On Place-Names

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Declaration

I, Zak Hayden Szadowski, declare that the following dissertation, except where otherwise stated, is entirely my own work; that it has not previously been submitted as an exercise for a degree, either in Trinity College Dublin, or in any other University; and that the library may lend or copy it or any part thereof on request.

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Summary

In my dissertation, I examine the problem of representing the relationship between place-names and the locations that they represent in modal logic. This problem stems from the fact that existing modal logics constrain the usage of names to refer to unique instances, where in reality we find that place-names can represent multiple locations at the same time.

In order to solve this problem, I construct my own multi-modal logic that is capable of representing this and other such complicated naming situations that the existing logical frameworks cannot handle.

My model of reference was constructed using the Causal Theory of Reference proposed by Saul Kripke after an examination of other competing theories. This theory proposes that place-names are rigid designators that do not imply any conotative aspects and that names are given to objects by an initial baptism and then that reference is given to other people by an ordered series of events.

This model of reference is then combined with an existing spatio-temporal framework, $ST^+_2$, to form the final multi-modal logic. $ST^+_2$ was chosen for its relation to the high-level spatial language RCC-8, which allows people to express notions of connectiveness between spatial regions. $ST^+_2$ combines RCC-8 with a points-based temporal logic that has additional characteristics that I desire.

On investigation, my logic can be shown to be consistent as well as descriptively adequate and is able to describe all possible relationships between toponyms and the regions. However further research is required to properly explore how decidable and tractable my logic is.
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Chapter 1

Introduction

1.1 Background and Context

Place-names are an important part of everyday life. They define where people come from and where they are going, as well as every location along the way. Place-names are an integral part of the fabric of mankind’s cultural identity and a reminder of the distant past. As various civilisations rose to power and colonised other lands, they spread their way of naming places across the regions that came under their dominion; the Greeks, the Romans and the British, amongst others, have all left their signatures on the geography of the planet. Even now, centuries later, their influence can still be seen in the names of places across the globe (Swadesh, 1959).

It is not surprising then that place-names are of interest to several different academic disciplines, such as Anthropology and Computer Linguistics (Swadesh, 1959; Garbin & Mani, 2005). In these scholarly works, place-names are often referred to as toponyms (Buscaldi & Rosso, 2008; Jackoway, Samet, & Sankaranarayanan, 2011). This more formal name is derived from the academic title given to the study of place-names, Toponymy.

There are several different areas of research within Toponymy, but there is one area of particular interest to Computer Science. The challenge is to find ways of tapping into the potential of the modern connected world to instantly and accurately track the location of events of interest as they occur across the globe. It is possible to manually identify and process toponyms, embedded in on-line documents but only up to a point. The overwhelming volume of information currently submitted in news reports, presented on social
media sites and available from other sources of citizen journalism makes this unfeasible. My aim is for individuals or groups to be able to accurately pinpoint global news events and identify those relevant to their interests in real time.

In order to efficiently evaluate and process toponyms contained within the vast amount of articles, reports and other documents that are now available on the internet, man has already turned to electronic systems in an attempt to overcome human limitations. These systems can be quite complex and most of them have been designed purely to examine certain types of documents, such as tweets (Sankaranarayanan, Samet, Teitler, Lieberman, & Sperling, 2009), news articles (Garbin & Mani, 2005) or historical documents (Smith & Crane, 2001). Despite being programmed to process specific material, these systems are usually consistent in comprising of two distinct stages; they first identify possible place-names associated with the document or snippet of text and then determine the locations to which the toponyms refer (Smith & Crane, 2001).

As the identification phase is the first part of the process, it has naturally received more attention from researchers. Consequently a myriad of ways to identify place-names in text have been developed. The most common of these are a domain specific heuristics system citeravin0 or a self-learning concept spotter trained to locate place-names in text (Strzalkowski & Wang, 1996; Bikel, Miller, Schwartz, & Weischedel, 1997). The use of a hidden-Markov model to identify toponyms has also seen considerable success (Zhou & Su, 2002). In general, any system designed to identify concepts in text can, with some retooling, be used to identify place names. However ambiguous cases, such as confusing Man City with the city of Manchester, when in fact it refers to the English Premier League football club are still a concern.

The next step in evaluating toponyms is to remove any such ambiguities, in order to tie them to a concrete location. This will allow the mapping of events that are occurring in the text to the real world. There are several different techniques that can be used to perform this disambiguation. The first step is to examine the content of the article and take advantage of the conventions used when referring to place-names to correctly identify them (Garbin & Mani, 2005). Alongside this, a map of unambiguous and already disambiguated toponyms can be built from the document. This map can then be filtered, so that it only contains place-names which are close to the place-name of interest contained within the text; these names in many cases refer to locations that surround the candidate
Finally, in documents such as tweets where such context may be lacking, it is possible to cross-reference similar documents, i.e. tweets with the same hashtag, in order to build up a richer picture with which the program can perform the disambiguation (Sankaranarayanan et al., 2009).

1.2 The Problem

Unfortunately the current automated disambiguation processes have several shortcomings that are not easily resolved. They are not able to achieve the desired level of precision; in fact they perform significantly below human ability, making them totally unsatisfactory.

The primary difficulty to be overcome is the problem of distinguishing between and representing places that have been given the same name. This is particularly common in former British colonies such as the United States of America where homesick settlers named their new surroundings after the places that they came from “to remind them of the familiar” (Bean, 1980, p. 312). But this problem is not exclusively colonial in origin; even within individual countries there is considerable replication of place names.

It is possible to try to mitigate the problem of identifying which place is being referred to when there are several places with the same name during the disambiguation phase. This is done by examining how well each of the alternatives fit according to the rest of the content (Smith & Crane, 2001). The question of how to represent a multitude of such places is another matter. To assume that they can be simply represented in a database would be a mistake; a database cannot capture the complex nature of how toponyms behave.

In addition to the dilemma of multiple places with the same name, there is the problem of changes to names and regions. Over time a place may have more than one name, the region covered by the name may shrink or increase and in extreme cases the region may be relocated entirely (Smith & Crane, 2001). These aspects make it difficult to capture
the essence of the relationship between names, locations and location name pairings in a database in a logical consistent manner, which is what is desired from such a system.

These problems combined pose a difficult question to anyone who seeks to design and build a program that attempts to evaluate, process and store toponyms and the places to which they refer. The question I shall consider is: how to represent place-names, the locations to which they refer and their behaviour with respect to each other and other toponym-location pairings in a logical, consistent manner once a program has distinguished between them and stored them for reference.

1.3 Scope and Objectives

In this work, I will examine the problem of creating a formal logic system that will adequately describe the complex relationships between geographical locations, the names that are given to them and changes that have occurred over time. This system will be developed using Modal Logic, a type of Formal Logic extended from classical predicate and propositional logic.

