric field integral equation-based methods with the path loss predictions from the Okumura-Hata model.

9. Implement Localization method based on the Reciprocity Theorem using signal strength measurements to predict the location of a non-cooperative transmitter using.

1.4 Research Challenges

- Limited literature available on the topic and the newness of the domain was difficult to get a good grasp of, in the early days.

- In the electric field integral equation-based method the numerical optimization methods are dealt with which are quite tedious and even the slightest of error can make results go haywire.

- Limitation of computational resources at disposal, made it really difficult to implement some of the more time-consuming tasks and was a challenge to get past.

- COVID-19 situation had a huge impact and limited the resources to a great extent:
    - Lack of access to library resources impacted considerably.
    - Lack of access to the terrain data, measurements, and proper working algorithm impacted in the following way:
        - Still, some dilemma around the exact method to calculate k for FExM.
        - Error in the results of the Intutive Iterative Forward-Backward Sweeps method couldn't be rectified.
    - It was hard to get the code cross-checked when required.
    - Even though the weekly meetings were scheduled with the professor, but efficiency reduced quite significantly.

1.5 Dissertation Structure

The rest of this document is organized as follows: Chapter 2 lays down the basics of the domain and puts light on the existing literature explaining current state of the arts. This dissertation then progresses into Chapter 3 which explains the methodology adopted in the dissertation and explains the algorithms in detail. In chapter 4 the results obtained after actualizing the methods in previous section are shown. Chapter 5 provides the Summary of New Contributions made by the work, Applications of devised methods in 5G, Limitations of the work and Future Scope of the study.

Chapter 2

Background and Related Work

2.1 Wireless Channels and Noise

Wireless channels are the main component in the implementation of wireless communication systems. Unlike the channels in wired communication, understanding the behaviour of channels in wireless communication is very tedious. Different environmental topographies and the mobility of users make these dynamic channels very complex to understand.

The figure below shows the architecture of the generic communication system as described by Claude Shannon in ‘A Mathematical Theory of Communication’ [6]:

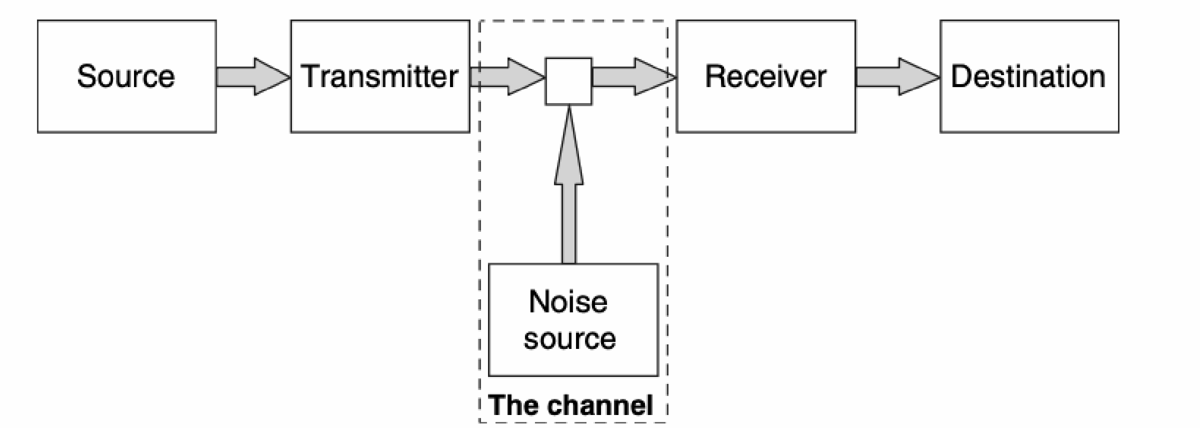


Fig 2.1: Architecture of a generic communication system

The transmitter converts the data generated from the source and then sends it via the channel to the receiver. The noise persistent in the channel modifies the signal in an unpredictable fashion. This persistent noise can be categorized into, additive and multiplicative. Additive noise is generated due to the receiver itself and also because of external effects like cosmic radiations, interference, etc. Multiplicative noise is caused due to fading in the channel and also due to some processes in transmitter and receiver antenna [7]. Here we will focus on the fading caused in the channel.

There are numerous factors that contribute to the multiplicative noise in the channel which causes fading. These factors include reflection from smooth surfaces, transmission through objects, diffraction around sharp edges, and scattering on rough surfaces due to the topographic features of the environment. This implies that we don't need the line of sight between transmitter and receiver.

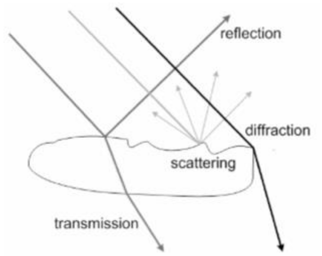


Fig 2.2: Phenomenons acting on the typical propagating electromagnetic wave

2.1.1 Fading

Let's take a situation where a receiver mobile is in the car moving with a constant velocity $v$with respect to the transmitting antenna which is separated from the car by the distance$d$.

Fig 2.3: Radio Channel

The thing which we are most concerned about while analysing the wireless network is the power of the signal. Let's say that the transmitted power from the antenna is P_t and the received power at the receiver is P_r. The plot shows the received signal as a function of distance [8].

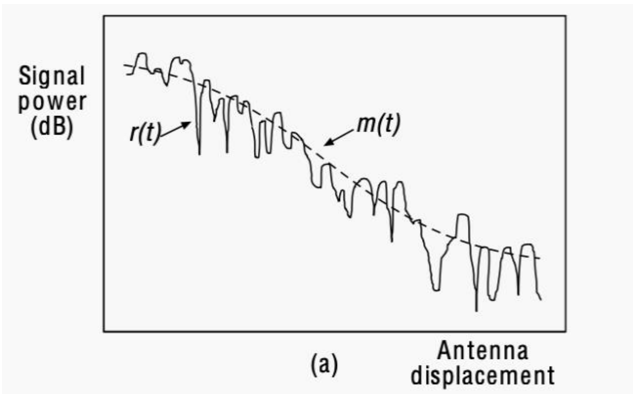


Fig 2.4: Small and Large scale fading

Since we have assumed above that car is moving with a constant velocity, i.e. we can say that the distance is directly proportional to the time elapsed. This gives the liberty to right the received signal as a function of time. In the figure above $m(t)$ is the localized mean signal which decays with distance and has this relatively slow change as compared to $r(t)$. Similarly, $r(t)$ denotes the power received instantaneously, and we can say that deviation. This takes us to the very important classification of the fading effect which divides it into two main categories, i.e. large scale fading effect which contains the signal power decay of the form $m(t)$ and small scale fading effect which deals the variability of instantaneous signal power about the mean signal power.

2.1.1.1 Large-Scale Effects

The localized mean power of the signal is found by averaging the measured power taken on many points within the range of few wavelengths. This localized mean signal power decreases as the distance between the transmitter and receiver increases. This gradual reduction in localized mean power also happens because of the obstructions in the path of transmitter and receiver, along with the increase in distance between them. Based on this criteria large scale fading can be divided into two.

Path loss

It is the reduction in signal power which is attributed to the distance between the receiver and the transmitter and it increases with an increase in distance. It is proportional to d^{γ}. This γ is called path loss exponent and is 2 for free space. It is empirically identified for different environments. Path loss causes variation in the received power over long distances, i.e. in the range of $100 - 1000\, meters$. The variation caused is around 150 dB over the area of propagation of the signal [6][9]. This is discussed in more detail in the sections to come. Generally speaking, the path loss is the ratio of the power of the transmitted signal to that of the power of the received signal.

$$L = P_t / P_r \tag{2.1}$$

Shadowing

Normally there are numerous things in the propagation path which causes decay in signal strength. This can be due to absorption caused by each individual leaves of trees, diffraction by overbridges, etc. These things are difficult to account for individually. So, these are clubbed into one called shadowing. Shadowing causes variation in the received power over the distance which is proportional to the size of the obstructing object, i.e. in the range of $10 - 100\ meters$ in outdoor environments. The variation caused is up to approximately $20\ dB$ over the area of propagation of the signal [6][9]. The small change in position does not change the signal strength dramatically. Hence sometimes it is also called slow fading.

2.1.1.2 Small-Scale Effects

The signal received at the receiver is the convolution of the transmitted signal and the impulse of the channel which varies with time and distance. Since we have assumed that the mobile is moving with a constant velocity, we can say that the impulse of the channel evolves with time.

This impulse of the channel is characterized by reflection from the smooth walls and other objects which are larger than a wavelength and scattering caused due to the objects close to the receiver with either dimension of the order of wavelength or with rough surfaces [8].

The phenomenon of scattering and reflection makes signal transfer over different paths and then meets at the receiver, where the net power of the signal received is the superposition of these signals. The interference between the signals can either be constructive or destructive depending upon the phase of the interfering signals. The multipath travel of signals is so sensitive that traveling over a few wavelengths can change the phase difference between the interfering signal and can turn constructive interference to destructive and vice versa. This causes the rapid fluctuation of power in the received signal.

Considering a mobile in motion it will pass rapidly from the area of constructive and destructive interference, receiving a rapid time-varying signal with respect to the time. This phenomenon of rapidly changing instantaneous power of the signal is referred to as fast fading or multipath fading.

Fast fading causes variations of the order of $35 - 40$ dB. These variations are caused on the scale of half-wavelength e.g. 50 cm at 300 MHz or 17 cm at 900 MHz [7]. This rapid variation in the power of the received signal impacts hugely on the minimum signal power attributed to the receiver for clear signal reception.

