ST3009 Mid-Term Test 2016

Attempt **all** questions. Time: 1 hour 30 mins.

1. Suppose we roll a red die and a green die.

(i) What is the sample space for this experiment? [5 points]

(ii) What is the probability that the number on the green die is larger than the number on the red die? [5 points]

(iii) Define what it means for two events E and F to be independent. [5 points]

(iv) Let event E be that the sum of the dice equals 2 or 3 and event F be that the sum equals 3. Are E and F independent? Explain with reference to the definition given above. [10 points]

2.

(i) State Bayes Rule. [5 points]

(ii) Suppose 1% of computers are infected with a virus. There is an imperfect test for detecting the virus. When applied to a computer with the virus the test gives a positive result 90% of the time. When applied to a computer which does not have the virus, the test gives a negative result 99% of the time. Suppose that the test is positive for a computer. What is the probability that the computer has the virus ? [10 points]

3. You invent a game where the player bets €1, and rolls two dice. If the sum is 7, the player wins €k, and otherwise loses their €1 bet.

(i) Define the expectation and variance of a discrete random variable. [5 points]

(ii) What is the expected reward in this game ? [5 points]

(iii) What value of k makes the game fair (i.e. makes the expected reward zero) ? What is the variance of the reward in this case ? [10 points]

(iv) For two independent random variables X and Y show that Var(X+Y)=Var(X)+Var(Y). Hint: Recall that E[X+Y]=E[X]+E[Y] and that when X and Y are independent then E[XY]=E[X]E[Y] [10 points]

(v) Suppose that you play the game 2 times in a row with k=5. What is the expected value of the reward (i.e. of the aggregate winnings after playing 2 times)? What is its variance ? What is the expectation and variance of the reward after 100 plays ? [5 points]