The logical framework that I will create will be a polymodal logic system developed by extending an existing suitable logical framework; I have chosen the framework created by Brandon Bennett, et al in “Multi-dimensional modal logic as a framework for spatio-temporal reasoning” (Bennett, Cohn, Wolter, & Zakharyaschev, 2002). This resulting logic will combine spatial and temporal modal models with a system of reference, designed with place-names in mind.

To do so, I shall first examine some of the several different views on how toponyms are connected to the locations to which they refer. These theories range from the view that place-names have both connotative and denotative aspects to the view that it can only be said that they may refer to a definite location. I shall discuss the merits of each in Chapter 2. Out of all of these theories discussed, I will be utilising Saul Kripke’s Causal Theory of Reference (Kripke, 1980).

Next I will investigate modal logic and its potential to resolve my problem. In particular, I will examine previous attempts to implement naming systems and how they have
failed to adequately capture the vagaries of the real world. Additionally I will discuss spa-
tial and temporal models for model logic that I can use as a basis for the final multi-modal
logic than I will produce.

I will then go onto formally define the desired characteristics of my logic based on
observations during my research and already existing spatial logic desiderata developed
by Oliver Lemon (Lemon & Pratt, 1999). After that I will then proceed to define the log-
ical system and its characteristics in depth. Finally, I will examine the logic and see how
closely it adheres to what I initially desired, as well as discussing the possible directions
of future research.
Chapter 2

Proper Names

2.1 Overview of Theories of Reference

Before I can create a system of logic that will adequately describe places, toponyms and the relationships between them, I have to be able to define what toponyms are. A toponym is a noun phrase that refers to a definite location, as opposed to a type of location. Like all names that refer to definite objects, they can be considered a class of proper names.

Over the last century, the question of how proper names should be defined has been considered by many different philosophers and even today, it is still very much a subject of debate. There are two sides to the debate; those that believe that a proper name has both connotative and denotative aspects and those that believe that a proper name has only denotative aspects. Currently the latter is the more supported of the two sides, mainly due to the strong rebuttal of the dominant descriptivist theories published by Saul Kripke in the 1980s (Kripke, 1980).

In this chapter I will discuss the merits of the more prominent theories from both sides and choose a theory that best captures the nature of toponyms to be the basis for my logical system. This examination shall first prove the unsuitability of the descriptivist theories to be used as the basis for my system. Next I will examine Mill’s Direct Reference Theory and the value of its observations about toponyms. Finally, I will explain in detail Kripke’s Causal Theory of Reference as a template for our logic.
2.2 Mediated Theory of Reference

The first descriptivist views to be released were those of Gottlob Frege. They were published in the paper “On Sense and Reference” (Frege, 1948). In this paper, Frege argues that proper names not only have a denotative aspect, but that they have a connotative aspect as well, which he refers to as a sense. Frege’s theory is built upon Immanuel Kant’s seminal work, “The Critique of Pure Reason” (Kant, 1999). He utilises Kant’s theories about the difference between analytic and synthetic truths to reason about the nature of the relationship that the equality symbol conveys, which he uses as the foundation of his theory.

Frege assumes that the equality symbol describes the relationship between signs of objects; that the statements $a = a$ and $a = b$ have different cognitive values. This agrees with the fact that $a = a$ is something that, according to Kant, holds a priori and can be considered analytic while $a = b$ is an extension of knowledge that cannot be known a priori. Otherwise if the equality symbol is believed to describe a relationship between objects, “it would seem that $a = b$ could not differ from $a = a$ (i.e. provided that $a = b$ is true)” (Frege, 1948, p. 209), which would contradict Kant’s beliefs.

Following on from this, Frege states the difference between the signs $a$ and $b$ can then be considered to be a difference in the mode of presentation of that which is designated. Using this piece of information, Frege then defines proper names not just as referring to a definite object, but as having a sense in which the mode of presentation is contained as well. This separation between references and senses offers me lots of expressibility; it allows me to reason about objects, the proper names to which they refer and their mode of presentation individually or as a group.

However there are several issues with this theory. Firstly Frege states that a sense should be able to be “grasped by everybody who is sufficiently familiar with the language or totality of designations to which it belongs” (Frege, 1948, p. 210); sufficient understanding of any natural language can be difficult for a computer program to obtain. Secondly, Frege notes that people’s opinions on what the sense is may differ. As I am trying to describe the complex nature of place-names as concisely as possible, it is clear that this is not the direction that I want to take. Finally, as I will show in the next section, even some supporters of the descriptivist viewpoint have found some issues with it.
However this does not mean that Frege’s theory is irrelevant. Frege’s view that proper names have both connotative and denotative aspects forms the basis of the opposition to proponents of the denotative only viewpoint in the debate on proper names; it has been interpreted in several different ways, which I will discuss in the later sections of this chapter.

2.3 Frege-Russell Descriptivist Theory

Like Frege, Bertrand Russell does not believe in the view that proper names only have a denoting element. However, he also states that after examining Frege’s theory, Frege’s distinction between sense and reference does not satisfy him either (Russell, 1905). Instead he believes that a meaning is assigned to every proposition in which a denoting phrase occurs but that a denoting phrase has no meaning by itself.

Russell’s examination of Frege’s theory turns up several problems. Foremost among these is the issue of what occurs when a case is encountered where the denotation of a proper name appears to be absent. In the case of “the present King of France is bald” and “the present King of France is not bald”, both could be considered to be false, as there currently is no King of France (Russell, 1905, p. 485). To avoid the contradictions that are inherent in such cases, Frege defines a null class to which “the King of France” belongs. However Russell regards this procedure as artificial and states that it “does not give an exact analysis on the matter” (Russell, 1905, p. 484).

This undesired behaviour of Frege’s theory, led Russell to set out to define his own theory on proper names. This theory is defined in his paper “On Denoting” (Russell, 1905), where he pursues a more natural resolution of the anomaly through the detailed analysis of denoting elements. In his work, Russell sees proper names as shorthand abbreviations for descriptions in denoting phrases and distinguishes between primary and secondary occurrences of proper names, where a secondary occurrence is considered to be “one in which the phrase occurs in a proposition $p$ which is a mere constituent of the proposition we are considering and the substitution for the denoting phrase is to be effected in $p$, not in the whole proposition concerned” (Russell, 1905, p.489).
This separation allows him to resolve the artificiality that he encountered in Frege’s theory. It allows him to declare that the statement “the King of France is bald” is false and that the statement “the King of France is not bald” is true (Russell, 1905, 490). This is because in the former proposition, “the King of France” is a primary occurrence of the proper name, while in the later it can be considered a secondary occurrence. This allows Russell to define the latter as “It is false that there is an entity which is now King of France and is bald” (Russell, 1905, p.490).

While this theory purports to have resolved some of the weaknesses of Frege’s theory as I have already noted, it still has the weakness that when two people think of connotative aspects of a name, they can come up with two different solutions. For instance, to one person, Aristotle could be shorthand for the pupil of Plato, while to another he could be the teacher of Alexander the Great.