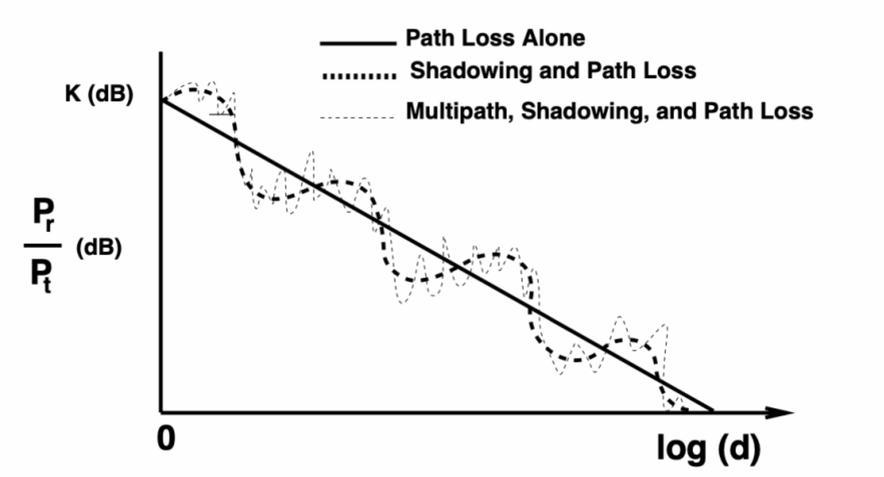


Fig 2.5: Multipath, Shadowing and Path Loss plotted against Distance

2.1.2 Free Space Path Loss

As the name implies it is the propagation of the signal in the space which does not have any sort of obstruction, i.e. there is no reflection, absorption, etc. due to objects. Putting it in easy words it can be said that there is no obstacle in the line of sight which can cause obstruction.

Two assumptions:

- The transmitting and receiver antennas are assumed to be isotropic, which implies that it radiates equal power in all directions, i.e. the antenna gain G_t and G_r both are 1.
- The fact that we are considering free space implies that there is conservation of energy, as there is not a sort of absorption. This leads us to the fact that power spread across the spherical spreading wavefront is the same, but the power density reduces as we go away from the source [8].

So now the area of the sphere at a distance d from the transmitter and enclosing it is:

$$4\Pi d^2 \tag{2.2}$$

And hence the power density is:

$$P_t/4\Pi d^2 \tag{2.3}$$

Power received at the antenna with effective area A_e and distance d away from the source is:

$$A_e P_t / 4\Pi d^2 \quad (2.4)$$

The antenna gain G_r (of the receiver antenna) and the effective area A_e are related as:

$$G_r = 4\Pi A_e / \lambda^2 \quad (2.5)$$

Since the receiver antenna is also isotropic. So,

$$A_e = \lambda^2 / 4\Pi \quad (2.6)$$

Hence the received power is:

$$P_r = P_t \lambda^2 / (4\Pi d)^2 \quad (2.7)$$

This is represented with the figure shown below in which the transmitter is at the center of the sphere and the shaded portion on the boundary represents the receiver antenna [9].

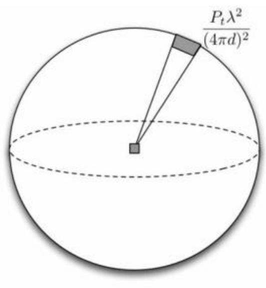


Fig 2.6: Depiction of the transmitter, receiver and the effective power at the receiver

On the contrary, if they were to be the real antennas rather than isotropic than G_r and G_t would have been nonzero. There is also a factor called system loss factor which is specified by L and is 1 unless specified. So,

$$P_r = G_t G_r P_t \lambda^2 / (4\Pi d)^2 L \quad (2.8)$$

This is known as Frii's free space equation.

2.1.3 Decibels

The bel is a relative unit of measuring power. 1 bel indicates an increase in the power by a factor of 10 relative to the reference power. It is expressed as:

$$Power\ ratio\ in\ bels = log(P/P_{ref}) \quad (2.9)$$

That is why it is also known as the logarithmic unit of the ratio of powers. Decibels are one-tenth of the bel and is used more commonly than bel. It is expressed as [7]:

$$Power\ ratio\ in\ decibels = 10 * log(P/P_{ref}) \quad (2.10)$$

So Frii's equation for path loss in free space can be modified to express path loss in decibels with isotropic antennas and unit channel loss as:

$$P_r(dBm) = P_t(dBm) - 21.98 + 20log_{10}(\lambda) - 20log_{10}(d) \quad (2.11)$$

$$L(dB) = P_t(dBm) - P_r(dBm) = 21.98 - 20log_{10}(\lambda) + 20log_{10}(d) \quad (2.12)$$

2.1.4 Simplified Path Loss

There are numerous path loss models which can be used to compute the path loss of a propagating signal, but even the most accurate of models, i.e. ray tracing (out of the scope of the paper) and empirical models (discussed later in more detail) are nothing but just the better approximation for the path loss. In most of the cases where the objective is trade-off analysis, it is a better choice to use the simple path loss models which captures the general nature of the path loss over distance. This simplified model looks like this:

$$P_r = P_t K(d/d_0)^{\gamma} \quad (2.13)$$

It can be written in decibels as:

$$P_r(dBm) = P_t(dBm) + K(dB) - 10\gamma log_{10}(d/d_0) \quad (2.14)$$

$$L(dB) = P_t(dBm) - P_r(dBm) \quad (2.15)$$

K in this equation is a far-field reference distance and is characterized by the characteristics of the antenna and the average attenuation of the channel. This model is valid only for the field that is at the distances greater than d_0. It is because of the presence of scattering in the near field. This d_0 is $10 - 100\ meters$ for outdoors and $1 - 10\ meters$ for indoors. K can be measured using the MSME fit [11] or K is expressed as the free space path loss at distance d_0 when its magnitude is less than 1. It is expressed as:

$$K(dB) = 20log_{10}(4\Pi d_0/\lambda) \quad (2.16)$$

Gamma is the path loss exponent whose value depends upon the nature of the environment. For free space its value is 2. The value of gamma can be found by fitting the MMSE to the real-world measurements or it can be found using the empirical models. These models consider the height and frequency of the source. Few of these models are discussed later in the report. The path loss exponent increases with an increase in frequency and it tends to decrease with an increase in the antenna height [11]. The equation below shows the MMSE equation [12]:

$$F(\gamma) = \sum_{i} \quad [L_{measured}(d_i) - L_{model}(d_i)]^2 \quad (2.17)$$

where,

$L_{model}(d_i) \rightarrow are\ the\ approximations\ to\ path\ loss\ made\ using\ the\ simplified\ model$

$L_{meausred}(d_i) \rightarrow are\ the\ real - world\ path\ loss\ measurements$

Setting the first derivative of the equation equal to zero yields the value of gamma.

2.2 Cognitive Radio Networks

A cognitive radio network is a wireless communication network in which cognitive radio devices that can sense which channels are empty to establish communication are used. Based on this information it can alter its operational parameters to select the vacant channels. This sensing and decision making on its own or based on the interaction with other nodes helps to maximize the utility of radio frequency spectrum, while also keeping the interference in check [13][14].

These cognitive radios or nodes with cognitive abilities are the very basis of cognitive radio networks. This when clubbed with intelligent network access nodes helps in the deployment of various cognitive technologies such as smart grids, smart homes, smart offices, smart transportation systems, etc.

Traditionally the architecture of cognitive radio networks sought cognitive radio nodes to have all the cognitive abilities and expected them to make decisions on their own. This process of making decisions included the process of analyzing the spectrum for other primary or secondary users, learning from this analysis, and then after deciding when to transmit the information over the network. It is not sensible and rather not feasible to enable all these functionalities in mobile devices due to their hardware limitations and difficulty of governance. This is when the Radio Environment Map (REM) assisted CRNs come into play.

The REM is an extraction of the real-world scenarios which is being shared throughout the region of operation of the wireless network. It includes the information about the radio frequency emission, geographical features of the radio frequency signal environment, policies, historical performance, limitations of the network, and residing nodes. The REM provides network support for CRs. The network support once available will have multiple implications on the performance of the cognitive radios. These are as follows:

- Load on the individual mobile devices will decrease as several computationally exhaustive tasks will be offloaded to the network.
- The use of distributed computing will accelerate the processing and make things available to each node in the network more rapidly, thereby improving the performance of the entire network.
- Network support to the cognitive radios will help in its commercialization by lifting off the computational limitations of the mobile devices which are imposed due to their small form factor, limited battery capacity, performance, and memory.
- These limitations of the mobile devices act as a bottleneck when detecting the incumbent’s signal, say it be a primary user or another secondary user. The most important issue associated with such signal sensing is a hidden node problem. The radio takes the aid of a radio environment mapping server to resolve such issues efficiently [15].
- The long-foreseen future to merge all different sorts of networks to the CRNs can only be achieved by providing the support of network to the cognitive radio at the moment. The REM is one such enabler.

- Cognitive radio with network support allows dynamic spectrum access policies to be made and maintained by governments, service providers, and the regulators very efficiently.

The REM is to CR what city map is to a traveller. It provides the CR with the information such as spectrum opportunities, policies regulating spectral use, presence of hidden nodes, signal to noise ratio and expected path loss, primary and secondary users' usage pattern, etc. In fact, there are lots of parallels that can be drawn between both, and the existing concepts can be used for the added advantage in development. These include concepts such as signalling, capacity, throughput, routing, etc.

The REM provides the prior knowledge to the CRs which help them give a heads up and avoid the overhead of random scanning. Take for example spectrum sensing, where if CRs query the REM's archived historic and current knowledge, they can scan more efficiently rather than starting the scan blindly. This helps us design a more cost and time-efficient CRN by reducing the minimum requirements of dynamic range, transmission power, battery, and sensitivity of each of the CR devices [16].

