Graphic Interfaces with Semiotic Mediation for Learning Algebra

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A dissertation submitted to the University of Dublin,
In partial fulfilment of the requirements for the degree of
Master of Science in I.T. in Education

2007

Declaration

I declare that the work described in this dissertation is, except where otherwise stated, entirely my own work and has not been submitted as an exercise for a degree at this or any other university.

Signed ____

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14th May 2007

Acknowledgements

The author wishes to express thanks to a number of people who assisted in many ways in the completion of this dissertation. Thanks are due to the students of Columba College who were very cooperative participants in this study. Many thanks to my colleagues in the school, the secretary and particularly the deputy principal who all took on an extra workload over the past two years to enable the author to complete this project.

Thanks to my mother, my husband and family for their support and encouragement during the past two years.

Thanks are due to Brendan Tangney for his inspiration, advice and guidance throughout the project. Thanks to all the members of the College Staff who offered guidance, help and support during the last two years. Sincere gratitude is due to all fellow students in MITE05 for their friendship and support.

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Abstract

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Mathematics is considered to be difficult to learn and difficult to teach and has traditionally been taught by developing a set of skills and procedures which are then applied to solve problems (Wood 1988). Learning mathematics is difficult for many but algebra seems to present a particular problem because of its abstract nature.

Many researchers have examined the learning of mathematics and algebra but Bruner's(1967) Theory of Instruction presents a model which he used to teach quadratic functions and which was adopted for use with ICT in this study. The research was also informed by the work of Mariotti (2002) who advocates the use of an artefact as a channel of communication between student and artefact and student and teacher.

In this research using graphical interfaces, to replicate Bruner's model enabled the user to move from the enactive to iconic to symbolic representations. This artefact was not designed for independent learning but to be used with the teacher's intervention for the purpose of abstracting and developing concepts. The study was implemented in a second level school with five 12/13 year old students.

Qualitative research was the most suitable in this context and the case study design was adopted. The testing sessions were conducted with individuals to enable video recording of participants' interaction with the interfaces and of the dialogue between teacher and student. This evidence was supported by a post test and an online questionnaire which formed the basis for individual taped unstructured interviews. These instruments provided data which was analysed and triangulated to produce reliable findings.

The findings clearly indicate that the students learned some algebra in a very short time. The artefact helped them to overcome the common obstacle of closure in algebra. They were assisted in arriving at an understanding of the source and use of symbolic notation. Meaningful dialogue, which contributed to meaning making and abstraction of concepts, occurred between student and

teacher using the graphic interfaces as a tool of semiotic mediation. The experience gave the researcher insight into the need for the adaptation of pedagogy when integrating ICT into teaching and learning.

Chapter 1 Introduction

1.1 Background

Mathematics is a subject which children are usually introduced to in their own homes before ever attending school. The idea of number arises in everyday life when a child knows how many are in his family or how many sweets he has got and several other simple examples. The primary School Curriculum introduces mathematics from the very beginning and it is a core subject in all second level schools. Many students experience difficulties in learning maths and teachers find it difficult to teach. The aim of this study is to examine the effectiveness of using graphic interfaces to assist students in understanding mathematical concepts.

1.2 Literature Review

The difficulties with mathematics and more particularly with algebra have been acknowledged by many researchers who have explored the areas of teaching and learning maths and understanding mathematical concepts(Papert, 1993; Skemp, 1986; Wood, 1988). The mere verbal transmission of information is not considered sufficient and students interpretation is often not that intended by the teacher. Strategies which make the experience of learning maths and algebra more meaningful were explored and among them the use of technology.

The methods by which technology could be incorporated in teaching and learning was of particular interest in order to assist in the design of an artefact to teach maths. Tall and Thomas (1989; , 1991) carried out research into the use of ICT(Information and Communication Technology) in mathematics and it clearly recommends the benefit of another facility added to textbook and teacher for getting the maths from the mind of the teacher to the mind of the student. They described this method as the Enhanced Socratic Mode where the technology provides the enhancement. In the context of integrating technology into maths teaching and learning the role of the teacher is significant.

The role was examined as the teacher could no longer be considered to be instructing or transmitting information but was to be a party to teaching and learning which used an artefact as a teaching tool. Mariotti’s (2002) theory of semiotic mediation has a twofold aspect (a) the role of the artefact (b) the role of the teacher in assisting the student with meaning making while using the graphic interfaces. This theory informed the author when deciding on a teaching strategy to adopt when using the artefact with participants.

1.3 Research Questions

From the literature reviewed the model employed by Bruner using Dienes blocks to teach quadratic functions influenced the author to replicate this model using graphic interfaces. Bruner(1967) in his Theory of Instruction stated that any domain of knowledge can be represented in 3 ways : a set of actions (enactive representation); a set of summary pictures or graphics (iconic representation); a set of symbols which are governed by rules or laws(symbolic representation).

The research questions which the researcher examined with this model are:

How does an ICT replication of Bruner's Enactive, Iconic to Symbolic representations assist students' understanding of mathematical concepts?

Can the researcher get insights into Marotti's two-fold view of semiotic mediation regarding

The role of the artefact?

The role of the teacher?

1.4 Design of the Artefact

The artefact was designed in Multimedia Flash which was embedded in a website. Users were in control and the interfaces provided the user with shapes for construction. From the construction of squares and rectangles symbolic notation was developed and explored. The store of images created enabled dialogue to occur between student and teacher which facilitated making meaning and generalising from the patterns observed. The artefact was designed to enable learners to construct knowledge actively and develop understanding with the assistance of the teacher.

1.5 Implementation

The implementation occurred in a small rural second level school in Ireland. The students were 12/13 year olds and they were selected from a first year mixed ability group. There were 2 boys and 3 girls. The researcher, a teacher in the school, though not of this group, carried out the research with each student individually over a five week period. Students participated in two or three sessions depending on their pace of learning.

1.6 Methodology

The nature of the research was qualitative and the case study was the design adopted. Data was collected primarily on video tape during each session with the student in order to gain insights into the level of interaction between student and artefact, the role of the artefact as a tool for semiotic mediation between teacher and student and the role of the teacher. A post test questionnaire was completed at the end of the sessions with each student. These provided a basis for individual taped interviews. Post test questions provided the only element of quantitative data which showed what actual learning, if any, occurred during the research. These sources of data were used to triangulate the evidence and thus provide reliability within the research.

1.7 Findings

The students were successful in learning algebra in a short time which was evident from their scores on the post test. The artefact enabled the participants to achieve clarity, from their constructions in distinguishing the concept of length and area and writing them appropriately in algebraic notation. The students, through their constructions, were provided with a store of images which they could refer to when explaining emerging patterns and generalizations. The artefact enabled the teacher to engage in dialogue with the student in order to clarify and make meaning at all stages of the learning process. The role of the teacher was different when teaching with this medium and necessitated the adoption of a different pedagogy to that which would be used when transmitting knowledge in a more traditional setting.

1.8 Conclusions

In a case such as this it is not possible to generalise because the sample was so small. The evidence suggests that this was a successful way to teach algebra and it assisted students in understanding the concepts but the reality in schools is that classes are taught as a unit. Further study could be done with this artefact to examine if it would be suitable for use in a whole class situation with an interactive whiteboard as a tool for mediation and collaboration.

Chapter 2 Literature Review

2.1 Overview

This study aims to examine the effectiveness of the integration of ICT in teaching mathematics to assist student's understanding of concepts. The areas of relevant literature to be examined as a background to this study cover the philosophy of constructing knowledge, the difficulties of teaching and learning mathematics and in particular algebra and the formation of mathematical concepts. In order to inform the researcher in creating an effective artefact the impact of integrating ICT into the teaching and learning of maths and the role of the teacher in this process was explored.

2.2 Methods

A review of the literature was conducted from books, journal articles and conference proceedings papers sourced from the library of Trinity College, Dublin.

On-line journals and articles were sourced from the internet and keywords used were: learning, mathematics, algebra, technology, teaching, ICT, semiotic mediation.

2.3 The Constructivist Approach

Constructive practices are described by many as student centered teaching as distinct from the teacher dominated practices and instruction should be the process which supports construction rather than communicating knowledge. During the construction of knowledge learners are actively engaged in exploring, creating, recreating and interacting with the environment. They learn to build structures which lead to critical thinking and an ability to solve problems. This constructivist approach enables students to learn content and process at the same time(Malabar & Pountney, 2002; Marlowe & Page, 1998). These are the opinions of recent researchers but the notion of constructing knowledge has been well established by theorists such as (Bruner, 1967; Piaget, 1967; Vygotsky, 1978). Construction and discovery were at the core of Bruner's Theory of Instruction and Bruner and Piaget stress that procedures must be grounded in practical activities if they are to be meaningful. Vygotsky (1978)concurred with the constructivist theory of learning with the addition of social interaction for meaning making (Piaget, 1967).

