

Let  $X$  be the number that comes up on a fair 6-sided die. What is  $E[X]$  and  $\text{Var}(X)$ ? Let  $I$  be a random variable which takes value 1 when the die value is greater than 3 and  $I$  equals 0 otherwise. What is  $E[I]$  and  $\text{Var}(I)$ ? Now suppose we throw 10 dice. Let  $X_i$  be the result for  $i^{\text{th}}$  die. What is  $E[X_1+X_2+\dots+X_{10}]$ ?

A computer program crashes at the end of each hour with probability  $p$ , if has not done so already. What is the expected time until the program crashes? Express in terms of  $p$ . Useful fact<sup>1</sup>:  $\sum_{i=1}^{\infty} ix^{i-1} = \frac{1}{(1-x)^2}$ . Write a Matlab simulation and compare its estimate with your calculation.

The time to run a cloud computing task is  $X = 1/N + Y$ , where  $N$  is the number of servers allocated to the task and  $Y$  is a random variable with  $E[Y]=0$ . Suppose  $n=2$  servers are allocated, what is  $E[X|N=2]$ ? Suppose the number of available servers has PMF:  $P(N=1)=0.5$ ,  $P(N=2)=0.2$ ,  $P(N=3)=0.2$ ,  $P(N=4)=0.1$ . What is  $E[X]$ ?

Suppose the number of people using a mobile app in a day is a random variable  $N$  with PMF  $P(N=n) = \frac{e^{-n}}{1-1/e}$ . On average each person pays €1 per day to use the service. What is the expected revenue for this app? Useful fact:  $\sum_{i=0}^{\infty} ix^i = \frac{x}{(1-x)^2}$

A tout is buying tickets for a concert. The price of the tickets is €50 and the tout sells them for €100, making a profit of €50. However, if he can't sell a ticket then he makes nothing from it (so it costs him €50). Let random variable  $N$  be the number of people who will buy tickets from the tout.  $N$  has PMF  $P(N=n) = (1 - e^{-0.5})e^{-0.5n}$ . Suppose the tout buys  $m=10$  tickets, what is his expected profit or loss? Express in terms of  $m$ ,  $n$  and constants. Using Matlab plot the expected profit vs  $m$ . What value of  $m$  maximizes profit?

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<sup>1</sup> E.g. see [https://en.wikipedia.org/wiki/Arithmetico-geometric\\_sequence](https://en.wikipedia.org/wiki/Arithmetico-geometric_sequence)