The concept of modern REM server uses technologies such as database management, data mining, artificial intelligence, cross-layer adaptation and optimization, positioning and estimation, detection and classification, site-specific propagation prediction, etc. The technologies like data mining and artificial intelligence, when implemented properly, can be used as leverage to use the traditional radio nodes, which do not have the cognitive capability, to be included in the CRNs. This is made possible by offloading the cognitive decisions to a REM server. Along with this REM also gives a system-level solution to the typical CRN problems like hidden node problem, load balancing, dynamic spectrum regulations, policy management, etc.

2.3 Dynamic Spectrum Access

To understand dynamic spectrum access, it is must to have an idea of what is radio spectrum. If we were to write two messages on top of each other such that they're sharing the exact same area, but they're differentiated in colour, so that we can recover both messages. In a similar way, the radio spectrum is shared by everyone, but people use different frequencies. So, if you share the same frequency as if those two words have been in the same colour it would overlap, and it would destroy the information.

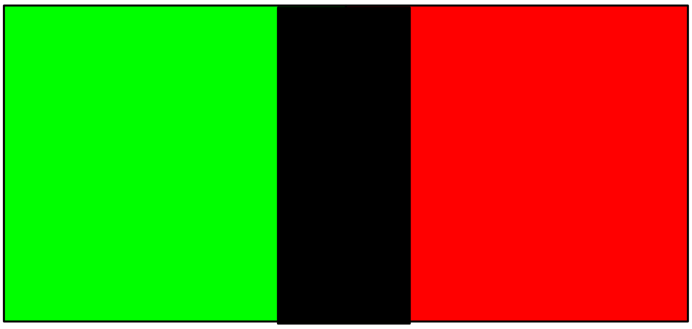


Figure 2.7: Green: Message 1, Red: Message 2 and Black: Distorted message due to overlap

Formally speaking the process of increasing spectrum efficiency via the real-time adjustment of radio resources, i.e. via a process of local spectrum sensing, probing, and the autonomous establishment of local wireless connections among cognitive nodes and networks is called Dynamic Spectrum Access.

So, when we have a radio, we tune between frequencies to filter out the information we don't want to receive. Who can use what frequency is set up by the government and it is then allocated. So, for example, AM radio can only lie in a certain band. There's no other use of that radio spectrum except for AM broadcasts. AM radios are normally allocated very high-frequency bands. What's of more interest to us is a comparatively lower frequency band where a lot of high data rate stuff occurs. These frequency bands are very crowded and that's because there's a lot of potential users for it. In the city, you might see more people with cell phones and more TV broadcast stations but out in the country even though it's the same allocation there won't be as many users. So, this opens up the idea of dynamic spectrum access. Where we sense the radio is not being used and we use that spectrum to get even more available to us. We have to be careful to not only avoid those people that are destined to use the spectrum, but other users doing dynamic access as well.

For example, taking an analogy from shipping there's a phenomenon known as radar assisted collisions that occurred when radar first came out. This ship the Andrea Doria was subject to it. So, if there are ships without radar and a ship with radar. The ship with radar can dart in between them safely because the other ships can't see it in dense fog but two ships with radar might steer into each other because they're both trying to avoid a collision [17].

There are a lot of challenges in dynamic spectrum access but if we can successfully master the technical and governmental and regulatory challenges there's a real opportunity here to open up a huge range of products, for the future use of this radio. The main challeng out of all is propagation modeling which we will look in detail in the sections to come.

2.4 Propagation Modeling

Propagation modeling is the computer-based simulation of the radio waves. Radio waves are nothing, but the electromagnetic waves emitted from a radio transmitter. In the case when there is nothing in the space in which the radiation has been emitted, it is said to be in ‘free space’. This type of perfect line-of-sight propagation results in the path loss which is proportional to the square of the distance, as explained in earlier sections. The modeling in this sort of ideal condition would have been the easiest to do. But the real-world scenarios are not so ideal, since most of the communication that occurs is land-based, i.e. close to the earth. This implies that the communication will be impacted by the environment in between because of the phenomena like refraction, absorption, reflection, and refraction. Even the communication paths which are unobstructed are not ‘free spaces’ because they are impacted by the reflection from the ground and the atmospheric conditions.

This propagation of waves through the atmosphere is analogous to the ripples in water. There are factors from the climatic condition and the conductive properties of the earth to the man-made objects and vegetation which impact the propagation of the waves. One thing is their presence and then the other factor is their time-varying nature. These factors include variation in weather conditions over the course of the year which include the change in temperature, wind speed, humidity, etc. The nature of the reflectors affects the propagation depending on whether they are static or moving, for instance, the varying density of vehicles on the road. The seasonal crops and their movement with a change in the direction of the wind [18].

These factors are really complex to take into account while modeling the propagation of the radio waves. The work on such models has continued to evolve over several decades, but it is not until recently when a lot more models started to come into existence. It is due to the increase in the computing power at hand. There are basically two types of models, empirical and deterministic. Empirical models are based on extensive real-world observation and statistical analysis. Okumura-Hata is a widely used empirical model. Deterministic models try to simulate the link with the math and physics of wave theory to find the path loss. Even though deterministic models sound more accurate, but they only started gaining popularity in the recent past, because of their high computational requirements.

For instance, deterministic models take into account the underlying terrain profile to make predictions as opposed to empirical models which are normally defined for categories of profile and struggle when the profile underlying is a combination of more than one profile. Later in this work, the Okumura-Hata model (empirical) and Electric Field Integral Equation based models (deterministic) are discussed.

2.5 Related Work

The need to establish the crisp and clear wireless communication among devices, even though very important, but hasn't been the easiest of things to implement because of drastically varying electromagnetic coverage. This is because of the impact of the underlying terrain profile. There have been numerous attempts to make it computationally feasible and few of the methods devised have been serving the purpose of network planning really nicely.

There are basically two kinds of models, empirical and deterministic. The deterministic models which are integral equation based are very reliable and efficient when it comes to taking into account the underlying terrain profile, and as the name suggests, the electromagnetic field is formulated as an integral equation. The work done in [19] suggest that integral intervals should be of the order of the quarter of the wavelength. This obligation is to make sure that the integral equations converge in time, else it would take even more computational power. The downside of these methods is the high computational time, which is of the order of the square of the number of integral intervals under consideration.

It is because of this computational complexity that the integral equation-based models have been edged by the empirical models, and integral equation-based methods have been confined to the small and medium-sized bodies. This has resulted in the playground being established in the large bodies, like terrain profiles, for the empirical and loosely deterministic models. The introduction of deterministic results help to overcome the complexity of moving the propagation models from one terrain to the other, which is not possible with the empirical models.

The most famous empirical model used is the formulation of the path loss formulas by Hata [20] based on Okumura's observations [21] in and around Tokyo. The integral equation-based methods are advantageous as compared to this simple yet effective parametric model, in terms of the inclusion of slow fading components of the signal, like shadowing and diffraction [19]. As compared to the models based on the geometric and uniform theory of diffraction [24][25] and knife-edge diffraction [22][23], integral equation-based methods give the full-wave solutions without the need to reduce them to the canonical forms.

The integral equation-based models started their journey in the domain of propagation modeling of electromagnetic wave over terrain profiles with a two-dimensional case at grazing incidence on rural terrain profile [26][27][28]. Even though the results produced were very good and in sync with the measurements, but the computational speed was the main issue that was supposed to be resolved.

The work done in [29][30] takes into account two signal fading components, large scale, and small scale. The devised method involving only the large-scale fading component reduces the dependence from the square of the integral intervals to the square of the groups of integral intervals and when compared to the methods before, is very fast. The second contribution of this work was the introduction of the statistical method to incorporate small scale (multipath) fading, which improves the accuracy of the model. This can be incorporated into the former without any significant computational overhead.

Since the development of this relatively quick implementation of the electric field integral equation method, there has been research in the application area of such propagation modeling techniques. The work done in [31] by Eamonn O Nuallain suggests how integral equation-based methods can be used to solve the hidden node problem and address the salient security threats when implemented alongside REM (Radio Environment Map) in CRNs (Cognitive Radio Networks) and thereby allaying the concerns of the primary users.

The work done by Eamonn O Nuallain in [32] suggests that because of the ‘automatic’ nature of the integral equation-based methods they remain one of the most suitable candidates for propagation modeling and are normally implemented to give results close to that of the actual measurements. Further in his work, he stated that FExM (Field Extrapolation Method) is a suitable way of implementing the integral equation-based method because of its high speed and accurate predictions. This has been tested and shown to be effective for rural/suburban profiles. Along with this in his work, Eamonn O Nuallain also mentions that it can be used to deal with the security threats inherent to CRNs (Cognitive Radio Networks) [31][32].

Eamonn O Nuallain in [33] devised a method for the localization of the non-cooperative transmitter using the Helmholtz Reciprocity Theorem implemented alongside integral equation-based propagation modeling. This method is entirely based on the received signal strength and gives fairly accurate results, with an error of around 20 meters in a quick time. The main advantage is that unlike other signal strength-based localization methods it is significantly inexpensive.

The method used in [34] to take into account the backscattering magnetic field and yet keeping the computational complexity of the integral equation based method to the square of the integral terms is inspiring and can be used in a similar manner, by implementing the iterative forward-backward sweeps to take into account the backward scattering electric field. This will improve the accuracy of the predictions even further.

Chapter 3

Methodology

3.1 Electric Field Integral Equation

Electric Field Integral Equation as the name suggests is an integral equation based deterministic method to solve the problem of finding the coverage of the transmitter. Deterministic propagation modeling methods draw their inspiration from the physics and maths of the wave theory. This is a comparatively easy to understand method as compared to the other popular ray-tracing techniques.