Sfard (1991) considers that the major aim of teaching maths is to develop understanding and he suggests that the constructivist approach with teacher interaction enables learners to build understanding.

2.4 Difficulties with mathematics

In the case of mathematics there are a plethora of theories and opinions as to why difficulties exist. They can be divided into teaching issues and learning issues. Papert(1993) referred to the difficulty with learning maths as mathphobia which he explained as a widespread fear of mathematics. He claimed this belief in individuals was very deep and as individuals encountered failure when attempting a mathematical task the consequence was further reinforcement of fear. He argues that this type of experience prevents people from doing anything that they recognise as maths related

Wood (1988)accepted that mathematics was difficult to learn and difficult to teach. He argues that the difficulty arises as a result of traditional mathematics methodologies which are instructivist and where attention is given to drills and procedures while neglecting conceptual understanding. Skemp (1986) stated that training students to manipulate symbols and memorise a set of rules with little or no meaning attached was what was inflicted on students when learning mathematics. He argues that this unconnected material is hard to remember and boring. This approach involves rote learning of rules and procedures and Tall(1986) would hold that rote learning of facts may assist to build a foundation but meaningful learning of facts is necessary for flexible thinking which is needed in mathematics. He argues that this procedural thinking gives short term results but is likely to lead to failure in the long term. These short term results may well be what motivates teachers to pursue this approach as it may enable students to pass examinations at school but will it permit them to pursue maths at any higher level or to choose a career with a well developed mathematical requirements? Kaput(2000) argues that the procedural approach drives people away before thy have an opportunity to experience any construction of mathematical knowledge or to realise the importance of mathematics and its usefulness in their lives.

2.5 Difficulties with Algebra

This difficulty with mathematics becomes more apparent with algebra and Kaput(2000) refers to algebra as the topic that almost everyone, it seems, loves to hate. In relation to algebra he comments on how it has been taught and its disconnectedness from other mathematical knowledge and from the real experiences of the students.

Skemp(1986) referred to algebra as generalised arithmetic and he claims that to understand algebra one must first understand arithmetic and to understand algebra without first understanding arithmetic is impossible. This gap between arithmetic and algebraic concepts is very difficult to bridge. It is essential that when skills are acquired that they are understood as the acquisition of skills and knowledge does not guarantee understanding which is essential for the critical and creative use of that skill (Gardner, 1993; Perkins, 1993). Tall and Thomas (1991)explain how some of the difficulties arise for learners of algebra. There is a conflict between natural language and the symbolism of algebra. Firstly, reading is from left to right but in algebra following this may lead to wrong interpretation of an expression. Also students in arithmetic had an emphasis on arriving at an answer and that brought closure but in algebra they have difficulty understanding for example 3 +5x and they attempt to bring closure to such an expression by tidying up to one piece of notation, in this case 8x (Collis, 1972; Tall & Thomas, 1989). Mc Gregor and Stacey (1997) suggest that success or failure in algebra may be due to the way the subject is introduced, the teaching materials, teaching styles and the learning environment. Initial difficulties if they are not discovered and corrected may persist for years and lead to an inability to make sense of algebra. Hart (2004) argues from evidence of the CSMS(Concept in Secondary Maths and Science) programme in the UK that achievement in the area of algebra is closely linked to the IQ scores of the individuals and he emphasises that the progression is at a different rate as children grow older thus making it very difficult to teach algebra at the same pace to a whole class. Shayer and Adey(2002) too are concerned about the number of students who do not reach the level of formal operations which would enable them to understand algebra. Kaput(2000) claims the challenge is finding ways to make the power of algebra available to all students by creating a classroom environment which enables students to learn with understanding. To infuse algebra throughout the mathematics curriculum and to alternate algebra and arithmetic all through school is his recommendation.

2.6 Mathematical Concepts

The development of a concept is something which has occupied the minds of many researchers. Bruner's (1967) theory is that concepts are developed through three modes of representation the *enactive* which allows the students to manipulate objects – manipulating, the *iconic* images are created from the building of shapes, diagrams, pictures or graphs –depicting, the *symbolic* when symbols are used in an abstract way – denoting. Mason et al.(2005) extended Bruner's work to develop the spiral MGA which is to *manipulate, get a sense of and articulate*. Tall and Thomas (1991) also identified with Bruner's theory by labelling the stages as *enactive, visual and symbolic*. Individuals can mentally construct concepts, according to Skemp(1986) by:

1. interaction with the real world
2. interaction with other people
3. internal reflection on concepts and the relationship between them.

Later Tall and Thomas (1991) divided the interaction with the real world into the manipulation of inanimate objects on one hand and cybernetic systems(computer software) on the other.

A concept is an idea and for a concept to be formed it requires a number of experiences which have something in common and these experiences enable generalisation. Kieran(1997) tells us that generalising takes place in very young children before they ever go to school. When children count objects they become aware that the order in which they do this does not affect the result and then they have engaged in generalisation. Generalising is an essential element in the formation of mathematical concepts. According to Mason et al(2005) the power to generalise is the root of all algebra learning.

A particular difficulty with algebra is the concept of a variable. If during the introduction to algebra an object is assigned to an alphabetical symbol rather than a numerical value the result can be difficult to undo. Crowley et al(1994) acknowledged this difficulty when they described the short term strategy, often adopted by teachers, of the fruit salad approach. This arises when a teacher may explain 3a + 2b as representing 3 apples and 2 bananas which works in addition but creates problems when 3ab has to be explained. Crowley et al(1994) proceeded to develop a new idea of a procept which is a combination of process which enables students *to do* maths by following procedures, and a concept which enables students *to think and manipulate mentally*. A procept represents the process and the concept. This procept is common in algebra where two different symbol strings represent the process and the product of the process as in $2(x+3) = 2x+6$. They emphasise that the mental manipulation of procepts gives the learner great power (Crowley, Thomas, & Tall, 1994; Kramarski & Hirsch, 2003). Skemp (1986)suggested that the computer created a suitable environment for building and testing mathematical concepts.

2.7 Impact of ICT

Pea(1985) believes that computing should not just allow learners do traditional tasks more effectively but it should change the tasks we do so that the software provides the students with tools for supporting and reorganising their thinking. When computers were first introduced in schools they were usually said to carry out tasks which might have been traditionally done with pen and paper, or access to computers was as a reward for completing other tasks successfully (Pea, 1985). Somekh(1991) and others would be supportive of this idea of pea's and would be very critical of an approach where computers are used as an add on or bolt on instead of the computer as a cognitive tool to support cognition and meta cognitive processes.

Kaput (2000)states that the three aspects of electronic technology which impact on mathematics are:

1. interactivity
2. control by designers of the learning environment
3. connectivity

Thompson(1992) did a comparative study which supported this view and he comments on the difficulty that exists when using technology to inject meaning when students have already learned and automised procedures meaninglessly. Clements(2000) advocates a problem centered approach which is motivating for learners. Students are encouraged to explore ideas engaging knowledge they already have, to make decisions and then receive feedback and this experience is removed from drill and practice. It must be acknowledged that students may not always learn what the teacher intends and using techniques in the traditional methods learners may arrive at the correct answer but with little understanding. Students are less likely to be able to hide their lack of understanding in the computer environment (Clements, 2000; Laborde, 1993). There are commercial computer applications available for use in schools and Lagrange when commenting on Computer Algebra Systems (CAS) states that they have great potential to contribute and improve student learning in maths because of the speed of calculation and capabilities of presentation of representations but they may be lacking he considers in enhancing student reflection and understanding (Lagrange, 2005). Oldknow and Taylor(2003, p. 58) state that ICT has great potential to make a significant contribution to the teaching and learning of mathematics in secondary maths. They bring three key principles to our attention which should be applied before deciding to use ICT in teaching a subject or sections of a subject.