Additionally, Russell’s theory is often not used independently. Instead, Frege and Russell’s views on proper names are often referred to as one common view in contemporary debates on the subject despite their differences (Kripke, 1980; Wettstein, 1990). This line of thinking is referred to as Frege-Russell Descriptivism and it was considered the dominant theory of reference for the first half of the 20th Century, until Saul Kripke published “Naming and Necessity” (Kripke, 1980).

2.4 Direct Reference Theory

John Stuart Mill was one of the first modern philosophers to examine the way in which objects are referenced. Mill proposes the view that “Proper names are not connotative: they denote the individuals who are called by them; but they do not indicate or imply any attributes as belonging to those individuals” in a book entitled “A System of Logic, ratiocinative and inductive: being a connected view of the principles of evidence and the methods of scientific investigation” (Mill, 1846, p. 40). This is known as the Direct Reference Theory.

While this theory is not directly the theory that I will be using as a basis for my logic, it was the first to argue that toponyms only have denotative aspects and thus can be
considered the theory upon which all the other theories that argue against proper names having any connotative aspects are based. It is therefore a theory that should definitely not be overlooked.

This theory offers us a few key observations about place-names. It notes that it cannot assumed that a location’s name describes a place’s surroundings, “a town may have been named Dartmouth, because it is situated at the mouth of the Dart. But it is no part of the signification of the word ... Dartmouth, to be situated at the mouth of the Dart. If sand should choke up the mouth of the river, or an earthquake change its course, and remove it to a distance from the town, the name of the town would not necessarily be changed” (Mill, 1846, p.41). Alternatively, the name of the location itself can change, while the its surroundings stay fundamentally the same. For instance, the Irish national rugby and football teams’ stadium was originally known as Landsdowne Road as that is where it was situated. When this stadium was torn down and then rebuilt on the same site in 2010, it was renamed the Aviva Stadium, a name that is not connected with the local area at all.

In a general sense, these examples mean that while descriptive appropriateness is somewhat important in naming systems; it is not necessary for it to be part of such a system in order for that system to function correctly (Bean, 1980). Therefore no logical system can rely on the idea that toponyms actually describe their surroundings. .

The main arguments against using this theory as the primary basis of our logical system are twofold. Firstly, not long after Mill’s rejection of any connotative aspects of proper names was published, Frege and Russell’s views on place-names were released. The ideas contained within these papers, as I have already stated, were viewed as compelling arguments against Mill’s assertions and became the dominant theory during the first half of the last century, known as Frege-Russell Descriptivism. Secondly and most importantly another paper called, ”Naming and Necessity” was published by Saul Kripke (Kripke, 1980). Kripke’s paper extolled another theory which addressed the concerns that Frege, Russell and others had raised with Mill’s anti-descriptivist theory. Kripke’s paper managed to comprehensively refute the widely accepted Frege-Russell Descriptivist Theory and offered up a compelling alternative, the Causal Theory of Reference. This anti-descriptivist theory is the currently most widely accepted theory about how proper names should be defined.
2.5 Causal Theory of Reference

As I have just mentioned, the paper "Naming and Necessity", outlining Saul Kripke's Causal Theory of Reference was published in the 1980s (Kripke, 1980). Kripke's theory was developed from a series of lectures which he had given on the topic of proper names. It can be regarded as as an improvement upon Mill's anti-descriptivist theory. In his paper, Kripke begins by rejecting the Frege-Russell Descriptivist view. He then turns to the views of those like Searle who advocated a cluster of descriptions model (Searle, 1969) and states that they “abandoned the letter while retaining its spirit” of Frege and Russell's views (Kripke, 1980, p. 290). Instead of either of these theories, he proposes his own; that proper names are rigid designators.

He begins in the first of his lectures by noting that descriptivist theories, whether they advocate a single description or a cluster, and even some of the theories that deny that names have meanings at all, can be viewed as saying that the description of the name somehow determines its reference. These theories, as they are presented as theories of reference rather than meaning, suffer somewhat when presented with some of the problem cases at which Kripke looks. In particular, Kripke states that they cannot be used to easily analyse statements that contain a name that is not synonymous with a description, even if the resulting statement would be materially equivalent. With this in mind, he states that the singular description theories are not viable (Kripke, 1980).

Secondly Kripke examines the notions of knowledge known a priori that Kant proposed as it is important to his methodology; he states that just because something can be known a priori does not mean that it is necessary that is known a priori.

These distinctions feed into his description of proper names as rigid designators; that proper names designate the same object in all possible worlds in which that object exists, unlike descriptions that could potentially describe a different object in another world, but still be considered to be known a priori in this world. To use Kripke's example, “Someone other than the U.S. President in 1970 might have been the U.S. President in 1970 (e.g., Humphrey might have)” but “no one other than Nixon might have been Nixon” (Kripke, 1980, p. 293-294). A similar example in terms of toponyms could be said to be, Edinburgh could have been designated the capital of the United Kingdom instead of London, but no location could have been London other than London.
On first glance, those examples could seem to argue that Kripke believes proper names are unique, something that is definitely not the case in our world. Indeed if proper names were unique, there would be no need for me to devise this logic system. It is the very fact that proper names are not unique, which presents the problem I am trying to solve. Instead what Kripke is stating is that when referring to an object or in our case, a location, its name identifies it in all possible situations, even if in the world under consideration, it is not known by that name or the inhabitants speak an entirely different language (Kripke, 1980).

Following on from this, Kripke turns his focus towards refuting the cluster descriptivist theories in his second lecture. He begins by stating 6 theses that he believes to capture the properties of such theories:

1. To every name or designating expression “X” there corresponds a cluster of properties, namely the family of those properties φ such that A believes “φX”

2. One of the properties, or some conjointly are believed by A to pick out some individual uniquely.

3. If most, or a weighted most of φ’s are satisfied by one unique object y, then y is the referent of “X”.

4. If the vote yields no unique object, “X” does not refer.

5. The statement, “If X exists, then X has most of the φ’s” is known a priori by the speaker”.

6. The statement, “If X exists, then X has most of the φ’s” expresses a necessary truth (in the idiolect of the speaker)”.

This is followed by the condition on the satisfaction of the other theses, C “For any successful theory, the account most not be circular” (Kripke, 1980, p. 294).

Of these theses, I have already encountered (6) when discussing Kripke’s definition of proper names. As a single description of “X” does not express a necessary truth and can be false in a counterfactual situation, I cannot regard an extension of such a statement to be true. Furthermore, Kripke notes that theses (2)-(4) “have a large class of counter instances” and that in reality, “the fact that the referent coincides with that determined
by (2)-(4) is an ‘accident’, which we were in no condition to know a priori” (Kripke, 1980, p.296).