To start with, integral equations are basically mathematical equations that contain an unknown function under an integral sign [35].

$$y(x) = \int_0^x g(x)h(x)dx \tag{3.1}$$

Solving this equation for unknown $g(x)$ looks a very tedious task. Something which we not generally encounter. This brings us to the very definition of integration. The integration below:

$$\int_0^x f(x)\, dx \tag{3.2}$$

If $f(x) = cos(x)$ is nothing but the sum of the product of $cos(x)$ and dx in the interval 0 to x. This integration looks easy to solve because of the availability of explicit formula which reduces it to $sin(x)$. But these explicit formulas are not available for all the integrations, these are solved by replacing the sign of $\int$ by $\sum$ and dx by Δx. These substitutions reduce the above equation to:

$$\sum^N f(x)\, \Delta x \tag{3.3}$$

This makes it suitable for implementation on the computer. The accuracy of this increases, as we increase the value of N. Now coming back to the integral equation with unknown $g(x)$, this becomes easier to solve. As this is nothing but a set of N simultaneous equation with N unknowns. This is the sort of equation that is required to be solved while solving EFIE.

Assumptions:

- All the surfaces considered are perfect electrical conductors (PEC), i.e. all the traditions incident on the surface are reflected.
- All the radiations incident propagates away from the source, called Forward Scattering Approximation.

Now getting back to Electric Field Integral Equation, the source transmits an electric field which when incident on any surface induces an oscillatory current J on the surface. From the concepts of electromagnetics, any surface carrying oscillating current acts as the source of the electric field. This source also radiates the electric field which causes the current to be induced on other parts of the surface due to this secondary source as well. The picture below shoes the geometry of this phenomenon.

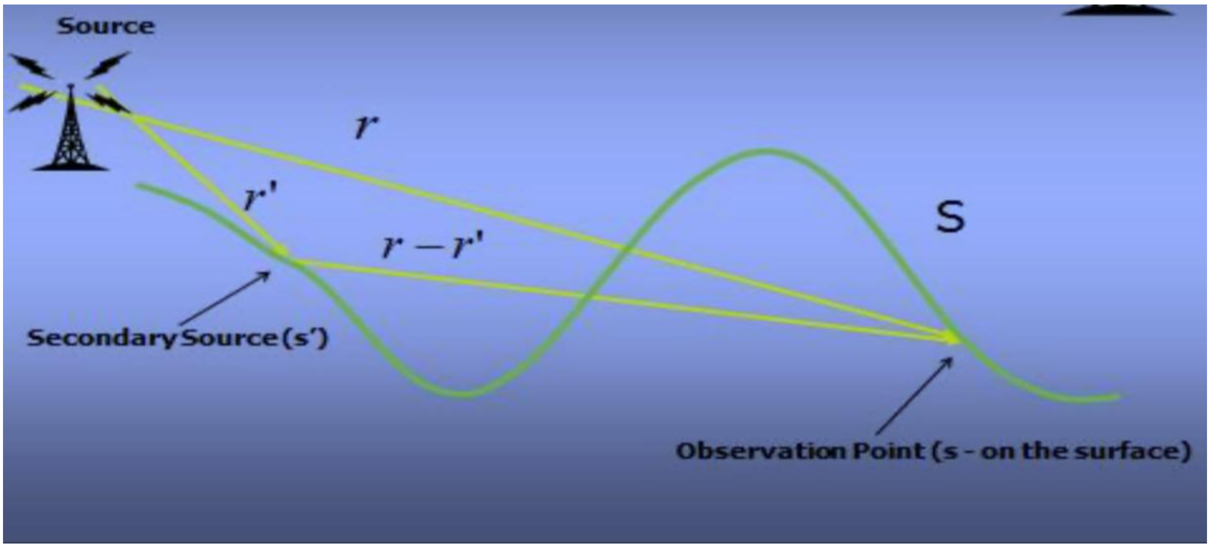


Fig 3.1: Geometry depicting the primary and secondary user

The current J induced on the surface satisfies the below mentioned Electric Field Integral Equation (EFIE) [31]:

$$E(r) = \int_s \left(\frac{\beta\eta}{4}\right) J(r') {H_0}^2(\beta|r - r'|) dr' \tag{3.4}$$

where,

r and r' are vectors whose ending points are receiving and scattering points respectively.

$s \in S$, which denotes the set of points on the surface.

$E(r)$ is the electric field incident from the source at r.

η is the wave impedance of the medium of propagation.

β is the wavenumber of the wave emanating from the source.

${H_0}^2$ is the zero-order Hankel function of the second kind.

We know all these quantities and the objective is to find the function $J(r')$. The equation above denotes our first assumption, i.e. field incident on the point r, which is $E(r)$, is equal to the field reflected from it. The term $(\beta\eta/4)J(r'){H_0}^2(\beta|r - r'|dr'$ denotes the electric field scattered from the point r' to r. The integration denotes nothing but the summation of such scattered fields.

Now let us assume the same surface discretized into N portions. Each of those N portions are of length Δs. The center points of each of these portions/lengths are indicated by r_i and r_j depending upon whether they scatter or receive respectively. The EFIE above can now be rewritten by replacing the integral by summation as:

$$E_j = \sum_{i=1}^{i \le N} J_i Z_{ij} \tag{3.5}$$

where,

$E_j = E(r_j) = {H_0}^2(\beta|r_j|)$

$Z_{ij} \approx \Delta s(\frac{\beta\eta}{4}) J(r') {H_0}^2(\beta|r_i - r_j|)$

$$Z_{ii} \approx \Delta s(\beta\eta/4)((1 - j\frac{2}{\Pi} ln(\frac{1.781\beta\Delta s}{4e}))$$

$$J_i = J(r_i)$$

Normally the discretization made are of the quarter of the wavelength of the signal from the source. The term Z_{ii} is called the self term. $J_i Z_{ii}$ represents the field scattered from i to itself. The term $J_i Z_{ij}$ represents the field scattered from i to j.

Expanding the equations given by $E_j = \sum_{i=1}^{i \leq N} J_i Z_{ij}$ will give N simultaneous equations with N unknowns, as follows:

$$E_1 = J_1 Z_{11} + J_2 Z_{21} + J_3 Z_{31} \; .... \; J_N Z_{N1}$$

$$E_2 = J_1 Z_{12} + J_2 Z_{22} + J_3 Z_{32} \; .... \; J_N Z_{N2}$$

$$\vdots \qquad\qquad \vdots$$

$$E_N = J_1 Z_{1N} + J_2 Z_{2N} + J_3 Z_{3N} \; .... \; J_N Z_{NN}$$

The above equations can be represented in a compact matrix equation form as:

$$E = ZJ \tag{3.6}$$

Solving N simultaneous equations with N unknowns is a computationally intensive task and rather than finding the exact solution of the equation by finding the inverse of matrix Z. We use the forward scattering assumption. This reduces the N equations to the form which can be solved by forward substitution and reduces the computational time significantly. The equations look like [31]:

$$E_j = \sum_{i=1}^{i \leq j} J_i Z_{ij} \tag{3.7}$$

Their expansion looks like:

$$E_1 = J_1 Z_{11}$$

$$E_2 = J_1 Z_{12} + J_2 Z_{22}$$

$$\vdots \qquad \qquad \vdots$$

$$E_N = J_1 Z_{1N} + J_2 Z_{2N} + J_3 Z_{3N} \; \ldots . \; J_N Z_{NN}$$

So, from the first equation:

$$J_1 = \frac{E_1}{Z_{11}} \tag{3.8}$$

Substituting the value of J_1 to find J_2:

$$J_2 = \frac{E_2 - J_1 Z_{12}}{Z_{22}} \tag{3.9}$$

Similarly,

$$J_j = \frac{E_j - \sum_{i=1}^{i<j} \quad J_i Z_{ij}}{Z_{22}} \tag{3.10}$$

After finding all the Js. The electric field at each point can be calculated as:

$$E^{total}(r_{obs}) = E^{inc}(r_{obs}) + \int_s \quad (\frac{\beta\eta}{4}) J(r') {H_0}^2(\beta|r' - r_{obs}|) dr' \tag{3.11}$$

The integral equation-based methods are superior to Okumura-Hata model in terms of the standard deviation from the measured results and it also ends up predicting large scale fading signal more accurately, but the thing of concern is the computational complexity which is of the order of $O(N^2)$, where N is the sub-wavelength in dimension [32].

3.2 Field Extrapolation Method (FExM)

The computational complexity of the integral equation-based methods has always been the reason that they are not widely used. The work done in [30] where rather than dividing the surface into N sub-wavelength divisions, the surface is divided into groups of sub-wavelength divisions. Each group contains M such sub-divisions, which means the surface is divided in N/M groups.

So, the normal Electric Field Integral Equation mentioned above can now be rewritten as:

$$E_i = \sum_{l'<l} \sum_{j \in G_{l'}} J_j Z_{ji} + \sum_{j \in G_l} J_j Z_{ji} \tag{3.11}$$

where,

$G_{l'}$ is the scattering group.

G_l is the receiving group.