1. *Decisions about when, or when not, and how to use ICT in lessons should be based on whether the use of ICT supports good practice in teaching the subject. If it does not, it should not be used.*
2. *In planning and in teaching, decisions about when, when not, and how to use ICT in a particular lesson or sequence of lessons must be directly related to the teaching and learning objectives in hand.*
3. *The use of ICT should either allow the teacher or the pupil to achieve something that could not be achieved without it; or allow the teacher to teach or the pupils to learn something more effectively and efficiently than they could otherwise; or both.*

Practical considerations play a significant part when considering when and how to include ICT in teaching a subject. As a result of inclusion there is a diversity of learners in classrooms and teachers must challenge all students in their class regardless of their current level of performance. Using the computer in the class may enable the teacher to be differentiating instruction as part of her preparation (Thompson, 1992). A recent study in the UK MathsAlive which aimed to embed ICT in teaching and learning maths proved that the impact of ICT on both teachers and students was significant in the context of enjoyment and understanding of maths (Oldknow, 2005). This supports research on higher order thinking skills by Tall and Thomas (1991). Having taught and tested two groups of students, one experimental and the other a control group taught by traditional methods. Their findings in the post test did not display significant variation between the two groups in performance on tasks but when tested some six months later the experimental group scored significantly better and displayed superiority in the higher order thinking skills. John and Sutherland(2005) emphasise that there is nothing inherent in technology that generates learning but rather it is distributed between the technology, the learner, the context and the teacher (John & Sutherland, 2005) When Rutheven and Hennessy et al.(2004) carried out their studies with both teachers and pupils in Britain the teachers ‘ comments were largely to do with computers as sources of motivation for students, their speed, fostering independence in students and supporting trialling and correction. It was the pupils who while appreciating all of the benefits mentioned by the teachers emphasised the importance of a well planned task and the significance of the involvement of the teacher in the learning process.

2.8 The Role of the Teacher

It is argued that integrating technology fully to create valuable learning experiences for students requires a change of pedagogy on the part of teachers. Davis(1997) states that the quality of learning that students experience with computers depends on the opportunity provided for meaningful engagement and quality communication between teacher and student. This supports learners in what Vygotsky calls the zone of proximal development which is the gap that exists for the student in making sense of a topic but this gap can be overcome by the intervention of the teacher(Davis et al., 1997). Scrimshaw(1997) too speaks of the teacher as a stage manager of classroom activities where he directs the processes of learning rather than its product. He also mentions scaffolding the learner where the teacher would have a very specific goal in mind for the lesson and would be able to concentrate on bringing the student, by intervention, to get a sense of achievement (Scrimshaw, 1997). Listening only to a clear presentation of mathematical content from a teacher does not guarantee learning. Interaction is necessary and if this is allowed with the teacher and the computer the construction of meaningful mathematical knowledge will be fostered (Laborde, 1993). The common philosophy for teaching in the traditional system as described by Tall was the didactic triangle which consisted of the pupil, the text book and the teacher. The maths to be communicated was in the head of the teacher and the method employed was largely verbal apart from whatever diagrams or presentations placed on the board. With the introduction of the computer he described the didactic tetrahedron where the teacher has the facility of software which provides a dynamic representation while still maintaining the teacher, student and textbook. In this scenario the speed of action may be under the control of the user but mediation and negotiation is essential. Tall referred to this use of the computer as providing the Enhanced Socratic Mode of teaching as computers provide more freedom for the teacher to work with individual students in order that students may have a better opportunity to learn what the teacher intended.

Vygotsky's (1978, p. 53)perspective of semiotic mediation distinguishes between the function of mediation of *technical tools and psychological tools(or signs or tools of semiotic mediation)* Through the complex process of internalization, a tool may become a psychological tool and it may function as a tool of semiotic mediation. Marotti(2002; , 2001) further developed this theory and has looked at the teacher and the learner using the screen and the mathematics presented thereon as instruments of semiotic mediation. While working in the mathematical domain using l'Algebrista and Cabri she emphasises the clarity that must exist when designing a piece of software for use in a classroom as a tool of semiotic mediation. Firstly, there must be a clear mathematical notion as an objective and it must be possible to use this software as a tool of semiotic mediation which will enable meaning to emerge. Therefore the artefact has a double function as a semiotic mediator:

1. The learner uses the artefact for a certain activity and some meaning may emerge from that
2. The teacher uses it, through dialogue with the learner, to assist in the drawing out and construction of meaning.

In order to assess the usefulness of an artefact as such a tool Mariotti (2002) recommends that the dialectics between actions and constraints is crucial as to what is automatic, what is left to the user to control, what the environment controls and what is left without any control.

During the mediation process Schon(1988) argues that the teacher needs to be capable of dealing with the *surprise* which he terms as a reflection in action. This arises when a student may give a reply to a question which the teacher has not anticipated and therefore necessitates the teacher reflecting on the spot to make meaning for the learner. For mediation to be worthwhile, whether it is semiotic or otherwise Clements(2000) presents this analogy students motor is always running, our job as teachers is to build roads, place signs, direct traffic, teach good driving but not to drive the car.

2.9 Summary

This review of the literature has examined the difficulties which have existed in maths teaching and learning and it has informed the design of the artefact which will be used to examine the effectiveness of integrating technology into mathematics.

Chapter 3 Design of the Artefact

3.1 Overview

Bruner (1967) in his Theory of Instruction has outlined a structure and sequence of 3 stages of representation when dealing with a domain of knowledge. He applied this himself in an experiment which he carried out in 1966 and in this study these stages of representation are to be replicated using technology. The research question which the researcher wants to explore is:

How does an ICT replication of Bruner's Enactive, Iconic to Symbolic representations assist students' understanding of mathematical concepts?

Within that question further questions need to be addressed

1. *does the artefact help the learner moving between Bruner's levels of representation*
2. *does it provide insights into Marotti's two-fold view of semiotic mediation which covers the role of the artefact and the role of the teacher.*

3.2 Bruners' Model

Bruner (1967) carried out a study on 4 eight year old children with an IQ of 120 – 130 in 1966. The topic which he introduced to the children was quadratic functions. They were given an hour of instruction each day, 4 times a week, for a period of 6 weeks. Dienes blocks were used allowing the users to initially play with the blocks but then to build squares according to specific instructions. They used three types of blocks – squares measuring "x" by "x", squares measuring "1" by "1", rectangular blocks measuring "x" by "1". They were instructed to build squares with sides(x+1)(x+1), (x+2)(x+2) and in so doing to be able to calculate the amount of wood used. The theoretical framework which Bruner followed was his theory of instruction which is made up of 3 stages. The Enactive Representation or manipulative stage which was the constructing of squares which would allow the children form mental images of the shapes constructed. This was the Iconic Representation. During the building of various stages notation was introduced and the children were guided to see commonality in the examples worked through and this enabled them to reach the third stage which is Symbolic Representation(Bruner, 1967).

3.3 *The Artefact*

The artefact, being designed here is a simulation and adaptation of Bruner's work using the computer as the medium (http://www.cs.tcd.ie/~mcaulim/Quadratics). It was used as an introduction to the topic of quadratic functions for 12 – 13 year olds who are in 1st year in an Irish second level school. It was essential in the replication to be true to Bruner's original study and the table below displays the key elements of each model.

Table 1

Representations	**Bruner's Model**	**Artefact**
Enactive	Dienes Blocks used for physical manipulation.	Shapes manipulated on screen by user through drop and drag.
Iconic	Squares and alternative shapes created from blocks according to specific instructions.	Squares and shapes created on screen according to specific instructions.
Symbolic	The use of algebraic notation to represent the amount of wood used in the creation of the shape.	The use of notation throughout to calculate the length of each side or the area of the shape created.

This artefact is created within a website using "Macromedia Flash". It consists of 5 lessons. The first lesson begins with revision frames regarding the square and the rectangle which are the shapes to be used throughout the activities. The approach is very much one of constructing knowledge based on past experience and the interfaces enable students to start exploring by creating a new square from the squares and rectangles provided. Through this playful engagement students will reflect on what they are doing and then a question regarding the area of the square created will allow them to attempt answering. Feedback is provided when answers are filled in boxes and if incorrect further attempts may be made following dialogue between teacher and student about what part of the answer is wrong. Squares and rectangles are shapes that students are familiar with from childhood and most children will have played with these as blocks at that time. Mason et al.(2005) would make the point that any manipulation of familiar objects inspires confidence in the student and they will be encouraged to interact with them. The need to use notation was introduced throughout the lesson when writing down the total number of blocks used in the construction. At the end of the first lesson the students are requested to attempt 3 questions based on what they have just completed which was solely dealing with $(x+1)^2$ and $(x+2)^2$. Students receive immediate feedback on each question which John and Sutherland(2005) tells us is vital to support the construction of knowledge.

Lesson 2 commences with 2 screens of revision of work completed in Lesson 1 and then progresses to further activity based learning.