Examples of how thesis (2) fails are fairly straightforward; for instance, if I describe Saul Kripke as a philosopher and do not attribute anything else to him, there is no property of Saul Kripke that can be used to pick him out uniquely.

The fact that thesis (2) fails severely weakens the logic behind thesis (3). Even if, thesis (2) and (3) are true, the theory can still fail. Kripke gives a rather involved example of this; a fictional account where Gödel’s incompleteness theorem could have actually been developed by a man named “Schmidt” but was mistakenly attributed to Gödel. According to thesis (3), this would mean that when we talk about Gödel, we are “in fact always referring to Schmidt” (Kripke, 1980, p. 298). As Kripke states, this is simply not the case.

Finally thesis (4) can be shown to be false by the fact that even if all that you have are false beliefs about someone, so that none of the properties of a name actually refer to a unique object, you still are using that name to refer to someone (Kripke, 1980).

This leaves us with theses (1) and (5). Thesis (1) can be described as a “definition” and on its own does not constitute a theory of reference (Kripke, 1980). As for thesis (5), I can refer back to the statement already made by Kripke about the necessity of knowledge known a priori. The fact that something can be known a priori does not mean that it must be, as it is required for thesis (5) to be true.

This compelling critique of cluster descriptivist theories leads to the realisation that it is not how the speaker thinks he got the reference, but the chain of communication which passed the reference of the proper name from the object itself to the speaker that is relevant.

This realisation forms the basis of Kripke’s theory. The Causal Theory of Reference that Kripke proposes can be defined by the following: An initial baptism takes place where the object in question is given a name. This name is then passed from link to link of the causal chain. When received by a member of the chain, that member must intend to continue to use it with the same reference as the person that gave it to him did, if he is to pass it on to someone else. A failure to satisfy this condition breaks the chain and
describes how simple it is to go from a singular use of a name to many diverging uses. This failure to pass on the reference also explains the fact that proper names can over time become common names; for example ‘hill’ could once have been considered to be a proper name, but the fact that mounds are a common feature of the earth’s surface and the failure to pass on the reference to the original ‘hill’ led to it becoming a common name.

Out of all the theories of reference that I have examined, Kripke’s theory certainly holds the most appeal. It does not get hung up on the ambiguities of how a name is given its meaning and concisely describes how people receive and create references to objects. For that reason, this is the theory of reference that I will use as the basis of my naming model for my multi-modal logic system.
Chapter 3

Previous Work

3.1 Introduction to Modal Logic

Before I describe my naming model and how it interacts with the rest of my system in detail, an explanation of what modal logic is and an examination of previous work on the subject is necessary.

A modal logic is a system of formal logic that is extended from classical predicate and propositional logic systems. This extension is formed primarily by the inclusion of operators that express qualifying statements, which allow for reasoning about philosophical concepts. These expressions of modality allow those who utilise such logic systems to express not only the truth of propositions, but also to allow for qualifications and judgments about such statements. In this way the statement, ‘Pigs can fly’ can be expressed as false and at the same time, the qualifying statement, ‘It is not possible for pigs to fly’ can be expressed to be true.

The defining operators of such systems are ‘necessarily’ and ‘possibly’. They are generally considered to be the most basic of the modal operators. They are unary in nature, with possibly being represented by a ♦ and necessarily, a □.

These operators describe modalities of the truth; ♦φ can be defined as ‘it is a possibility that φ is true’, while □φ is defined as ‘it is necessary for φ to be true’, that φ cannot be false. The necessity operator can be considered to be the dual of the possibility operator; their relationship can be expressed as the following formula, □φ := ¬♦¬φ (Blackburn,
In general, modality operators work well in conjunction with the more classical operators such as: \( \neg \), \( \rightarrow \), \( \land \) and \( \lor \) and add much to a logical system’s expressability.

There are a couple of existing modal systems that deal with names and I shall elaborate on their shortcomings in the following section, before explaining in detail the spatio-temporal logic model that I intend to use as the basis for my logic.

### 3.2 Logical Naming Systems

There have been several previous attempts to implement a general purpose naming system in modal logic. The more prominent of these naming systems were both published around the same time, “Nominal Tense Logic” by Patrick Blackburn and “Modal Logic with Names” by George Gargov and Valentine Goranko (Blackburn, 1993; Gargov & Goranko, 1993). These approaches are somewhat similar; they both define an additional set of variables that represent names that are constrained to be true at exactly one point in their model.

Patrick Blackburn’s paper can be considered the simpler of the two. In his paper, he introduces the concept of nominals, a new type of atomic variable to Priorean Tense Logic. These nominals can be combined with ordinary propositional variables to create formulas. Nominals differ from these ordinary variables because they are constrained to be true at precisely one point in any frame; the instant in which they are true is considered to be the instance at which it ‘names’ (Blackburn, 1993).

The language itself is defined in terms of fairly standard frames and models. Like the propositional temporal logic that I will discuss in Section 3.4, Blackburn utilises a frame that consists of a pair \( \langle T, < \rangle \) where \( T \) is a non-empty set of points of time and \( < \) is the precedence relation. This is then used as part of the model \( M = \langle T, V \rangle \) where \( T \) is a named frame and \( V \) is a valuation on \( T \). \( V \) provides a mapping of atomic variables to \( \text{Pow}(T) \) with the restriction that nominals can only be assigned singleton subsets of \( T \).

Gargov and Goranko take a similar approach. However unlike Blackburn, they intro-
duce the concept of the universal modality operator $\blacksquare$. In their language, they define a named model as a quadruple $< W, R, V, \chi >$ where $< W, R, V >$ is a classic logic model and $\chi : C \rightarrow W$ is a valuation of the names in the countable set $C = c_0, c_1, \ldots$.

Both of these languages are insufficient. As they constrain names to be true at one point only, they cannot be used as part of logic I am creating. If they were to be used, they would not be able to represent the real life situation of multiple places being referred to by the same name. This limitation is at the core of both logics and it cannot be easily ignored or changed. Instead of trying to adapt either one of these or another naming logic as a basis for my system, I will attempt to form my own naming logic, on top of an existing spatio-temporal framework.

### 3.3 Spatial Logic

Numerous spatial logic systems have been developed. However the one in which I am interested is Region Connected Calculus developed by Randell et al. (Randell, Cui, & Cohn, 1992). This logic is based on interval logic and allows me to reason about spatial regions rather than just abstract points. This is something that is highly desirable in my system since I wish to reason about arbitrary locations that may consist of any number of points.

Region Connected Calculus or RCC is based upon “Clarke’s calculus of individuals based on “connection” and is expressed in the many sorted logic LLAMA” (Randell et al., 1992, p. 1). The complete calculus of RCC is capable of expressing the relationship between regions by utilising its many relations. All of the relations are formed from an initial basic relation $C(X, Y)$ that states region $X$ is connected to region $Y$.