$j = 1,2 \ldots . M$

$i = 1,2 \ldots . N$

l' is the center point of the scattering group, such that $l' \in \{j\}$

l is the center point of the receiving group, such that $l \in \{i, j\}$

Now if we consider that each of the scattering groups $G_{l'}$ induces a current $J_{j_{l'}}$ on the receiving group G_l, this allows us to write the above equation as:

$$E_i - \sum_{l'<l} \sum_{j \in G_{l'}} J_j Z_{ji} = \sum_{l' \leq l} \sum_{j \in G_l} J_{j_{l'}} Z_{ji} \tag{3.12}$$

where,

$$J_j = \sum_{l' \leq l} J_{j_{l'}} \tag{3.13}$$

In the work done in [30] it is suggested to deal with the discontinuity towards ends of the finite surface profile, by extending the profile and hence artificially enforcing the continuity. This gives the following equation based on the assumption of the locally constant amplitude surface current:

$$\sum_{j \in G_{l'}} J_j Z_{ji} \approx J_{l'} Z_{l'l} \tag{3.14}$$

By substituting we can write the equation involving E_i as:

$$E_l - \sum_{l'<l} J_{l'} Z_{l'l} = \sum_{l' \leq l} \sum_{j \in G_l} J_{j_{l'}} Z_{jl} \tag{3.15}$$

Since the wave inducing the current $J_{j_{l'}}$ is a uniform plane wave, we can write:

$$J_{j_{l'}} = -J_{l'} \frac{Z_{l'l}}{Z_{jj}} e^{-j(\beta s_j cos\theta_{l'} + \varphi_{l'})} \tag{3.16}$$

Where,

s_j is the distance from the start of the group to the point j.

$\theta_{l'}$ is the angle of incidence of the wave on G_l which is being transmitted from $G_{l'}$.

$\varphi_{l'}$ is the phase shift.

So, after substitution, the equation incorporating E_i can now be written as:

$$E_l - \sum_{l'<l} J_{l'} Z_{l'l} = -\sum_{l' \leq l} \sum_{j \in G_l} J_{l'} \frac{Z_{l'l}}{Z_{ll}} e^{-j(\beta s_j cos\theta_{l'} + \varphi_{l'})} Z_{jl} \tag{3.17}$$

$$E_l = \sum_{l'<l} J_{l'} Z_{l'l} (1 - \sum_{j \in G_l} \frac{Z_{jl}}{Z_{ll}} e^{-j(\beta s_j cos\theta_{l'} + \varphi_{l'})}) + Z_{ll} J_l \tag{3.18}$$

At predominantly gazing angle incidence $\theta_{l\prime}$ is zero and $\varphi_{l\prime}$ can be reduced to zero without compromising the accuracy. So, now the equation becomes:

$$E_l = k \sum_{l\prime<l} J_{l\prime} Z_{l\prime l} + Z_{ll} J_l \tag{3.19}$$

where,

$$k = 1 - \sum_{j \in G_l} \frac{Z_{jl}}{Z_{ll}} e^{-j\beta s_j} \tag{3.20}$$

This value of k is nearly constant for almost all the scattering groups. The equation above is a huge improvement in terms of computational efficiency over the general EFIE which we discussed in the last section as it reduces the complexity from $O(N^2)$ to $O((N/M)^2)$ [30]. The total electric field above the surface at some point t now becomes:

$$E_t^{total} = E_t + k \sum_{l\prime<t} J_{l\prime} Z_{l\prime t} \text{ for } t = 1,2\ldots\ldots\frac{N}{M} \tag{3.21}$$

3.3 Integral Equation-Based Intuitive Forward Backward Scattering Sweeps

The two integral equation-based methods discussed above works on the assumption of forward scattering that is all the electric field incident will propagate in the forward direction but in the real world scenario that is not the case. The component of the field incident might also propagate in the backward direction known as backward. The method discussed in this section is the intuitive method to take into account the backward scattering field as well without enormously impacting the computational complexity.

The algorithm goes:

1. Create two separate arrays for the current induced due to forward and backward scattering fields and name them J_F and J_B respectively.

2. Forward sweep: Implement the EFIE method with forward scattering assumption and find the forward scattering currents J_F at all the N points.

Note: Apart from 1st iteration the J_F for the first element would be as updated in step 3.

Update the J_B on the last discretized part of the surface with J_F on that part.

3. Backward sweep: Starting from the last discretized element implement the EFIE method with forward scattering assumption in the opposite direction to that of the forward sweep, i.e. towards the source.

   Update the J_F on the first discretized part of the surface with $J_F + J_B$ on that part.

   Note: This sweep will not include the contribution of the source.

4. Repeat step 2 and 3 until $J_F + J_B$ when used to calculate the total field strength at all the points do not converge to the results obtained using the inverse of Z matrix (normally 5-7 iterations).

This procedure requires order N^2 or $(N/M)^2$ operations (depending upon the choice of the electric field integral equation-based method) in contrast to the matrix inversion requirement of the order of N^3 operations is deemed to be as accurate, which will be checked in the results and is way better suited to computational works done on the computers of modest cost.

3.4 Empirical Model: Okumura Hata

This is one of the most widely used microcell prediction models. The Okumura Hata model is the combined result of the work of two scientists, namely Okumura and Hata. This model is fully empirical, and it draws its formulation from the graphs made using the observations of the experiments conducted around Tokyo city by Okumura [39]. Later, out of these graphs, Hata formulated one of the most important ones [40]. The formulation of this method is completely empirical and has nothing to do with physical models.

This model is valid within the range of $1 - 100\ km$ and for the frequency range of $150 - 1500\ MHz$. The set of curves made by Okumura which gives the median attenuation with respect to the free space propagation of the signal in irregular terrain were all made using the various base station to mobile signal attenuation measurements which he made throughout the Tokyo.

The Hata's empirical formulation of the graph driven results of the Okumura model is very simplified and is in a closed formulation, which none the less is not based on the different empirical graphs for every different parameter. These things when combined make its application relatively simpler and straight forward [40].

This Okumura Hata model is the industry standard, normally against which most of the new breakthroughs in this field are compared. This model classifies the prediction area into three broad types, for which different factors are weighed in as required. These three terrain types along with their respective empirical formulation provided by Hata to calculate the path loss are [7]:

- Urban areas: Built-up metropolitan areas with close construction that include tall buildings and multistorey houses or villages with dense trees and closely built houses.

$$L_{dB} = A + B * log(R) - E \quad (3.22)$$

- Suburban areas: These are the areas with few obstacles around the mobile, which include villages and highways with scattered houses and trees.

$$L_{dB} = A + B * log(R) - C \quad (3.23)$$

- Open areas: This is the clear stretch of land without any obstacles for around 300-400 meters, which includes farmlands, open fields, etc.

$$L_{dB} = A + B * log(R) - D \quad (3.24)$$

where,

$A = 69.55 + 26.16 * log(f_c) - 13.82 * log(h_b)$

$B = 44.90 - 6.55 * log(h_b)$

$C = 2 * (log(f_c/28))^2 + 5.4$

$D = 3.2 * (log(f_c))^2 - 18.33 * log(f_c) + 40.94$

$E = 3.2 * (log(11.75 * h_m))^2 - 4.97$ for large cities, $f_c \geq 300\ MHz$

$E = 8.29 * (log(1.54 * h_m))^2 - 1.10$ for large cities, $f_c < 300\ MHz$

$E = (1.10 * log(f_c) - 0.70) * h_m - (1.56 * log(f_c) - 0.80)$ for medium to small cities

Along with the restrictions on the frequency of the transmitter, there are also restrictions on the base station height, mobile height, and the distance between the two. These empirical formulas suggested by Hata are only applicable when $30\ m \leq h_b \leq 200\ m$, $1\ m \leq h_m \leq 10\ m$, and $R \geq 1\ km$. h_b is defined as the average height above the ground within the range of $3 - 10\ km$ from the source.

The point of contradiction is normally the effective heigh of the base station to be considered. Consider two cases when the range of the terrain under consideration is less than 3 km or when the average height of the terrain between 3-10 km is greater than that of the height of the antenna. In both these cases, it is absurd to use the height of the base station over the average height of the terrain. Therefore, in these cases, we use the height of the base station above the local ground level [42].

3.5 Localization of Non-Cooperative Transmitter

Localization, in cellular communication, is the technique of pinpointing the exact region or geographic position. The method discussed here is the signal strength-based method which uses electric field integral equation-based method for propagation modeling and Helmholtz Reciprocity Theorem to establish the location of a non-cooperative transmitter.

The Helmholtz Reciprocity Theorem states that: 'A point source at P_o will produce at P the same effect as a point source of equal intensity placed at P will produce at P_o' [43].

If the transmitting power of the transmitter is known, then one can obtain a set of potential locations of the transmitter using the below-mentioned algorithm [34]:

1. Find the RSS at the location of the transceiver.
2. Use a propagation predictor (here FExM) to predict the signal strength at all the points across the surface with the transmitter with the same output power placed on the location of the transceiver.
3. The points where the computed signal strength is the same as the RSS at the observation point are the possible locations for the transmitter.
4. Introduce more such transceiver locations and repeat the above steps until left with just one transmitter location.

If the transmitting power of the transmitter is unknown then one can obtain a set of potential locations of the transmitter by using the Helmholtz Reciprocity Theorem to establish the concept that if rather than one transceiver location, two are taken then the difference in the uplink and downlink signal strength of the two will be same. The algorithm for this goes as mentioned below [34]:

1. The differences in measured RSS for each receiver pair in the downlink are calculated.
2. The differences in the uplink SS is calculated by using the propagation modeler (here FExM) by assuming the receiver pair, each transmitting with the same arbitrary power. The results are calculated as a function of location.
3. The residual function is calculated by subtracting the downlink difference from the uplink difference.
4. The residual functions for all the transceiver pairs under consideration are added.
5. The function so obtained is smoothed to eliminate false minima.
6. A search routine is implemented to find the absolute minima which will denote the location of the transmitter.