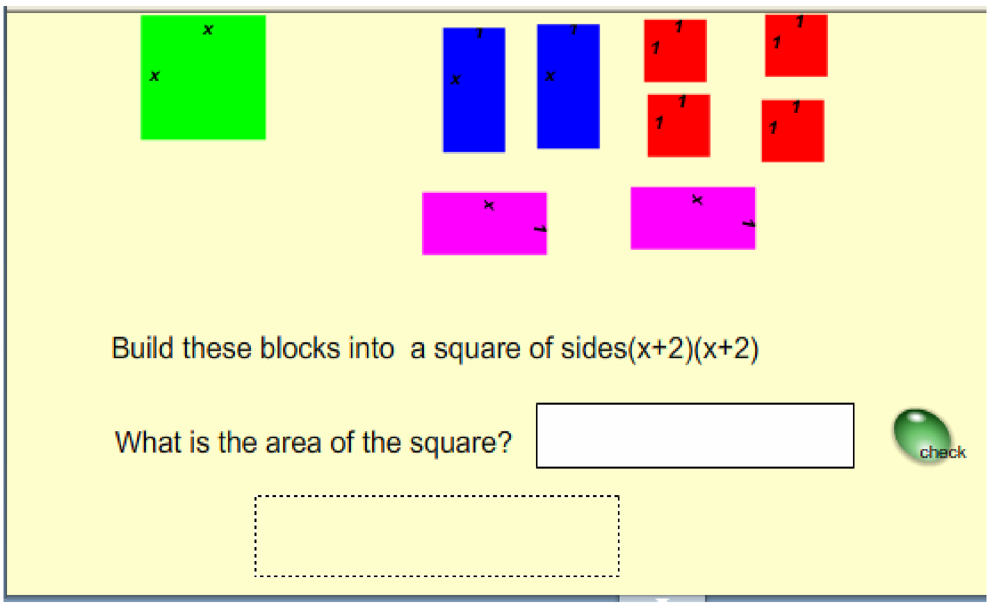


Figure 3.3.1

The student is again instructed to build up squares. Godwin and Sutherland (2004)were a little sceptical about activities of this type as it could lead to random playing but the inclusion of questions to be answered on each screen creates a clear focus for the student. As the students work through the enactive stage and through the process of manipulation of objects they should have arrived at the iconic stage. Images will have been formed from the four examples worked through in the two lessons. While notation is constantly being used a summary in notation format is presented in the middle of lesson 2.

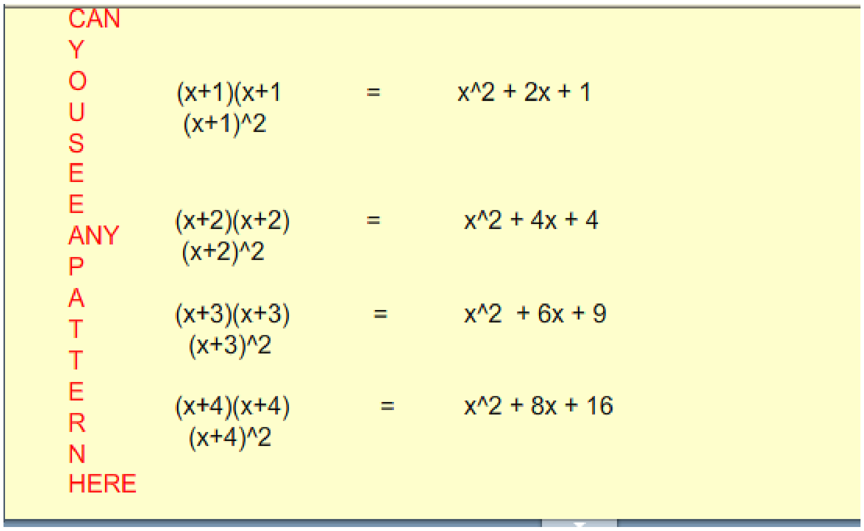
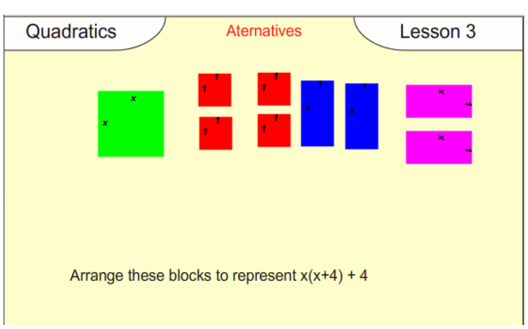


Figure 3.3.2

It is expected here that the student will see some pattern emerging and discover for himself/herself the rule that is emerging. Here students are looking for commonality and they may not be fully able to recognise the patterns but with the teacher's guidance and intervention they will be able to establish the relationships which will enable them to generalise. Questions are provided at the end of this lesson to allow the student to reflect further on the discoveries made and to test the knowledge acquired.

In Lesson 3 the participants are requested to arrange the blocks differently to enable them to see that the expression can be written in a different way which is the variation and contrast that Bruner favours for clarity to be achieved.



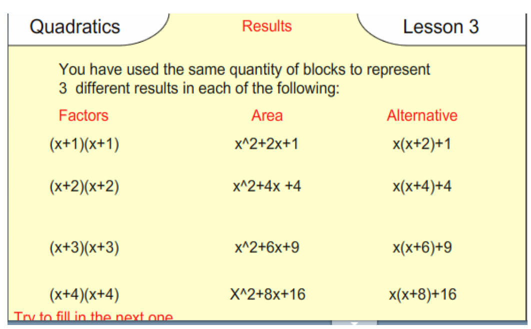


Figure 3.3.3

Figure 3.3.4

A summary is presented here to emphasise the results achieved in each of the examples and is used to assist generalising at the symbolic representation stage.

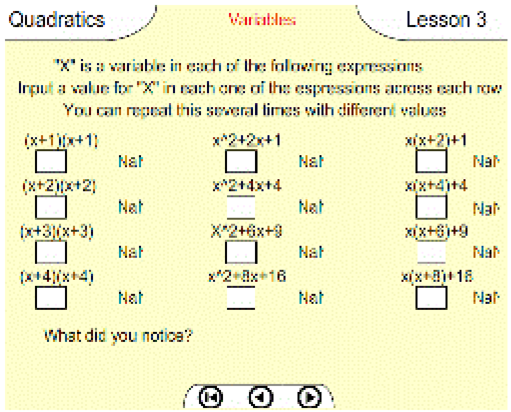


Figure 3.3.5

In this lesson the idea of "x" as a variable is introduced. The students here are presented with a screen for testing many examples of positive and negative numbers and this is the way in which the concept of a couple is introduced with an "x" and "y" value. The concept of relationship which is at the core of understanding a function is being built up. As this lesson progresses students are exposed to another way of representing these same functions – by graphs.

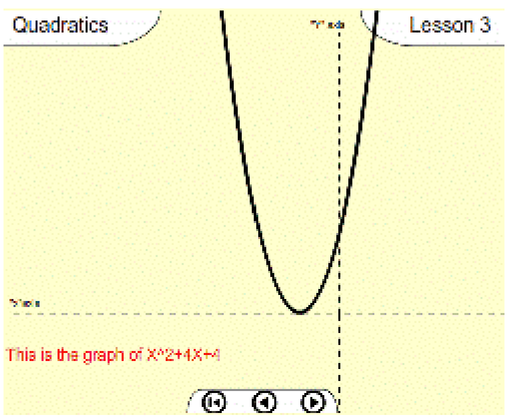


Figure 3.3.6

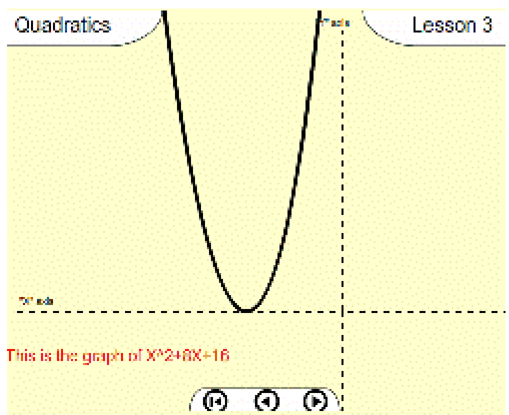


Figure 3.3.7

A graph is a representation or a relationship between "x" and "y" values and these screens should provide the learners with a clear notion of a parabola.

Activities need to be related to real life situations. It is essential that students can make sense and meaning in a real life context if they are to understand and it is with this in mind that Lesson 4 is created.