Unfortunately RCC is considered to be too expressive to be computationally decidable, which would cause complications with implementing my logic in a program, if I chose RCC as my spatial model. Fortunately however, there exists a fragment of RCC known as RCC-8 that is designed specifically for reasoning about spatial regions, which, unlike RCC, has been shown to be NP-complete (Wolter & Zakharyaschev, 2000).

RCC-8 is a high level spatial logic that does not take into account the topological shapes of regions (Wolter & Zakharyaschev, 2000). It can used to reason about regions in both
two dimensional and three dimensional space. It is known as RCC-8 because it consists of the following 8 basic relations:

- $DC(X,Y)$: $X$ is disconnected from $Y$
- $EC(X,Y)$: $X$ is Externally Connected to $Y$
- $PO(X,Y)$: $X$ Partially Overlaps $Y$
- $EQ(X,Y)$: $X$ Equals $Y$
- $TPP(X,Y)$: $X$ is a Tangental Proper Part of $Y$
- $TPPi(X,Y)$: $Y$ is a Tangental Proper Part of $X$
- $NTPP(X,Y)$: $X$ is a Non-Tangental Proper Part of $Y$
- $NTPPi(X,Y)$: $Y$ is a Non-Tangental Proper Part of $X$

These predicates are enough to express the relationships that two regions can have with one another as they are “jointly exhaustive and pairwise disjoint” (Wolter & Zakharyaschev, 2000, p. 3). This means that given any two regions in topological space, their relationship can be described by one of the eight predicates. For instance, given the Republic of Ireland and the United Kingdom, I could write $EC(Republic\_of\_Ireland,UK)$, which describes the Republic of Ireland and the UK as sharing a border. On the other hand if I wanted to describe the relationship between the island of Ireland and the United Kingdom, I would state $PO(Ireland,UK)$, showing that they overlap.
RCC-8 on its own is not good enough for my purpose as it does not give any option of reasoning about the relationship between names and locations. On the other hand, RCC-8 is a great candidate to use as the spatial model for my logic; it is expressive enough for the job that it is needed to do, without being so complex that it is unviable for use in a computer program. These characteristics also make the logic a popular choice when considering spatial frameworks which I can extend to include temporal concepts as we I will show in the next section.

3.4 Multi-Modal Logic

Creating a multi-modal logic system is much more complicated than formulating a single modal model. Not only is it necessary to form the models of the characteristics, to be expressed and reasoned about, but the way in which these models interact with each other must also be dealt with. There are usually complications, particularly if the two models are not particularly compatible with one another. These complications have to be overcome, if the resulting multi-modal logic is going to be fit for purpose.

Fortunately, I do not have to solve the problem of trying to combine RCC-8 with a temporal model to obtain my desired characteristics; instead there are several existing solutions that I can adapt to suit my purpose (Gerevini & Nebel, 2002; Bennett et al., 2002; Wolter & Zakharyaschev, 2000). Of these, the one that I am most interested in is “Multi-Dimensional Modal Logic as a Framework for Spatio-Temporal Reasoning” by Brandon Bennett et al which presents both a points-based and interval-based model paired with RCC8 (Bennett et al., 2002).

Since RCC-8 was originally developed from interval-based logic, it seems only natural to combine it with interval-based temporal logic (Randell et al., 1992). Out of all of the interval-based approaches available, Bennet et al chose to utilise Allen’s interval calculus as their temporal model (Allen, 1981). The resulting polymodal logic is called ARCC-8.

Allen’s Interval Calculus introduces 13 basic relations to describe the interactions between two time intervals. These temporal relations are: Before\((i, j)\), Meets\((i, j)\), Overlaps\((i, j)\), During\((i, j)\), Starts\((i, j)\), Finishes\((i, j)\), as well as their inverses and Equal\((i, j)\) (Allen, 1981).
Bennett et al combine the two models to create the logic ARCC-8. ARCC-8 is formed by adding a $HOLDS(R, i)$ clause introduced in one of Allen’s later works (Allen, 1984). This predicate is defined as ‘relation $R$ holds during some time interval $i$’. This clause is introduced to ARCC-8 formulas as $HOLDS(\varphi, i)$, where $\varphi$ is a formula from RCC-8. This version of $HOLDS$ is used by Bennett et al in conjunction with temporal predicates from Allen’s Interval Calculus to form ARCC-8 formulas (Bennett et al., 2002). The resulting modal model allows for the creation of statements like the one given in their example:

\[
\begin{align*}
&\text{Meets}(i, j), \text{During}(i, k), \text{During}(j, k), \\
&HOLDS(\text{TPP}(\text{Hong Kong, UK}) \\
&\land \text{EC}(\text{Hong Kong, China}, i), \\
&HOLDS(\text{DC}(\text{Hong Kong, UK}), j), \\
&HOLDS(\text{EC}(\text{UK, China}) \lor \text{DC}(\text{UK, China}), k) \\
&\models HOLDS(\text{EC}(\text{UK, China}), i)
\end{align*}
\]

The overall benefit of their approach is that it is a sufficiently high-level framework to allow me the freedom to choose how to model how intervals behave. However for simplicity of presentation, Bennett et al chose to represent intervals as “arbitrary non-empty subsets of time points of an arbitrary time flow” (Bennett et al., 2002, p. 247) and gave the following definition for the resulting model:

**Definition:** An it-model (interval topological model) is the triple $(M = F, T, a)$, where: $F = (W, <)$ is a strict linear order (modelling the intended flow of time); $T = (U, I)$ is a topological space; and assignment $a$ associates with every interval variable $i$ a non-empty convex subset of $W$ and with every region variable $X$ and every moment of time $u$ it associates a regular closed set $a(X, u)$ in $T$.

However, while I am interested in the interval-based RCC-8 for its focus on regions rather than individual points, the same cannot be said for temporal models. Instead of reasoning about intervals during which properties hold, I am more interested in reasoning about moments of change, points in time where the relationship between regions
has changed. For that reason, Bennett et al’s point-based approach has a certain appeal (Bennett et al., 2002).

In their points-based approach they utilise propositional temporal logic PTL. This logic is defined as being interpreted on the flow of time \( \langle N, < \rangle \) and having the temporal operators \( S(\text{Since}) \) and \( U(\text{Until}) \). They then define temporal versions of the standard modal operators to reason about the future:

- \( \diamond \varphi \equiv_{\text{def}} \varphi U \varphi \) (at the next moment \( \varphi \))
- \( \Diamond^+ \varphi \equiv_{\text{def}} (p \lor \neg p) U \varphi \) (at sometime in the future \( \varphi \))
- \( \Box^+ \varphi \equiv_{\text{def}} \neg \Diamond^- \neg \varphi \) (always in the future \( \varphi \))

and then state that similar definitions can be given for the past temporal modal operators \( \diamond^- \), \( \Diamond^- \) and \( \Box^- \). The initial formal definition for this model is then given as:

**Definition:** A topological temporal model (or tt-model, for short) based on a topological space \( T = (U, I) \) is a triple of the form \( M = (T, N, a) \), where \( a \), an assignment in \( T \), associates with every region variable \( X \) and every moment of time \( n \in N \) a non-empty regular closed subset of \( U \). For each \( n \), we take \( a_n \) to be the function defined by \( a_n(X) = a(X, n) \).