Chapter 4

Results

The methods discussed are applied at 970 MHz to 7.8 kilometres of terrain profile in Hadsund, Northern Denmark, and the first 700 meters of German terrain profile, both of which are shown below. On both of them, Exact Method using the inverse of Z, Electric Field Integral Equation with forward scattering assumption, and Field Extrapolation Method with forward scattering are applied. All the prediction models were run on a system with Macintosh operating system with 2 GHz Quad-Core Intel Core i5 processor and 16 GB 3733 MHz LPDDR4X RAM.

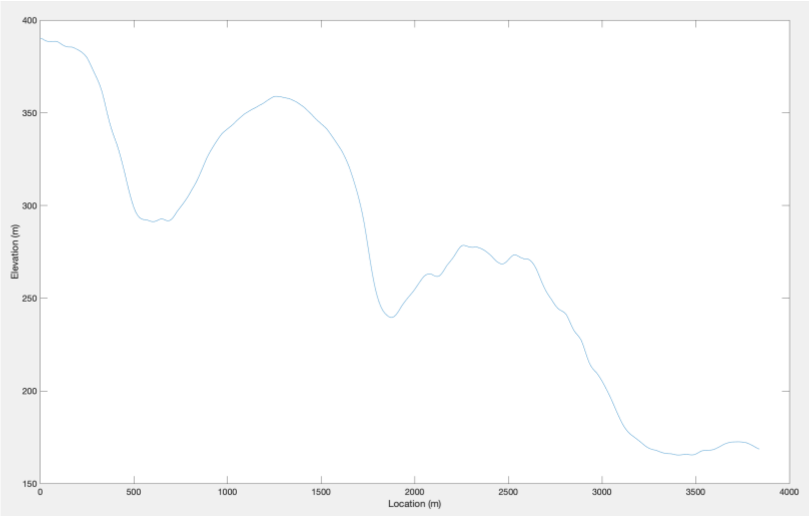


Fig 4.1: Plot of elevation of terrain vs location for German terrain profile

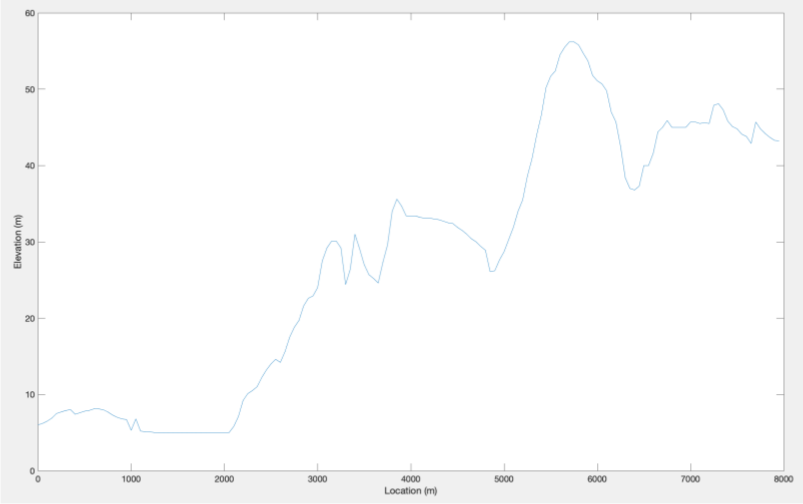


Fig 4.2: Plot of elevation of terrain vs location for Hadsund, Denmark terrain profile

4.1 Hadsund-Danish Terrain Profile

A superimposed graph of the results obtained using FExM, and the actual measurements on 7.8 $kilo$meters of terrain profile in Hadsund, Northern Denmark are shown in Fig. The source here is kept at the elevation of $16.4\ meters$, and the predictions made, and measurements taken are at $2.4\ meters$ above the surface. The group size of almost $20\ meters$ was taken for the implementation of FExM.

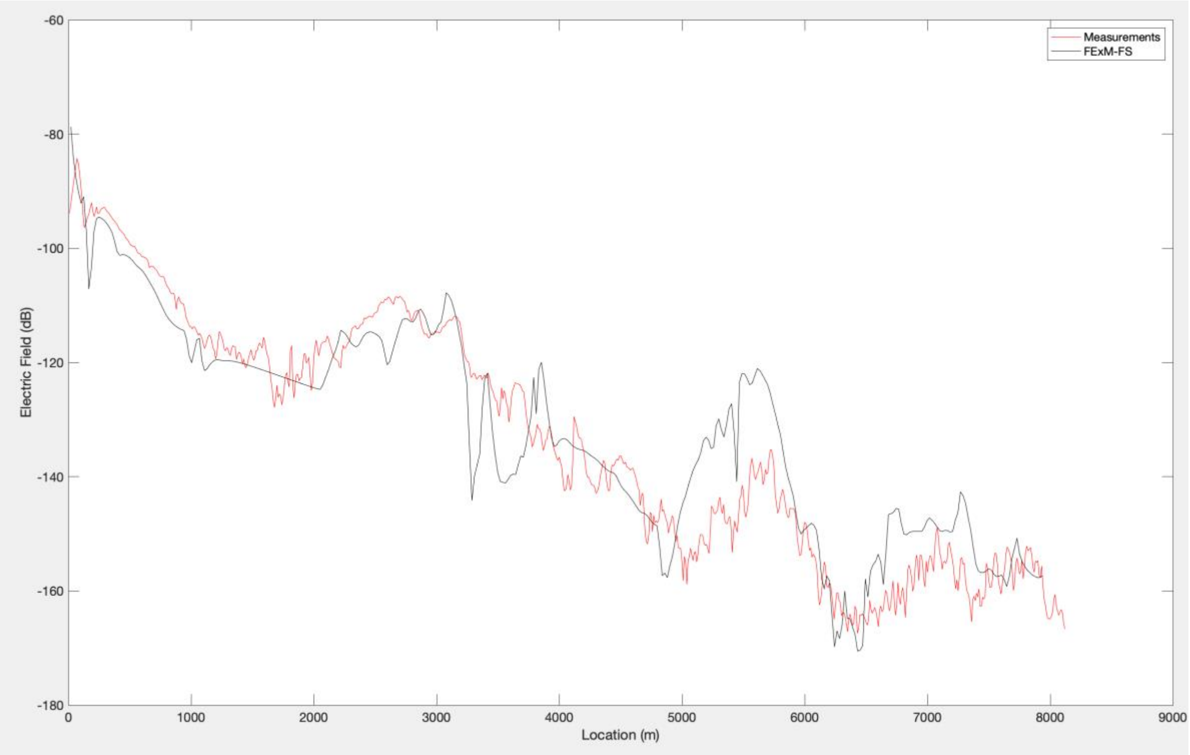


Fig 4.3: Comparative plot of Electric Field strength 2.4 meters above the ground on the Hadsund Profile as predicted using Exact Method (Red), and FExM-FS (Black) with the transmitter located at 442 meters above the ground at x=0 and transmitting at 970 MHz.

The graph shows that electric field strength predicted using FExM on Hadsund profile with a group size of $20\ meters$ produces a graph which is very close to the measurements. These results are in agreement with the results produced in [31] by Eamonn O Nuallain which were produced at $435\ MHz$. Similar to the work done in [31] signal envelope is taken to be a constant over the groups rather than Rayleigh distributed envelope shown in [30], which produces more accurate results without compromising the accuracy.

In the figure below the superimposed graph of the predictions made using FExM and Okumura-Hata's model on $7.8\ kilometers$ of terrain profile in Hadsund, Northern Denmark are shown. The source here is also kept at an elevation of $16.4\ meters$, and the predictions are made at $2.4\ meters$ above the surface. The Okumuar-Hata model was implemented in Suburban setting.

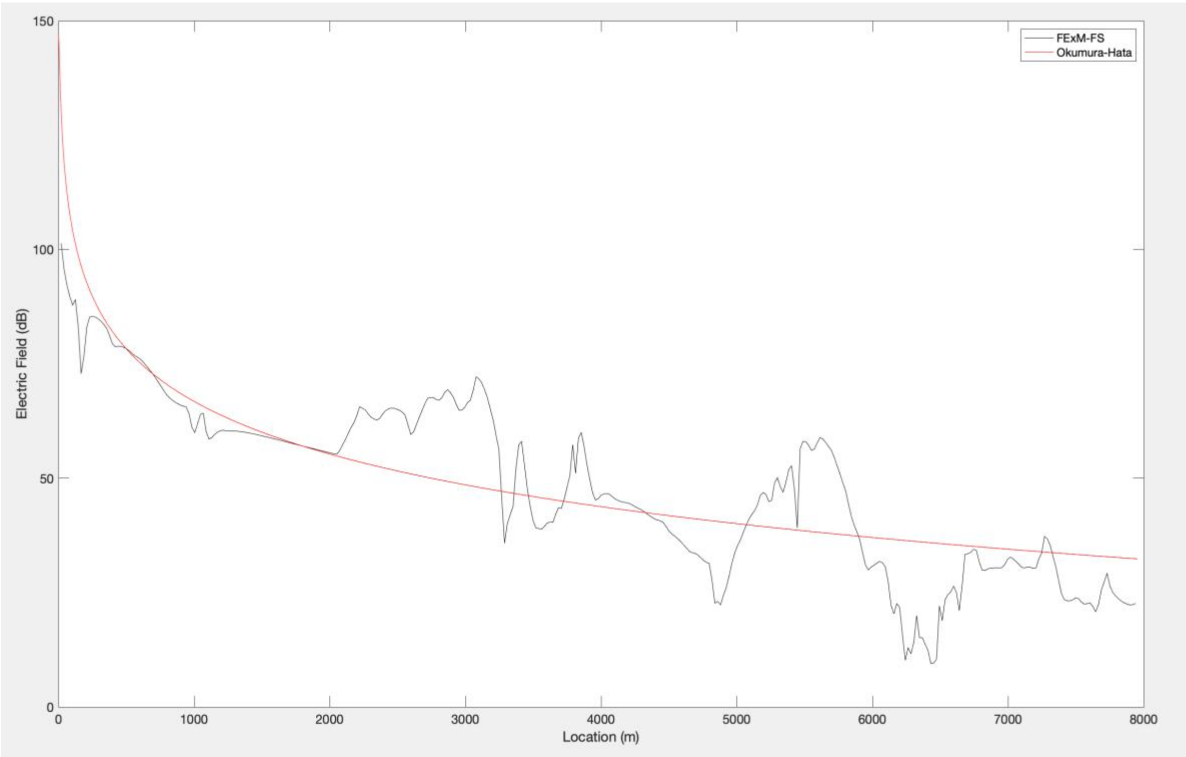


Fig 4.4: Comparative plot of Electric Field strength 2.4 meters above the ground on the Hadsund Profile as predicted using Okumura-Hata Model (Red), and FExM-FS (Black) with the transmitter located at 442 meters above the ground at x=0 and transmitting at 970 MHz.

