1. Suppose a string of *n* bits is sent across a lossy link. In how many ways can 2 bit errors occur ? When *n=3*, list the possible set of bit error patterns.

<u>Solution</u>

We're picking 2 items from a set of *n* items and we don't care about the order, so there are $\binom{n}{2}$ combinations.

When *n*=3, there are $\binom{3}{2}$ = 3 error patterns. The set of patterns is:

1 1 0

0 1 1

1 0 1

2. How many 3 digit numbers can you make using the digits 1, 2 and 3 without repetition ? How many 2 digit numbers can you make using the digits 1, 2, 3 and 4 without repetition ?

<u>Solution</u>

Pick the first digit in the number from 1, 2 or 3. Pick second digit from the remaining two, pick third digit from remaining one. So by product rule there are 3x2x1=6 possible 3 digit numbers.

In second case, pick first digit from available 4, pick second from available 3. So there are 4x3 = 12 possible 2 digit numbers

3. From a group of 8 professors and 9 students and admissions committee consisting of 5 professors and 3 students is to be formed. How many different committees are possible ?

<u>Solution</u>

We can pick 5 professors from 8 in $\binom{8}{5}$ different ways. We can pick 3 students from 9 in $\binom{9}{3}$ different ways. Using the product rule, the number of committees possible is $\binom{8}{5}\binom{9}{3}$.

4. How many ways can 12 people be seated in a row if:

(i) There are no restrictions on the seating arrangement ?
(ii) Two of the people A and B refuse to sit together ?
(iii) Two of the people A and B must sit together ?

Solution

(i) Pick first person from 12, second person from remaining 11, third from remaining 10 and so on. Using the product rule there are 12! permutations.

(ii) Suppose person A sits in seat 1. Then person B must sit in one of seats 3 to 12. So there are 10 different seats for B. The remaining 10 people can sit in 10! different permutations. Using the product rule there are 10x10! different arrangements when A sits in seat 1. When A sits in seat 2 then B can can sit in seats 4 to 12 i.e. 9 different positions. Similarly when A sits in seats 3 to 11. When A sits in seat 12, B can sit in seats 1 through 10 i.e. 10 different positions. So there are 2x10x10! + 10x9x10! arrangements in total.

(iii) Suppose person A sits in seat 1. Then person B must sit in seat 2 and the remaining people can sit in 10! different permutations. Suppose person A sits in seat 2. Then person B can sit in seats 1 or 3. So there are 2x10! different arrangements. Similarly for seats 3 through to 11. When person sits in seat 12 then person B must sit in seat 11, so there are 10! arrangements. In total there are 10!+2x10!x10+10! arrangements i.e. 22x10!

5. Consider an array x of integers with k elements, where each element has a *distinct* integer value between 1 and n inclusive and the array is sorted in increasing order i.e. $x[1] < x[2] < x[3]$ …. When k=2 how many such sorted arrays are possible ?

Solution

Pick the first element from the set {1,2,…,n-1} (it only goes up to n-1 because we need to allow at least one larger value for the second element. Select the second element from the set {$x[1]$+1,…,n}. Since $x[1] < n$ we know that there is at least one element in this set. So there are n-$x[1]$ ways to select the second element. So total number of arrays is $\sum_{i=1}^{n-1}(n-i)$.