Following on from this they create the spatio-temporal language \( ST_0 \) by allowing the operators of PTL to be applied to the spatial formulas of RCC-8. This resulting language \( ST_0 \) is “expressive enough to constrain movements to be continuous, in so far as one can describe possible continuous transitions among the RCC-8 relations that can hold between two regions” and allows me to express changes in spatial relationships that can happen over time (Bennett et al., 2002). This is formally defined as a change to the tt-model:

**Definition:** For a tt-model \( M = (T, N, a) \), an \( ST_0 \)-formula \( \varphi \), and \( n \in N \), define the truth-relation \( (M, n) \models \varphi \), meaning \( \varphi \) holds in \( M \) at moment \( n \), by induction on the construction of \( \varphi \):

- if \( \varphi \) contains no temporal operators, then \( (M, n) \models \varphi \) iff \( T \models^{a_n} \varphi \);
\( (M,n) \models \varphi U \psi \) iff there is \( k > n \) such that \((M,k) \models \psi \) and \((M,l) \models \varphi \) for every \( l \) such that \( n < l < k \);

\( (M,n) \models \varphi S \psi \) iff there is \( k < n \) such that \((M,k) \models \psi \) and \((M,l) \models \varphi \) for every \( l \) such that \( k < l < n \).

However, as noted by Bennett et al, as! ST_0 only allows the temporal operators to be applied to the spatial formulas of RCC-8 as a whole, it can only describe relations between regions that exist at the same point in time, e.g. I can state that \( \diamond^+ DC(Scotland, UK) \) (‘at some point in the future, it is possible that Scotland will not be part of the United Kingdom’) but not that the United Kingdom may shrink in size in the future. This oversight is corrected by the next iteration of their language,! ST_1 (Bennett et al., 2002).

\( ST_1 \) improves upon the expressability of \( ST_0 \) by allowing the next-time operator \( \circ \) to be applied not only to spatial relations but to region variables as well. \( ST_1 \) also changes the arguments of the RCC-8 predicates to be region terms, region variables that are prefixed by an arbitrarily long sequence of \( \circ \) operators. This is simplified by Bennett et al to define \( ST_1 \) as \( ST_0 \) extended by the following clause:

\[
a(\circ X, n) = a(X, n + 1)
\]

This extension allows us to express changes that can occur to individual regions over time. Bennett et al notes that the inherent restriction of only being able to compare regions over limited periods of time can be avoided by the use of auxiliary variables that are constrained to be constant over time. This leads to me being able to express formulas of the form:

\[
\Box^+ EQ(X, \circ X) \land \diamond^+ EQ(X, EU) \land P(Russia, X)
\]

where

\[
P(X,Y) = NTPP(X,Y) \lor TPP(X,Y)
\]

“which is satisfiable if ‘some day in the future the present territory of Russia will be part of the EU.’” (Bennett et al., 2002, p. 244).

Of course I would prefer that all the temporal modal operators could be applied to
region variables, so that I could reduce the complexity and artificiality inherent in the construction of such statements. Additionally the ability to construct regions like $\Diamond^{+}X$ that contains all the points that belong to $X$ at some point in the future or $\Box^{+}X$, encompassing all the points that will belong to all future states of $X$ is desirable (Bennett et al., 2002).

To this end, the examination of a further extension of Bennett et al’s spatio-temporal framework is necessary. This next iteration, $ST_2$, can be defined as the addition of the following clauses to $ST_1$:

\begin{itemize}
  \item $a(\Box^{+}X, n) = CI(\cap_{k>n} a(X, k))$
  \item $a(\Diamond^{+}X, n) = CI(\cup_{k>n} a(X, k))$
\end{itemize}

These additions allow me to now prefix the temporal modal operators $\Diamond^{+}$ and $\text{square}^{+}$ to region variables alongside the temporal modal operator $\circ$. Additional clauses can then be added to allow for the prefix of past temporal operators as well. This allows me to form a similar statement to that of $ST_1$:

$$P(\text{Russia}, \Diamond^{+}EU)$$

which states that “the present territory of Russia will belong to the EU at some time in the future” (Bennett et al., 2002).

Bennett et al then make one final modification to all of their languages, $ST_i$, for $i = 0, 1, 2$, that allows applications of the Boolean operations to region terms. This resulting family of logic is then referred to as $ST_i^{+}$ (Bennett et al., 2002).

Of the spatio-temporal languages that I have examined, $ST_2^{+}$ is the candidate that I have chosen for the basis of my Multi-Modal Logic as it already fulfills many of the criteria that I have set out for my logic; for instance like RCC-8 it can be shown to be decidable (Bennett et al., 2002). However, like many of the other state-of-the-art modal logics that I have looked at, the $ST_i^{+}$ family of logic as a whole still assumes that regions about which I am talking all have unique toponyms. As I have already noted, this is simply not the case in the real world situations that my logic system may encounter. Therefore I need to extend $ST_2^{+}$ further to include concepts from Kripke’s Causal Theory of Reference in order to be able to properly describe these situations.
Chapter 4

Modal Logic and Place-Names

4.1 Overview

In this chapter, I will formally define my multi-modal logic that which will be successful in describing the complex relationships between geographical locations, the names that are given to them and changes that have occurred over time.

In order to provide a way to determine whether this goal has been reached, I will first define the desired characteristics that I am are aiming to achieve. These characteristics can be considered to be the blueprints of my logic and a guide to those who desire to construct similar systems.

The first stage of my definition will consist of the formulation of a modal naming logic that adheres to Saul Kripke’s Causal Theory of Reference (Kripke, 1980). This logic shall attempt to encode the theory of reference and the definition of proper names that he proposed in “Naming and Necessity” into the $ST_2^+$ spatio-temporal framework.

I will then proceed to make adjustments to the resulting multi-modal logic system so that the concepts of the causal theory of reference that I have introduced to $ST_2^+$ are well integrated. This will ensure that the resulting logic will remain consistent once the models have been combined.

I will then make a few final adjustments to ensure that my naming model properly incorporates all aspects of the Causal Theory of Reference. This will be followed up by
an analysis of how closely the defined logic matches my desired characteristics.

4.2 Desiderata

For my logic to be considered adequate for reasoning about and representing named topological regions it must meet several desiderata. These desired characteristics represent an ideal logic for the task and allow me to measure the success of my logic by examining how well the desiderata have been achieved.