The results produced using Okumura-Hata's model in the suburban setting and FExM on Hadsund profile produced the results which support the inherent property of large-scale fading and path loss. The variation in FExM predictions (which includes large scale fading) is up to approximately $20\ dB$ over the area of propagation of the signal. These results are very strong evidence in support of the integral equation-based Field Extrapolation Method.

4.2 German Terrain Profile

A superimposed graph of the results obtained using the Exact Method, EFIE, and FExM on the first $700\ meters$ of German profile are shown in figure below. The source is kept at an elevation $442\ meters$ and all the predictions made are $2.4\ meters$ above the surface. The group size of almost $1\ meter$ and $20\ meters$ was taken for the implementation of FExM.

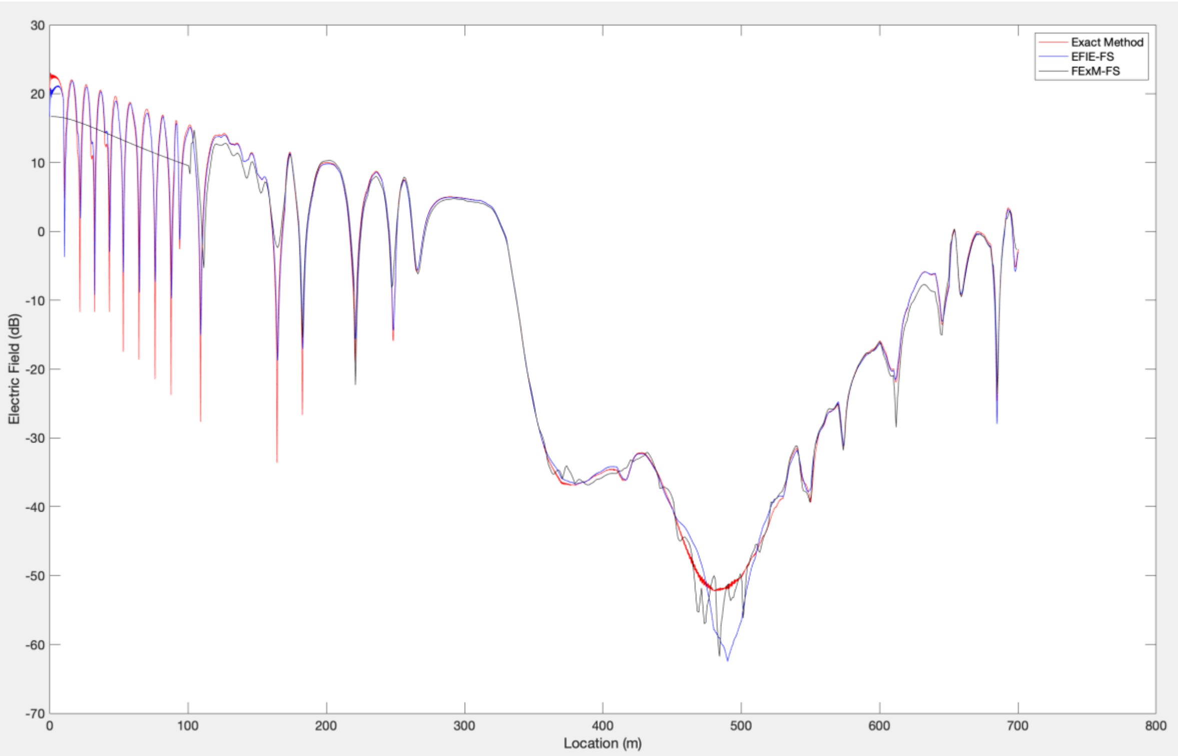


Fig 4.5: Comparative plot of Electric Field strength 2.4 meters above the ground on the first 700 meters of German Profile as predicted using Exact Method (Red), EFIE-FS (Blue), and FExM-FS (Black) with the transmitter located at 442 meters above the ground at x=0 and transmitting at 970 MHz.

The agreement of the predictions made using the Exact Method, EFIE, and FExM on German profile is very promising because of the slightly more rugged nature of the German profile as opposed to that of Hadsund profile. The group size used here is 1 meter. The smaller group size taken as compared to Hadsund profile is because of the more rugged nature of the profile.

There is a huge gain in the computational time in implementing EFIE over Exact Method. The computational time gained was by the factor of the number of integral divisions. Similarly, there was a gain in computational time in Implementing FExM over EFIE by a factor of $13 * 13$ (13 is the group size).

In the figure below the superimposed graph of electric field prediction using Exact Method, EFIE, and Iterative FS and BS sweeps using EFIE are shown. The source is at an elevation of $442\ meters$ and the observation points are $2.4\ meters$ above the surface. The total of 5 iterations were run for implementing Iterative FS-BS Sweeps.

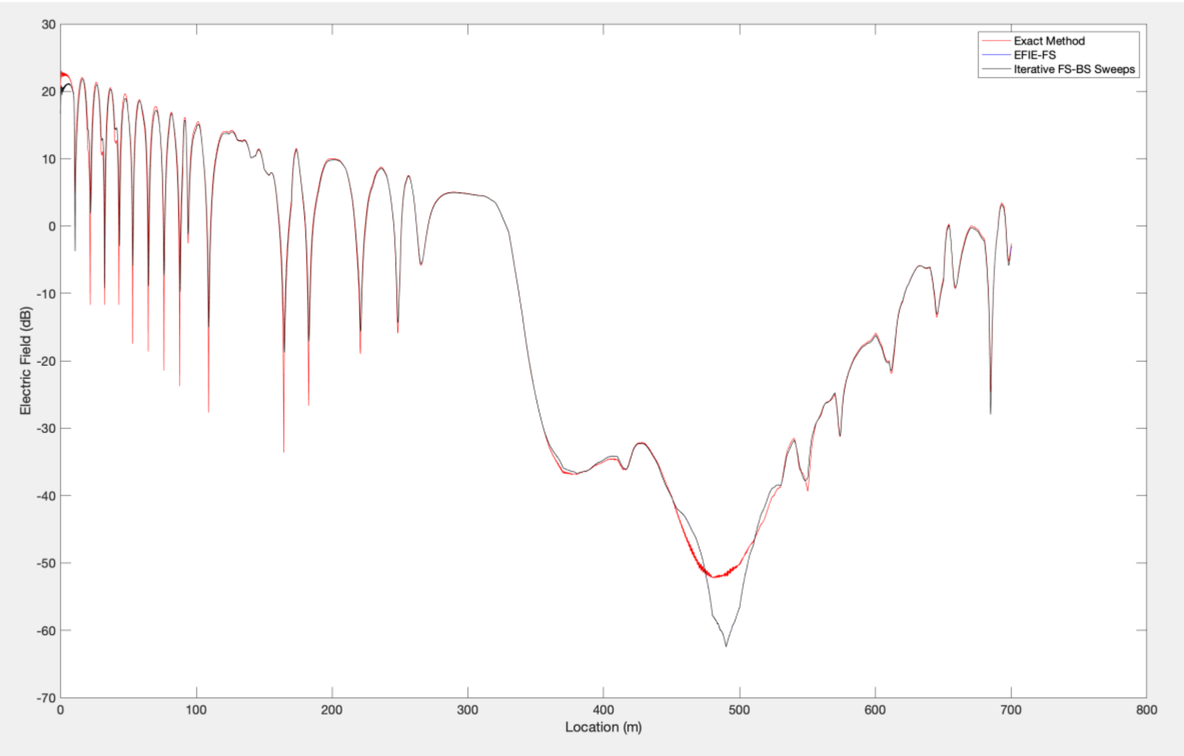


Fig 4.6: Comparative plot of Electric Field strength 2.4 meters above the ground on the first 700 meters of German Profile as predicted using Exact Method (Red), EFIE-FS (Blue), and Iterative FS-BS Sweeps (Black) with the transmitter located at 442 meters above the ground at x=0 and transmitting at 970 MHz.

In the graph above the results obtained using Iterative Forward Backward Sweeps are closer to EFIE as opposed to Exact Method shows that even though something is done right but there is something lacking. This is because of the fact that the Iterative Forward Backward Sweeps method takes into account the back-scattering field as well and so the graph for it should be closer to the Exact Method.

4.3 Hadsund-Danish Profile Localization

In Fig. a 3-D plot is shown which was made as a result of implementing the Localization Algorithm on the Hadsund terrain profile. In the algorithm, three pairs of locations ($3000\ and\ 5000, 3500\ and\ 7000, and\ 4000\ and\ 6000$) are taken. And a 3-D plot is plotted for location, the summation of difference in Uplink and Downlink electric field strength for each of the pair, and, elevation above the surface (0,1….20 *meters*).