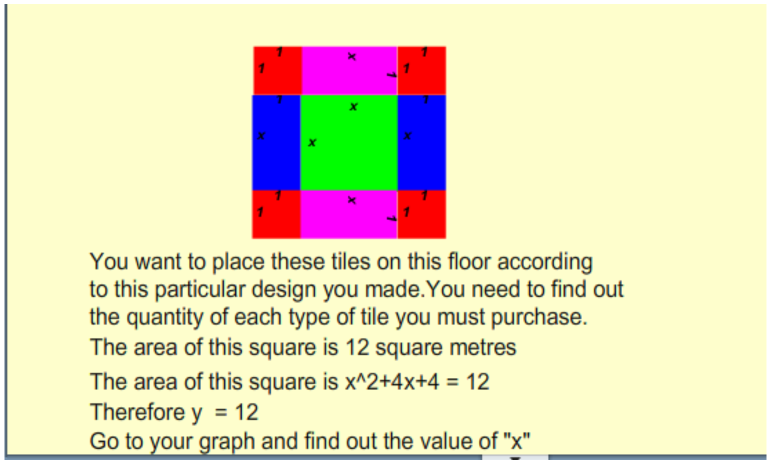


Figure 3.3.8

The final lesson Lesson 5 moves away from squares into functions of the nature of (x+3)(x+2) or (2x+1)(x+3). The students are provided with the procedures of the traditional method of solving quadratic functions.

This artefact is not designed for independent learning and therefore the role of the teacher is significant. In relation to students learning with graphic interfaces the teacher is present to assist students in abstracting the underlying concept. The computer screens may be used as tools of semiotic mediation for learner and teacher to engage in dialogue which will contribute to making meaning for the learner(Mariotti & Cerulli, 2001). Students have been led into difficulties when they do not construct their knowledge or get adequate time to reflect on what they have been learning. In relation to algebra the students must not only be able to manipulate symbols, which is just a surface skill but be able to connect knowledge of concepts which is what is required in order to understand algebra(James.J Kaput, 2000).

This artefact was tested with students to study their interactions with it and with the teacher in order to explore the benefit accruing to them in understanding quadratic functions.

Chapter 4 Implementation

4.1 Overview

The artefact has been designed and embedded in a Macromedia website for testing with a small group of students. The purpose of this process is to examine if the artefact enables students to move between the three stages of representation in order to assist them in understanding mathematical concepts.

4.2 Implementation

The implementation of the artefact took place over a five week period from the 6th March 2007 up to 23rd April 2007 in Columba College , Killucan which is a small second level rural school.

All classes are mixed ability groupings and 6 students were selected for testing. The students were from the first year group and they were all 12/13years old. Parental consent was sought in order for these students to participate in the research. As the researcher is a school principal the selection of students was left to the maths teacher of the 1st year class.

The sessions with the students were generally of 50 minutes duration and all students had two sessions to complete the activity and in some cases it was necessary to take a third session with a student to complete the process. The sessions were all held on a one to one basis in the dedicated computer room of the school. The participants were all capable of using the computer keyboard, the drop and drag facilities of the mouse and they displayed good manual dexterity during the implementation. During the testing the teacher was present with the student for the entire testing period.

4.3 Limitations of Implementation

The students were only available when a teacher of another subject was willing to allow them to be absent from his/her class. There were also restrictions with the use of the computer room which other class groups needed at times when it would have been suitable for research to be done. On a few occasions the researcher had made arrangements to work with a student but due to a minor crisis the research had to be postponed.

The result of these circumstances meant that some students were being tested for the last two classes of the day and in the case of 2 sessions with two different students the activity had to be done immediately after school. The consequence of these circumstances was that students' concentration and stamina were less than they might be if they could be tested earlier in the day.

The implementation, with its limitations, has been completed and the data which has been collected throughout the testing process needs to be examined and evaluated.

Chapter 5 Methodology

5.1 Overview

Qualitative research best suits the educational context and Merriam(2002) states that qualitative researchers want to find out how individuals interact and experience their social world and the meaning it has for them. Yin (2003, p. 13)writes that the case study is the most suitable strategy when "how" and "why" questions have to be answered and when "the focus is on a contemporary phenomenon within its real life context". The strength of the case study as stated by Bassey (1999)is that it is "strong in reality" but difficult to organise. Commenting on education research he states that "educational research is a critical enquiry aimed at informing educational judgements and decisions in order to improve educational action". Gall et al.(1996) tell us that in order to add validity to the research sufficient data must be collected if the researcher is to explore significant features of the case and to put forward interpretations for what is observed. This process of triangulation may not always produce convergence and there may be inconsistencies and contradictions among the findings. Cohen et al.(2000) state that case studies try to capture the close up reality of a particular situation and the thick description of participants experiences, thoughts and feelings of a particular experience. Yin(2003) would argue that significance rather than frequency is the hallmark of case studies and frequency of occurrences is replaced with quality and intensity. He emphasises too that in planning research the researcher must have a very clear proposition to test.

5.2 Research Questions

In this study the question being researched is:

How does a replication with ICT of Bruner's enactive, iconic to symbolic representations assist learners' understanding of mathematical concepts.

The focus was on mathematics which the researcher taught to 12 to 15 year olds for some time. The area of focus was further refined to algebra. The artefact was designed to allow the participant interact with the computer for the duration of the study sessions. The initial number of participants was 6 and one was eliminated on the basis that the period for which he was available for testing was not adequate.

5.3 Data Collection Instruments

In carrying out a case study emphasis is on the collection of rich data and therefore the decision was taken to use a video camera and to record each complete session with each participant. This involved the collection of 10 hours of video footage. At the end of the test the participants were requested to complete the online questionnaire.

When the questionnaire was being designed the age group of the participants and the time when it would be administered was considered. The questions were mainly to gather general information on the participants experience with computers, and applications previously used. One question designed according to the Likert scale was provided for commenting on the experience of working on this site and finally one open ended question allowing the student to provide whatever comment he wished on his experience of learning quadratic functions with this site. This questionnaire was administered immediately following the post test questions and in order to ensure accuracy in its completion with this age group it needed to be concise and capable of being completed in a short time.

The researcher followed up on that questionnaire by having unstructured interviews with each participant. It is important with participants of this age that the researcher can probe as the students are not very forthcoming with lengthy descriptive answers. Each interview lasted approximately 10minutes and they were taped and transcribed.

5.4 Difficulties with Data Collection

While it is essential to collect reliable data the researcher has to take cognisance of the ease with which the participant will be allowed to work on the tasks. The research was carried out in a dedicated computer room. The significant features which needed to be recorded were the participants' interaction with the programme, the dialogue between the teacher and participant, the use of the screen as a means of semiotic mediation and the way the screen was used by both teacher and participant for that purpose. While one video camera was considered to be the minimum requirement other considerations were: to have somebody present for the purpose of zooming in on the screen when appropriate, to set up a second camera in a stationery position, to have an independent observer in the room, or to download a programme called SnagIt which would capture screen movements. The availability of personnel to operate a camera was a problem and even, if available, the interference to the continuity of the exercise with the participant would be interrupted if the teacher were to attempt to indicate when to zoom. The layout of the room did not accomodate the setting up of a second camera which would provide any better footage from that already provided by one. The capture with Snag It programme which the researcher had on a laptop was attempted. Again this needed to be set up at the beginning of a session but in so doing there were problems recording at the end because of the unavailability of suitable recording space in the memory of the laptop. A couple of attempts were made at recording short snippets while participants were working but this was too intrusive and it was decided to abandon that source.

5.5 Data Analysis

As Bassey (1999)states the data which is collected in the course of doing a case study can be difficult to organise and this is particularly true in the case of video footage. The tapes were used for the purpose of getting a general overview. The primary data source was the video tapes and a coding and theming process was followed to cluster code words and to allow themes to emerge(Table 1). Transcription of dialogue took place in order to provide thick description for the reader to get a sense of interaction of student with programme and with participant researcher.

The themes that emerged were then connected with the data provided by the questionnaires and the data on the recorded interviews. If new themes emerged from the interviews as against what was provided by the footage it was added to previously analysed data. This process of triangulation was carried out in order to note where convergence was occurring and also to note any inconsistencies which emerged. Because the researcher was present as a participant observer during the whole study it was possible that moods or other aspects could be noted which might not be collected by the other sources.

Then the researcher examined what this analysis contributed to answering the research questions regarding the impact of graphic interfaces on the understanding of algebra.