My desiderata are refined from a proposed set of characteristics put forward by Oliver Lemon and Ian Pratt to aid the construction of logic for computational geography (Lemon & Pratt, 1999). Lemon and Pratt’s desiderata can considered to be a solid base from which to work, although they are focused on purely computational spatial logic. For my logic to be considered adequate for reasoning about and representing named topological regions over time, it must meet the following desiderata:

1. consistency
2. completeness
3. descriptive adequacy
4. decidability
5. tractability

As noted by Lemon and Pratt, the first desiderata should be fairly self-explanatory; that my logic must not be capable of making contradictory sentences true simultaneously. My logic must be capable of validly expressing the various complexities that it may encounter, such as different regions that have the same name consistently, otherwise it will not be any better than the previous systems that I have already described in chapter 3.

The second of the desiderata, completeness is where my interests begin to diverge. Completeness is described by Lemon and Pratt as the notion of a spatial logic having all its consistent statements considered to be geographically possible situations in euclidean space (Lemon & Pratt, 1999). While this is an important goal for any spatial logic to
have, it is not primary focus and I am satisfied if my logic expresses the minimal spatial concepts that they describe. Instead I shall define it to represent a similar notion, that all consistent statements of the logic can describe any topological reference situation that is possible according to the Causal Theory of Reference and my observations.

The third characteristic, descriptive adequacy, requires that the logic is ““up to the job” as far as cartographers and practitioners of GIS are concerned” (Lemon & Pratt, 1999, p. 77). For Lemon and Pratt that means that the logic must be able to at least express the following concepts of space:

- position: $x$ is at $y$
- connection: $x$ touches/contacts/meets $y$
- incidence $x$ is on/in $y$

They then state that further spatial characteristics that may be desired, such as direction, distance, betweeness and proximity can be addressed after these initial characteristics are formalised (Lemon & Pratt, 1999). For my purposes, the minimum concepts that Lemon and Pratt have identified are enough.

This notion of descriptive adequacy is then further extended to contain both temporal and causal theory of reference concepts. These criteria are necessary to achieve my aim of formally defining a logic not only capable of expressing relationships between geographical locations at one moment, but the changes that can occur to these locations over time and the relationship between a location and its name. To that end, I define the temporal concepts that I wish to express as:

- discrete temporal points: $x$ is true at moment $y$
- comparison between different temporal points: region $x$ at moment $y$ is different than region $x$ at moment $z$

These concepts allow me to reason about the spatial changes that have occurred in the past, what the relationship between regions is at the moment and what could or will change in the future.
The final part of descriptive adequacy are the concepts of naming that I wish to formally describe in my logic. When I am discussing regions in logic I need to be able to state that a name is being used to refer to a particular region, that we are using London to refer to a region in Canada rather than the United Kingdom. It is also desirable that I define a notion of naming or re-naming a region, so that I will be able to reason about events like the continent of America being given its name as well. Saul Kripke’s Causal Theory of Reference additionally proposes that after an initial baptism, the reference that the name makes is passed from person to person in a causal chain. If I take all these concepts together, then in order to have achieved descriptive adequacy, I need to have modelled the following concepts (in addition to the ones already named):

- reference: region x is referred to by toponym y
- baptism: region x is given the toponym y
- causal chain: the links between baptism and reference

Decidability and tractability, the final two desiderata, are important aspects to consider when designing a logical system that could potentially be used by a computer. Decidability can be described as whether or not there exists an effective method for determining whether an arbitrary formula is true or false in our logic. That my logic be decidable is important because when this concept is translated to computer science, as if the resulting program is decidable, then I can be assured that it will eventually return a true or false value and not be stuck in an endless loop. This is related to, but not the same as a tractable program, which can be described as a problem that can be solved in polynomial time (Garey & Johnson, 1979).

This is an extremely desirable characteristic for scalable computer programs. An intractable program, which provides a solution in exponential time, if at all, can easily exceed a human life span when working on larger problem sets. If we put these two characteristics together, we can see that for my logic to be useful in a computer program, it must be decidable in polynomial time.

4.3 Formal Definition

My multi-modal logic is an extension of $ST_2^+$ with the concepts of the causal theory of reference. To begin the formal definition of this logic, I will first define a modal model that
will extend $ST_2^+$ with the concepts of the causal theory of reference. This initial modal model shall have an additional set of ontological primitives, toponyms. This set of names can be used to denote specific objects from the object set. This initial model can then be defined as the following:

**Definition:** A temporal topological reference model (or ttr-model, for short) is extended from a tt-model $(T, N, a)$ and takes the form of a triple, $M = (W, N, a)$, where $W$ represents a world consisting of a topological space $T = (U, I)$ and a countable set of place-names $P = p_0, p_1, \ldots$ and $a$, an assignment in $W$, associates with every region variable $X$ and every moment of time $n \in N$ a non-empty regular closed subset of $U$ and zero or more toponyms from set $P$. For each $n$, we take $a_n$ to be the function defined by $a_n(X) = a(X, n)$.

To represent the relationship between a region and a name, I have implemented the predicate, $REF(X, Y)$ . This relation $REF(X, Y)$ is defined as ‘toponym X can be used to refer to region Y’. This signifies that there is a causal chain of reference that links the name and region back to an initial baptism. In the formal semantics of the language, $REF(X, Y)$ is treated exactly like the spatial relations from $ST_2^+$ and can be described as true at a moment $n$ if $a_n(Y)$ associates $X$ with $Y$.

This has several implications. Firstly it means that I can apply the temporal operators $\circ, \Diamond, \Box$, $\frown$, $\bowtie$ and $\square$ to the predicate. For example, $\Diamond \frown REF(Irish\_Free\_State, X) \land REF(Republic\_of\_Ireland, X)$, would mean that ‘the present territory of the Republic of Ireland was once known as the Irish Free State’. Secondly, it means that variable $Y$ in $REF(X, Y)$ should be considered to be a region term, which as I have already noted is a region variable that is prefixed by an arbitrarily long sequence of temporal modal operators. Thirdly and finally it means that boolean operators can be used in conjunction with region terms. This allows for the construction of more complicated sentences like, $REF(West\_Germany, X) \land REF(East\_Germany, Y) \land \Diamond \frown REF(Germany, \Box(X \land Y))$ which means that ‘at some point in the future, the permanent territories of East Germany and West Germany will be known simply as Germany’.

To alleviate the neccessity of explicitly using the $REF$ predicate, ordered pairs can be formed by combining toponyms and region terms that satisfy the reference predicate. It is important to note that an ordered pair is distinct from an unordered pair; for example,
given objects $A$, $B$, $C$ and $D$ that form ordered pairs $(A,B)$ and $(C,D)$, $(A,B) = (C,D)$ is only true if $A = C$ and $B = D$. In this case, the ordered pair of a toponym and a region term can be defined as a region tuple that takes the form:

$$(X,A)$$

where the first coordinate of the pair, $X$, denotes a toponym and the second coordinate of the pair, $A$, denotes a region term, which satisfies $REF(X,A)$.