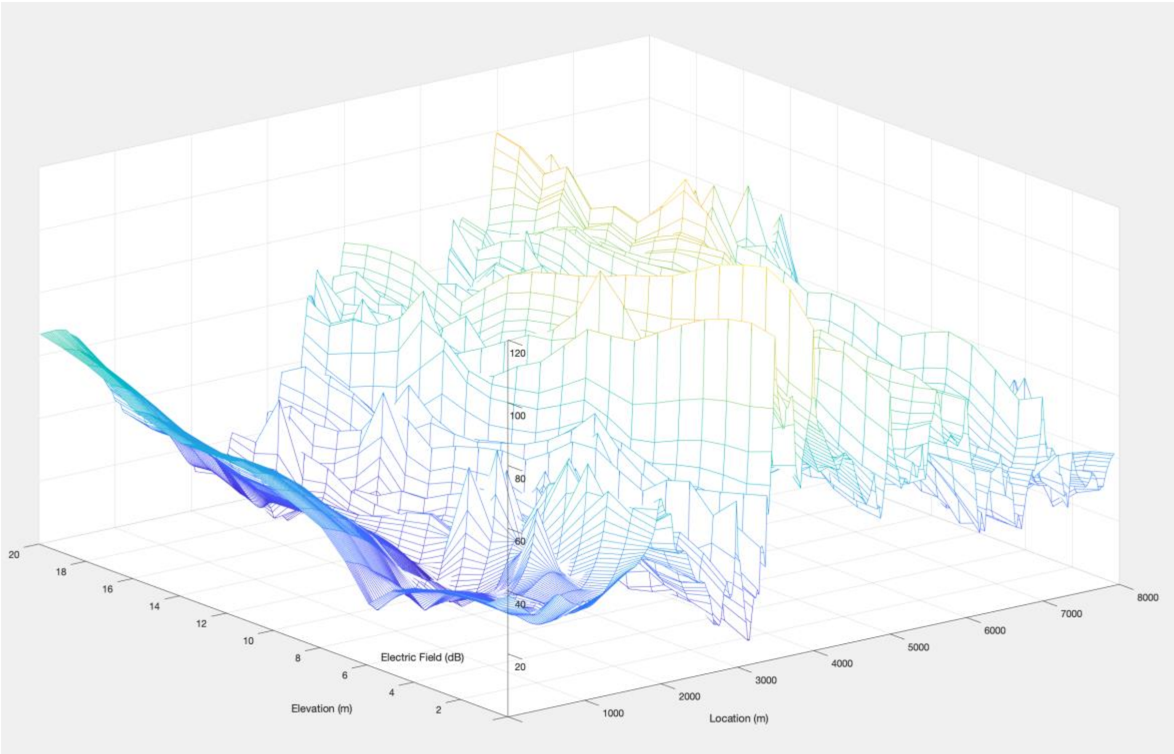


Fig 4.7: 3-D Plot of summation of the difference in the uplink and downlink electric field strength for three pairs of locations (3000 and 5000, 3500 and 7000, and 4000 and 6000), location, and the elevation above the surface.

The method used helped in the localization of the transmitter without its cooperation based on the signal strength readings taken using signal strength meters. The use of modest computational resources for propagation modeling. The location obtained is 147 *meters* accurate, which compares well with the accuracy of 2G and 3G systems, i.e. accuracy in the neighbourhood of 100 *meters*. Unlike the results obtained in [33] by Eamonn O Nuallain which had the accuracy comparable to GPS systems, which has an accuracy of about 10 *meters*. This might be because of the fact that few locations of the transmitter chosen are around 3500 meters and 6500 meters mark, where there is a significant difference in the simulated results and measurements.

Chapter 5

Discussion and Conclusion

5.1 Summary of New Contributions

The integral equation-based methods have been known to perform well only on the gently undulating profiles like Danish because of the fact that there is a possibility of a significant amount of backscattering. This work provides the implementation of EFIE on a rugged German profile with good predictions. The work done in [31] by Eamonn O Nuallain suggests that the backscattering effect from gently undulating profiles are very small. When it comes to more rugged profiles preferably smaller group sizes should be taken and the backscattering effects are there, but they don't appear to be that great because of the fact that there would be significant absorption in these cases. The results obtained here are in alignment with the work done in [31]. The fact that smaller group size of $1\ meter$ when used in FExM produced accurate results on German profile as opposed to Danish profile for which the predictions were good for even the group size of 20 *meters* justifies the conclusions in [31].

The Okumura Hata's empirical model is a state of the art and is adopted for most of the network planning because of its fast implementation. This work provides a comparison of the accuracy of results produced using electric field integral equation-based methods and specially FExM because of its comparable implementation time. The results produced by FExM are in sync with the Okumura Hata's prediction with an added advantage of the inclusion of the large-scale fading signal as well. Unlike other deterministic models that mostly depend on the line of sight communication greatly, electric field integral equation-based methods operate well in non-line of sight condition as well. So, deterministic FExM provides a strong alternative to the empirical Okumura Hata's model.

The electric field integral equation-based methods have always worked with forward scattering assumption. This work devises an intuitive iterative forward-backward sweeps method for the electric field which has been previously used for the magnetic field in [34]. This method keeps the computation complexity to the square of integral points and includes the backscattering field as well.

The localization method devised by Eamonn O Nuallain in [33] has been reimplemented in this work with different sets of locations for transceiver pairs, with few transceivers in the zone where there is a significant difference between the simulated and measured electric field strength, i.e. around $3500\ and\ 6500\ meters$ mark. The result produced is in sync with the observations made in [33] that choosing such a location for transceivers will reduce the locating accuracy of the procedure, and the accuracy got reduced from being comparable to GPS, to being comparable to 2G and 3G achievable accuracy. This implemented method provides a quick and inexpensive mode of locating a transmitter without its cooperation and the fact that the propagation modeling done, is based on the electric field integral equation-based method provides it the ability to work in NLOS condition as well.

5.2 Application in 5G

The 5G is the next big thing expected this decade which is deemed to be the enabler of IoT and other significant technologies but as discussed earlier the traditional spectrum allocation system might become the bottleneck for 5G, because of the fact that in most of the countries the sweet spot required for 5G, which is $1 - 6\ GHz$, has already been occupied. In the countries in which it is not occupied, it will get overcrowded soon because of the increased among device communication which will increase with the implementation of IoT. These things require for the new way of efficiently utilizing the spectrum.

This work provides a Cognitive Radio based solution to this problem and discusses the basics of Cognitive Radio Networks and how traditional nodes can be utilized in the CRNs by implementing a Radio Environment Map. The main contribution of this work is the efficient deterministic propagation modeling technique which is based on an electric field integral equation. This deterministic modeling method discussed here takes into account large scale fading signals as well and is not dependent on the line of sight communication.

The proposed propagation modeling mechanism in this paper can be used in Cognitive Radio Networks which can be used in 5G implementation by aiding in Dynamic Spectrum Access. This work further provides a localization technique for the non-cooperative transmitter which is fairly inexpensive as opposed to other signal strength-based methods. This can be of great benefit in locating malicious users for network security purposes, location-based marketing, and other services, localization in emergency scenarios, and majorly the localization of primary users in CRNs. These localization applications are of great use in 5G systems and their implementation using a signal strength-based method which is inexpensive can be of great advantage.

5.3 Limitations and Future Scope

In FExM the method to evaluate the value of k even though is theoretically correct but it still has some discrepancies. The results for the value of k were not consistent and sometimes the hit and trial method were used instead. This is something that could not be resolved and requires further attention.

The Intuitive Iterative Forward-Backward Sweeps approach to take into account the backward scattering field as well, is theoretically correct. The fact that the results produced on the first $700\ meters$ of German profile coincide with the results of EFIE rather than that of the Exact method suggests that we are halfway there. It is most likely that the method implemented is correct based upon the results in [34] where a similar approach is applied on the magnetic field but there is some minute error in the code.

The method for localization of non-cooperative transmitter used here suffers from the same limitations as do other methods because the propagation modeling technique used is based on the eclectic field integral equation.

5.4 Future Scope

There is a need for more stringent propagation modeling solutions to protect the primary users and ensure network security. This calls for more efficient methods in terms of accuracy and computational time.

All the electric field integral based methods work on two assumptions of forward scattering and Perfect Electrical Conductor (PEC) which actually is not true. These work really well with gently undulated profiles and even for the comparatively rugged profiles but with smaller group size in FExM. These should be built upon by implementing the full-wave solution on the forward scattering assumption and include the backward scattering terms as well. One method of doing this has been attempted here by trying to implement the Intuitive Iterative Forward-Backward Sweeps method. The other way which can be explored is by assuming the entire terrain to be a dielectric rather than a PEC. This will include implementing a CFIE.

In FExM the Rayleigh signal distribution can be taken over each group rather than a constant distribution which will help improve accuracy [30] without impacting the execution time. The group of groups method can be used to build upon the already existing FExM which will decrease the computational time even further. Apart from this the integral equation-based propagation modeling methods can be improved by exploring and taking into account the effects of side scattering, frequency-selective effects, channel variations, the effect of weather etc.

The method for the localization of the non-cooperative transmitter considered here does not consider the situation where the transmitter will lie between the receivers and when there is more than one transmitter in the same band. These things, along with a better definition of the error function to improve the accuracy of the method and reduce the spurious minima can be built upon in the future to make the method more robust and accurate.

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Appendices

Abbreviations

CRN Cognitive Radio Network

CR Cognitive Radio

DSA Dynamic Spectrum Access

REM Radio Environment Map

EFIE Electric Field Integral Equation

FExM Field Extrapolation Method

ISM Industrial Scientific Medical