Chapter 6 Findings and Discussion

6.1 Overview

The research question posed by this dissertation is:

How does an ICT replication of Bruner's Enactive, Iconic to Symbolic representations assist students' understanding of mathematical concepts?

When Bruner(1967, p. 69) had completed his test, which was replicated in this artefact, he stated that he had succeeded well as "the children learned some elegant mathematics in a fairly short time". If the success of this study, which produced mainly qualitative data, was measured by student performance on tasks in the post test questions it could be said they had learned algebra, in a short time, which they could not have attempted before. (See Appendix A & B).

The role of the artefact was:

1. to help the learner move between Bruner's levels of representation
2. to act as a tool for semiotic mediation
3. to provide insights into the role of the teacher in the process of semiotic mediation.

Numbers 1 and 2 above are very closely linked as the participants while manipulating shapes to create images which they were using to write symbolic notation needed the intervention of the teacher to draw out meaning through dialogue. Therefore these two aspects of the artefact will be linked together in looking at the evidence produced from the data analysed.

The evidence indicated that the students were able to:

1. write in symbolic notation the length of the side of the shape which corresponded with the factors of a quadratic function
2. write in symbolic notation the area of the shape which corresponded with the quadratic function written as a x^2+bx+c
3. observe patterns and generalise from the tasks completed which enabled them to work with the symbolic notation solely in the final post test.

The analysing process has resulted in the emergence of themes(Table 1) which will now be used to assist with the answering of the research questions.

6.2 Profiles and Attitudes

The analysis of data has produced some information in relation to the profiles and attitudes of the participants which impacted on their approach to the artefact and their engagement and interaction with it. All of the participants had access to a computer at home and used one in school two or three times a week. It was very evident that all of the students were competent in using the mouse and the keyboard which contributed to smooth operations throughout the tests. Generally the computer was used at home for entertainment or occasionally to get information on the internet for projects. In school its use was largely confined to English and once only had they used it for maths which was for puzzles and games. Four out of 5 students liked maths but only two liked algebra.

6.3 How does an ICT replication of Bruner's Enactive, Iconic to Symbolic Representations assist students' understanding of mathematical concepts?

The answers that students gave to the open ended question in the questionnaire "Write in the box how you felt about this experience with quadratic equations" indicates that, from their perspective, learning and understanding was present.

Michael: *"I feel like I have learned a lot from it"*.
Sheila: *"I found it helped me to understand algebra"*.
Kim: *"I thought it was very helpful in algebra"*.
Mary: *"I feel it helped me a lot to understand maths"*.
George: *"It was a good experience I learned a lot of stuff"*.

These comments suggest that they had learned something but the question still remains is there evidence that they understood the concepts.

6.4 Did the artefact:

1. **help the learner to move between Bruners' levels of representation**
2. **act as a tool of semiotic mediation.**

At the outset the participants were not comfortable with the notion of constructing shapes and they would have preferred to get a list of procedures to follow. This unease was evident from the fidgety way they acted when presented with the first screen which required manipulation and they needed to be reminded that they were required to build a square from the 4 blocks provided. Sheila verbalised her feelings "I don't get what you do". This indicates how foreign this experience was to them but they settled down and proceeded to work on the shapes and showed great ease as they moved through the lessons.

In the two revision screens of Lesson 1 they were clear about getting area of a square and a rectangle by multiplying length by breadth. In the context of the artefact, it was not possible to apply the formula to get area. They presented with issues around the distinction between length and area. As a result of much teasing out of meaning they grasped that the surface area of the big square which was covered by the small ones, was the same as the area of all the small shapes added together. When a construction was completed the participant was able to establish the area and fill in the symbolic notation in the box. When answering the post lesson questions on Lesson 1 all of the participants needed to go back to the previous screens and redo the constructions and then come forward and fill out the answers. This highlights their dependence on the creation of the image to assist in the completion of the task and the assistance that they got from the manipulations and images.

The students had a difficulty with what is referred to as the "obstacle of closure" which is considered to be a difficulty in the transition from arithmetic to algebra. This is evident from a sample extract of the interaction with George in Lesson 1 and this same difficulty would have arisen for others too. This issue of closure was overcome by the teasing out of the meaning using the screens using the constructions created by the participants.

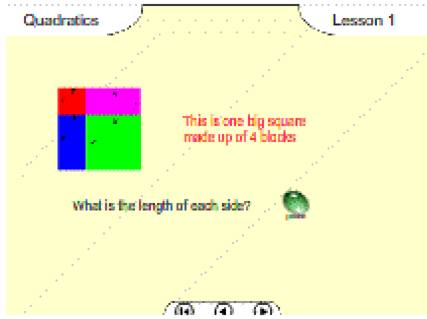


Figure 6.4.1

Teacher:	*How are you going to get the area?*
Gary:	*(1+x)(1+x)*
Teacher:	*Yes, in terms of what you have in front of you can you calculate it. (Long Pause)*
Gary:	*2x*
Teacher:	*What is the area of the green square.*
Gary:	*x*
Teacher:	*The length of it is x, what about the area?*
Gary:	*1x no* x^2
Teacher:	*Right, what about the pink and blue strips?*
Gary:	*1x*
Teacher:	*1x each and how many have you?*
Gary:	*2x*
Teacher:	*And what about the red one?*
Gary:	*1*
Teacher:	*So now the area of the complete big square is?*
Gary:	$4x^2$
Teacher:	*Now where did you get that from?*
Gary:	*1,2,3,x*
Teacher:	*Now show me where you are getting 1,2,3*
Gary:	*(pointing to blue, red, pink and green) like makes 1, that's 1x and that's 1x so 3x*

The dialogue continued and the desired result of writing area symbolically as the sum of the component parts was achieved.

The evidence in this example clearly indicates that the screen provided the means by which the teacher and student could engage with it to make meaning.

The students gradually were able to enter the notation in the box having completed the building of the square. But when they moved on to the post test questions and were asked, for example, “what is (x+2)(x+2)” the answer was given as $4x^2$. This indicates that the desire for closure was still evident when they moved from the image being present and it necessitated dialogue to eliminate the difficulty they had with this. All but one student was successful in overcoming this obstacle of closure as they moved through the lessons.

Before the summary was presented at the end of lesson 2 the participants had an opportunity to build and establish the length of side and the area of 4 different functions, (x+1)(x+1),(x+2)(x+2),(x+3)(x+3) and(x+4)(x+4). In the summary they were still referred to as sides and area and the array of information was presented on one screen. The artefact enabled the teacher and student to discuss this array by relating each of the elements of the functions back to the types of shapes that were used to create that element – green square or blue and pink strips or red squares.

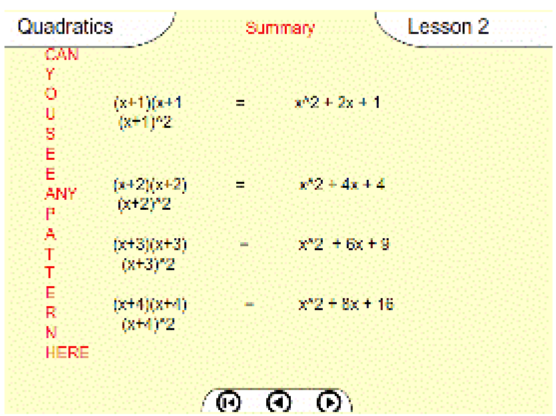


Figure 6.4.2

In this example Kim tries to make the connections

Teacher: *Can you see anything in particular as you look at this (pointing to the left side of the screen)*

Kim: *Yeah, they go up 1,2,3,4.*

Teacher: *Now can you notice anything about any section of this (pointing to the right side)*

(Long Pause)

Teacher: *What about the middle ones, do you see any connection between them?*

Kim: *They are 2s -2,4,6,8,*

Teacher: *What about these?*

(pointing to the last number in each row on the right)

Kim: *They (pause)*

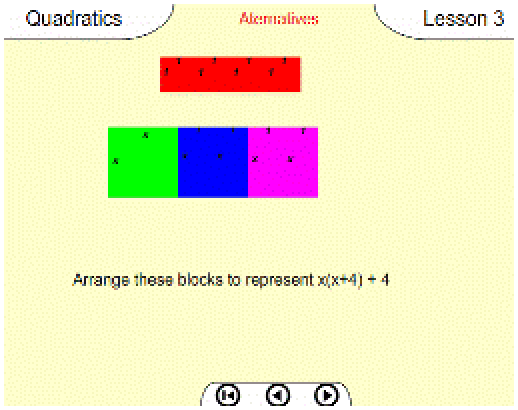
Teacher: *Looking at them where could they have got the last one?*

Kim: *Oh, its like 1 by 1is 1, 2 by 2 is 4, 3 by 3 is 9, (animatedly tapping them out on the desk)*

This example indicates that the student, and this was true for all the students, was not capable of inventing a rule to make a connection between the sides and the area but with the array available on the screen, the teacher and then the student could refer to certain sections and the student identified the patterns herself and she understood where they were coming from. This kind of pattern observation was explored further with each student as they moved through lesson 3.