The next step is to redefine the spatial formulas, $DC(X,Y)$, $EQ(X,Y)$, $PO(X,Y)$, $TPP(X,Y)$, $EC(X,Y)$, $NTPP(X,Y)$, $TPPi(X,Y)$ and $NTPPi(X,Y)$. Instead of just allowing region terms, they are extended to allow the use of region tuples in their place. When the spatial relations are applied to region tuples, they are applied only to the second coordinate, the region term. This can be formally expressed by the relationship:

$$REF(X,A) \land REF(Y,B) \land EC(A,B) = EC((X,A),(Y,B))$$

This allows me to refine my original examples, for instance I can now define ‘at some point in the future, the permanent territories of East Germany and West Germany will be known simply as Germany’ as $\Diamond^+ EQ((West\_Germany,\Box^+ X) \land (East\_Germany,\Box^+ Y), (Germany,\Diamond^+ Z))$.

However this does not fully capture the whole picture of the Causal Theory of Reference. While the predicate $REF(X,Y)$ being resolved as true does imply that there is a causal chain of reference that creates a link between an initial baptism of region term $Y$ by the toponym $X$ and its usage in the formulae of my logic (otherwise how would anyone know what the region is called), it does not offer a way to express that initial baptism directly, although one can be construct using a combination of $REF(X,Y)$ predicates.

To resolve this issue, I decided to introduce a baptism predicate, $BPT(X,Y)$ which can be defined as region term $Y$ is baptised with the name $X$ and can be represented in terms of $REF(X,Y)$ as:

$$BPT(X,Y) \equiv \neg o^- REF(X,Y) \land REF(X,Y)$$

This allows my logic to represent the discovery and naming of new regions, for instance
the discovery and naming of America can represented by \( EC((Pacific\_Ocean, X), Y) \land EC(Y, (Atlantic\_Ocean, Z)) \land BPT(America, Y) \). The addition of the baptism predicate also allows me to express moments in which a place is renamed, for instance the Irish Free State was renamed to the Republic of Ireland and never refered to as the Irish Free State again can be expressed as, \( BPT(Republic\_of\_Ireland, X) \land \neg \square^+ REF(Irish\_Free\_State, X) \).

### 4.4 Satisfaction of Desiderata

My multi-modal logic satisfies several of the desiderata that I defined as characteristics that are necessary for my logic to be considered adequate.

The first desiderata, consistency, can be seen to be a characteristic of my logic by inspection. Names can refer to no regions, one region or multiple regions without running into consistency problems or using artificial concepts. If a name does not refer to a region, it cannot be simultaneously said to refer to that same region at that moment in time and a name cannot be considered to belong to a region without being baptised in some fashion.

The completeness characteristic is perhaps the most important of the desiderata that I have defined. Achieving this characteristic can be considered to be the main goal of this dissertation. I believe that my multi-modal logic is complete. As I have already shown, it can describe the process by which a region is named and referenced, as well as a region begin able to have multiple names over time. Additionally my logic allows me to describe the relationship between regions that have the same name; for example I can state that ‘the former Soviet Republic of Georgia and the State of Georgia in the U.S.A. have no points in common’, \( DC(((Georgia, X), (Georgia, Y)) \land NTPP((Georgia, X), (USA, Z)) \land \diamond\neg \topp((Georgia, Y), (Russia, W)) \). From the basic concepts that I have introduced in my logic, more complex formulae can be constructed to represent all possible circumstances that can occur when reasoning about the naming and reference of topological regions.

Descriptive adequacy is the third characteristic and can be considered to be fully satisfied. Out of the spatial characteristics, all are covered by the RCC-8 logic that I use as my spatial model in my multi-modal logic. RCC-8 explicitly represents Lemon and Pratt’s concepts of connection and incidence with its 8 relations and implicitly represents position due to the fact of its regions are sets of points. If a stronger representation of position is
required the model can easily be extended.

The criteria for my logic to be considered temporally adequate are fulfilled by $ST^+_2$. The temporal aspects that it adds to RCC-8 allows my logic to be used to reason about specific moments in time, as well as make comparisons between specific moments in time. Additionally it permits statements about arbitrary points in time to be made.

The final part of descriptive adequacy is the question of whether the concepts of naming that I introduced in my logic are adequate to represent the defining concepts of the Causal Theory of Reference that I have identified. The first concept, reference, is adequately described by the reference predicate $REF(X,Y)$; it captures the notion that a reference just denotes a define object and does not allow for any conotative aspects, which is what Kripke argued in favour of. Baptism, the second naming characteristic is explicitly described by the baptism predicate $BPT(X,Y)$ and allows for new names to associated with regions. The only question then is whether my logic adequately represents the the causal links between baptism and the reference being recived by the logic.

The final two desiderata, decidability and tractability, are more complex. As I have already noted, when I initially defined the multi-modal logic $ST^+_2$, $ST^+_2$ can be shown to be decidable in EXPSPACE (Bennett et al., 2002). As my logic is based upon $ST^+_2$, it is as least as complex, but further research is required to prove exactly how complex it really is.
Chapter 5

Conclusions

5.1 Future Work

There are several directions that could be taken by those that seek to further my work. As well as the further examination of the complexity of my logic, there is also the option to take the logic that I have defined and utilise it in a toponym resolution system, a program that uses a toponym resolution system as the basis of a more involved system or even a general spatial program.

For instance, it could be implemented in a system that attempts to identify inconvenient and potentially life threatening events, such as motorway pile-ups, civic unrest, mayhem caused by roadworks or traffic jams as they are reported on Twitter or other instantaneous social network sites. Using my logic, the system could reason about the danger of the event and alert people who may be unaware of what has happened in the surrounding areas. Further utility could then be gained by analysing secondary reports to quantify the level of danger faced by specific locations in the general area.

Alternatively, there is the option to further expand the logic that I have defined. There are several areas that could be considered for extension; in particular one could expand the spatial model. One must bear in mind however, the trade-offs between expressability and decidability/tractability as the complexity of the logic increases.

Although my logic achieved the minimum requirements for being adequate to describe spatial relationship, some expanding of the logic may be considered desirable. In partic-
ular, the addition of explicit concepts of distance could be added. This modification has already been performed on the basic RCC-8 logic and would allow for reasoning about the distance between regions (Sridhar, Cohn, & Hogg, 2011). This addition would expand my logic’s usefulness in topological systems by allowing for more detailed examination of the relationship between unconnected regions.

5.2 Conclusion

Overall I believe I have achieved my goal of creating a formal logic system that will adequately describe the complex relationships between geographical locations, the names that are given to them and changes that have occurred over time.

My logical framework is extended from $ST_2^+$ and combines this spatio-temporal modal logic with a system of reference, that I consider to be descriptively adequate for use in topological resolution systems.

While further research is required to examine the complexity of my logic, I feel that my logic solves the inadequacies of previous logics and allows me to represent any relationship between toponyms and locations that can occur.
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