

1. In the Chinese Appetizer problem  $n$  people are eating  $n$  different appetizers arranged on a circular, rotating table. Someone spins the tray so that each person receives a random appetizer. What is the probability that everyone gets the same appetizer as before? How does this compare with the bound obtained using Markov's inequality?

2. Consider two games. In game A, each time we play we win €2 with probability  $2/3$  and lose €1 with probability  $1/3$ . In game B, each time we play we win €1002 with probability  $2/3$  and lose €2001 with probability  $1/3$ . What is the expected winnings in both games? What is the variance? Using Chebyshev's inequality, compute an upper bound on the probability that after playing 10 rounds of each game the winnings deviate by more than  $\pm 10$  from the expected value. Write a Matlab simulation to estimate the probability that make a loss after 10 rounds of each game.

3. In a poll  $n$  turkeys selected independently at random are asked whether they vote for Christmas or not. Let  $X$  be the number of yes votes in our sample and use  $S=X/n$  as our estimate of the actual fraction of turkeys who like Christmas. Let  $X_i=1$  when the  $i^{\text{th}}$  turkey likes Christmas and 0 otherwise and assume  $X_i \sim \text{Ber}(p)$ . Using Chebyshev's inequality, how big should  $n$  be to ensure that this estimate is within 0.04 of the true fraction at least 95% of the time?

4. Suppose you need to design a content delivery network. In a 5 minute interval you get  $N=1$  million content requests and each needs to be served from one of your 1000 servers. Suppose each incoming job is assigned uniformly at random to a server. What is the average load on each server? Use the Chernoff inequality to upper bound the probability that each server has no more than 1250 jobs. Recall that the Chernoff inequality for a Binomial random variable  $X \sim \text{Bin}(n,p)$  is  $P(X \geq (1+\delta)\mu) \leq \exp(-\mu((1+\delta)\log(1+\delta) - \delta))$  where  $\mu=np$ .