In Lesson 3 the students were introduced to the term factors rather than sides of shapes and during this lesson they got an understanding of the distributive law.

When asked for example to build a shape to represent $x(x+4)+4$ and to get the area they were able to connect that the area $x^2 +4x+4$ was the same as the result got when multiplying $x +4$ by x and adding on 4. Then they used these same blocks to build a square and the area of the square was noted to be the same as before x^2+4x+4.



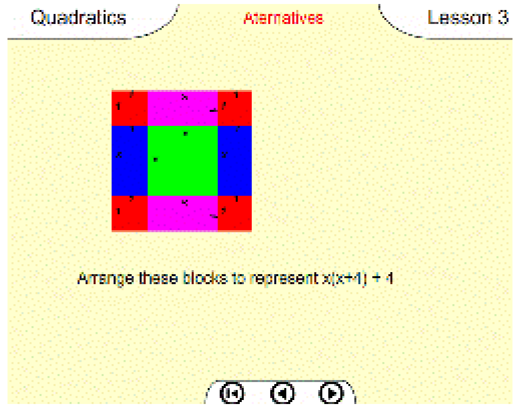


Figure 6.4.3 **Figure 6.4.4**

This and many other examples indicate that manipulation of graphics helped in understanding factors, distribution and the result.

In the summary of lesson 3 which now referred to factors, area and alternatives they had a clear understanding of what it related to and the relationship between each one. They were requested to continue with the pattern by filling in the empty boxes which every student was capable of doing without assistance.

The following extract provides evidence of how again the artefact enabled the learner and teacher to use an array of information, which has previously been worked through in Lesson 3, to relate back to images created and to make meaning of the patterns emerging.

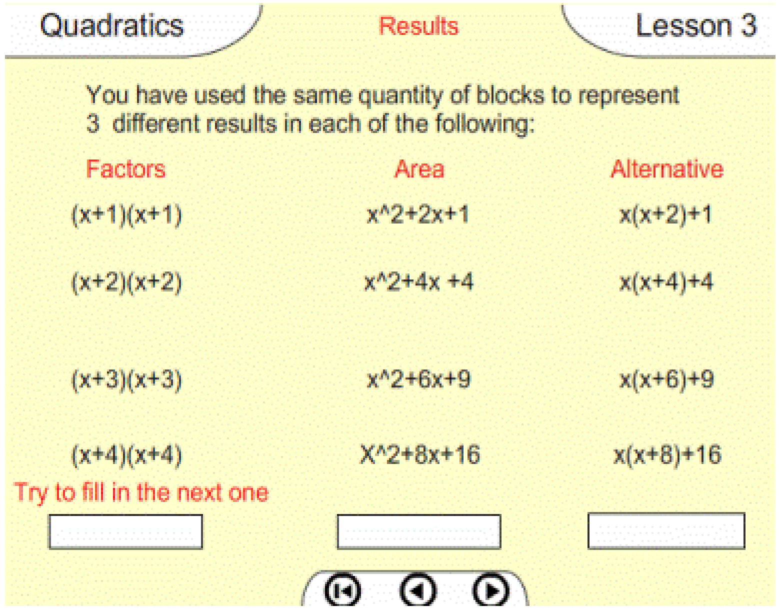


Figure 6.4.5

This is an extract from a session with Shelia who had become very successful at building the images and developing the appropriate notation from them.

Teacher: *(pointing to the middle row in area) where did we say the 2x, 4x,6x and 8x came from?*

Shelia: *the pink and blue shapes.*

Teacher: *What is the connection between the middle bit here and the factors?*

Shelia: *When you multiplied and added.*

Teacher: *Where did the 4, 9,16 come from?*

Shelia: *The red squares*

Teacher: *What is the connection between the factors and the 4, 9 and 16?*

Shelia: *When you multiplied them.*

Teacher: *What part of them?*

Shelia: *The 4 by 4 and the 3 by 3*

Teacher: *(moving to the right column) What do brackets mean?*

Shelia: *Multiply*

Teacher: *So if you have x outside the bracket what do you do*

Shelia: *x by x is* x^2 *and x by 4 is 4x*

Teacher: *+ 4*

It was evident from this extract that Shelia was immediately referring back to the images and also had a clear awareness of how each column had evolved.

There was further reinforcement of this relationship when the dynamic screen in lesson 3 enabled the student to input values for "x" and thus get a "y" value which was exactly the same across the rows. Again this and many other examples indicate that the students clearly understood where the patterns were emerging from and they were establishing a rule for themselves which they could implement in the solving of problems.

Evidence during lesson 5, which required building rectangles, suggested that students were now fully at ease building shapes. The students were building to specific instructions and as they built they detected very quickly if their construction was not exactly to instructions and they self corrected. In earlier lessons they would have built shapes and it was only when completed, or when the teacher intervened, that they would realise they were not correct. The following extract from a session with Michael in Lesson 5 indicates that he was in the process of building (x+2)(x+4) according to 4a (Figure 6.4.6) when he began to shake his head and started to undo it and began with 4b (Figure 6.4.6).

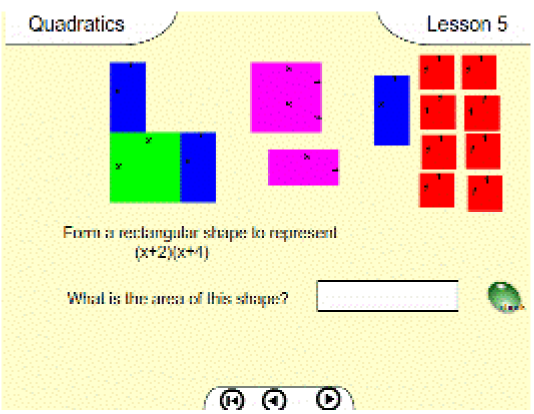


Figure 6.4.6

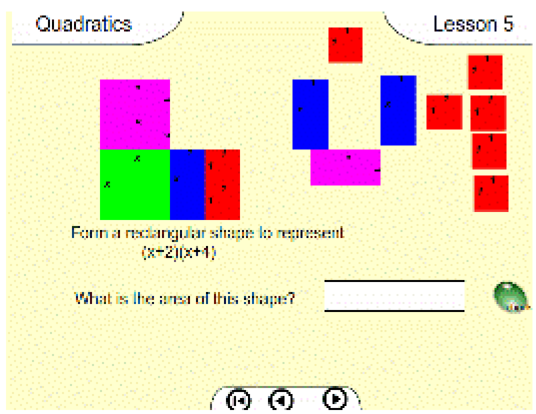


Figure 6.4.7

Teacher:	*What was wrong there, what dawned on you?*
Michael:	*It was 2x*
Teacher:	*What have you got on the left side now? (referring to Fig4b which is the beginning of the corrected version)*
Michael:	*x+2*
Teacher:	*What are you trying to get on the other side x+4*

Michael then continued to build very quickly and to correctly fill in the area in symbolic notation. The evidence produced by this and other extracts clearly indicates the merits of the artefact as a tool of semiotic mediation for making sense and meaning and clearly the student could not easily hide his lack of understanding. The student who was in control of the mouse used it to click on the screen when counting or referring to some element on it while the teacher was able to use the artefact to point to various parts during the dialogue. The artefact enabled the teacher to negotiate meaning with the student and enabled the student to grasp the abstract properties of algebra.

6.5 Insights into the teacher's perspective on the Role of the Teacher in Semiotic Mediation

Using the artefact as a teaching tool was a new experience for the teacher as was working solely with one student. The teacher was familiar with every aspect of the artefact which she had created but while reflecting on the experience it was the recognition of the variation in the students' *zone of proximal development* that was difficult. Using the screen as a tool for semiotic mediation was easier because the blackboard would be constantly in use in the classroom for reference. When students were building or filling in boxes and errors were evident to the teacher there was a strong temptation to intervene too soon rather than allowing the student time to reflect. When the student presented with a difficulty in relation to establishing the length of a side of a shape, the calculation of the area or in the use of symbolic notation intervention techniques such as the teacher adopting a socratic mode of questioning were more easily utilised. When the student remained looking at the screen sometimes motionless and speechless the temptation to interfere was strong and had to be consciously set aside. Reflection in action was necessary in order to achieve what the teacher intended when an unexpected construction arose. The variety of different ways which blocks could be arranged in a square sometimes needed quick reaction from the teacher in order to direct the student to the desired method of describing length. While reflecting on a session just completed or when reviewing video coverage missed opportunities for meaning making were clearly evident. From the extracts provided already the evidence is very clear that the role of the teacher is paramount in drawing out the meaning and abstracting the underlying concepts.

The positive outcomes relate to four out of the five students. One student approached the whole experiment with less enthusiasm than the others and a major focus on remembering. He had an anxiety about the result rather than the process and his understanding was limited though he still stated that "he learned a lot of stuff".

6.6 Discussions

The findings presented some outcomes which were to be expected from the review of the literature. The artefact provided: a means to manipulate and disassemble shapes very quickly, consistentcy in screen presentation, easy and speedy movement between screens and the presentation of arrays of information which enabled the teacher and student to work very efficiently which justifies the use of the artefact as a teaching tool. Oldknow and Taylor (2003)advocate the use of ICT when it allows the teacher and students to work effectively and efficiently which was supported in this instance.

The students displayed that they were capable of observing the emergence of patterns and generalising from them which is clearly referred to in the literature as being the root of all algebra learning(Kieran, 1997; Mason, Graham, & Johnson-Wilder, 2005)

Learning something by manipulation and conjecture, as this artefact demanded, was very foreign to students. They were not comfortable with the experience initially and they would have preferred if they had been given a mechanical set of procedures to follow which would provide an answer. They came with a focus on remembering, which would relate to their conditioning in maths classes, and they did not appreciate the challenge of this exploratory method. They did not treat the exercise as playful perhaps because there was constant contact with the teacher. If the researcher had more time available for testing it would have been interesting to allow the students explore playfully on their own followed by engagement and mediation with the teacher. Clements(2000) favoured enabling students to have this playful experience to encourage them to become mathematicians as distinct from learning how *to do* maths.

Bruner was concerned with the students' predisposition to learning and therefore in designing a task he emphasised the importance of structure and sequence. The importance of this was that the student needed to be in a position to assimilate the new information into their previously acquired knowledge. Vygotsky (1978)would refer to this as the zone of proximal development where the student is ready to be brought to the next stage of understanding by the material presented and by social interaction. In this study these concepts proved to be significant. There was an assumption that the students had a clear concept of area but they had knowledge only of how to calculate it by using a formula. This evidence further supports the theory that a teacher should never assume that topics covered on a curriculum are actually learned or understood. Students do not always learn what the teacher intended (Laborde, 1993).

This study took place over a short period of time and the findings which have emerged have proved its value, as a replication of Bruner's model, in allowing students to use their higher order thinking skills by moving from enactive to iconic to symbolic stages of representation. Engagement and interaction between student and teacher, enabled by the artefact, created the environment for moving from concrete to abstract.

It would be expected that this experience would have a long term value for these students. Tall and Thomas (1991)tell us from their study that those who learned with the computer had higher order thinking skills and the evidence was still apparent in six months after the post test.

This experience was difficult for the teacher in that it was a move away from the traditional role of presenting examples of procedures to be adopted, and encouraging practice on similar examples and correcting practiced material. In this case the researcher was reasonably comfortable with a Socratic mode of questioning as it would be a style that she would adopt to some extent in the course of teaching in the classroom.

The evidence, supported by the examples given and many others, indicated that the students learned algebra and understood the mathematical concepts. The artefact helped to deliver these results but the evidence is very clear that the role of the teacher was paramount in making meaning for the students and in helping them to abstract using the artefact. A recommendation to infuse algebra throughout the mathematics curriculum and to alternate algebra and arithmetic all through school is a worthwhile recommendation given by Kaput (2000).

Chapter 7 Conclusions

It has been established from the findings that the participants learned and understood algebra.

The artefact enabled this learning to occur with the assistance of the teacher. While learning by exploratory means was challenging for the students initially they eased into as they went through the lessons. They overcame the *obstacle of closure* , they understood the source of symbolic notation as applied to quadratic functions, they observed the emergence of patterns and found commonality in the examples which enabled them to invent rules. At the end of the experiment they were capable of working solely in symbolic notation.

The teacher found her role in the experiment challenging to get a balance between intervention as opposed to interference and mediation as opposed to instruction.

While inferences may be drawn from a small sample such as this Yin (1989) emphasises that in these circumstances it is not possible to generalise.

7.1 Future Research

This experiment would need to be repeated with several groups and ideally with a class group. In schools the reality is that teachers must teach the class as an all-inclusive unit. Research could be done by using the artefact with a class group who have access to an interactive whiteboard which could be used as a tool of semiotic mediation with discussion and collaboration leading to sense making.

During this study the students got an opportunity to learn algebra. The teacher was motivated to reflect on her practice now and into the future. The true test of understanding will be a long term issue for those students and the expectation would be that they would understand when they attempt algebra in the classroom.

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Appendix A Coding and Theming of Questionnaires

Themes	Codes
Uses of the computer	Myself, Games, Internet, Puzzles, English Projects
Perceptions of user	Hard, Little bit hard, Just Different, Challenge, Easy, Got used to it, How to do it, Remembering
Iconic recollection	Green Square $-x^2$, pink and blue squares – 1x, Red Squares – 1
Responses of Students	Helpful, Seemed Easier, Understand More, Understand Maths, Understand Algebra, Learned Alot

Appendix B   Post Lesson Questions

Lesson 1

1. What is the answer to (x+2)(x+2) ?
2. What is the answer to(x+1)(x+1) ?
3. What are the factors of $x^2 + 4x + 4$?

Lesson 2

What is:

1. (x+1)(x+1)
2. (x+2)(x+2)
3. (x+3)(x+3)
4. (x+4)(x+4)
5. (x+7)(x+7)
6. (x+10)(x+10)
7. (x+12)(x+12)

Lesson 5

What is:

1. (x+2)(x+2)
2. (x+3)(x+2)
3. $(x+3)^2$
4. $(x+6)^2$
5. (x+7)(x+2)
6. (2x+1)(x+6)
7. (2x+3)(x+4)
8. (x+5)(x+2)
9. (3x+2)(2x+2)
10. (2x+5)(3x+4)

Appendix C Questionnaire

QUESTIONNAIRE			
1. Number ID	Place your ID here		
2. How many correct answers did you get?	Result 1 Enter number of correct	Result 2 Enter number of correct	Result 3 Enter number of correct
Select one answer only in each of the following questions			
3.Do you use a computer at home?		Yes ○	No ○
4.How often do you use a computer in school ?		4/5 times a week	
5. Why do you use the computer?		Play Games	
6. Have you used a Maths site?	Regularly ○	Occasionaly ○	Never ○
7. Do you like Maths?		Yes ○	No ○
8. Do you like Algebra?		Yes ○	No ○
9. I found this website helpful.		Strongly Agree	
10 Write in the box how you feel about this experience with quadratic equations.		Write here	

Appendix D Results from Questionnaires

Questions

Question 1	How many questions did you get correct?
LA	Lesson 1
LB	Lesson 2
LC	Lesson 3
Question 2	Do you use a computer at home?
Question 3	How often do you use a computer at school?
Question 4	Why do you use a computer?
Question 5	Have you used a maths website?
Question 6	Do you like maths?
Question 7	Do you like algebra?
Question 8	Did you find this website useful?
Comments	Write in the box how you felt about this experience with quadratic equations

Results

ID N0.	Question 1			Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8
	L A	L B	L C							
1	3	7	4	Yes	2/3	Play games	Regularly	Yes	Yes	Strongly Agree
3	2	7	8	No	2/3	Play games	Occasionally	Yes	Yes	Agree
4	1	7	8	Yes	2/3	Other	Occasionally	Yes	No	Agree
5	1	5	6	Yes	2/3	Internet for projects	Occasionally	No	No	Strongly Agree
6	1	6	6	Yes	Never	Other	Occasionally	Yes	No	Agree

Comments

ID N0.	Comments
1	I feel like I have learned a lot from it
3	I found it helped me to understand algebra
4	I thought it was very helpful in algebra
5	I feel it helped me a lot to understand maths
6	It was a good experience,I learned a lot of stuff

Appendix E Contents of accompanying DVD

The DVD contains:

Transcripts of students' recorded interviews

Video Clips of interactions with students