TRINITY COLLGE DUBLIN

School of Computer Science and Statistics

Extra Questions ST3009: Statistical Methods for Computer Science

Question 1. Suppose a continuous valued random variable X has probability density function:

$$f(x) = \begin{cases} 0 & x \leq 0 \\ 1 & 0 < x \leq 1 \\ 0 & x > 1 \end{cases}$$

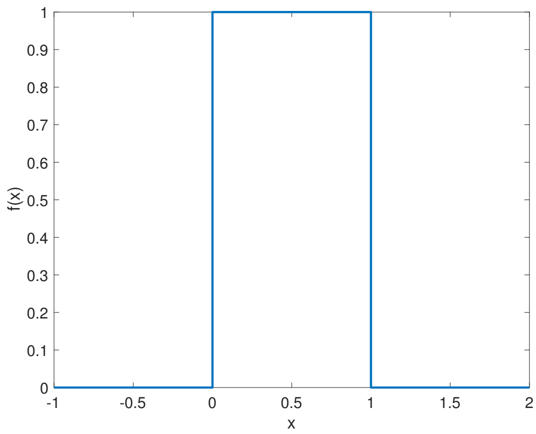

Figure 1: Plot of PDF $f(x)$

(a) Calculate $P(0 \leq X \leq 0.25)$
(b) Calculate $P(0 \leq X \leq 0.75)$
(c) Calculate $P(0.5 \leq X \leq 2)$
(d) Calculate the CDF for X

Question 2. Suppose a continuous valued random variable X has probability density function:

$$f(x) = \begin{cases} 0 & x \leq 0 \\ 2x & 0 < x \leq 1 \\ 0 & x > 1 \end{cases}$$

(a) Calculate $P(0 \leq X \leq 0.25)$
(b) Calculate $P(0 \leq X \leq 0.75)$
(c) Calculate $P(0.5 \leq X \leq 2)$
(d) Calculate the CDF for X

Question 3. Suppose a continuous valued random variable X has probability density function:

$$f(x) = \begin{cases} 0 & x \leq 0 \\ 4x & 0 < x \leq 0.5 \\ 4 - 4x & 0.5 < x \leq 1 \\ 0 & x > 1 \end{cases}$$

(a) Calculate $P(0 \leq X \leq 0.25)$
(b) Calculate $P(0 \leq X \leq 0.75)$
(c) Calculate $P(0.5 \leq X \leq 2)$

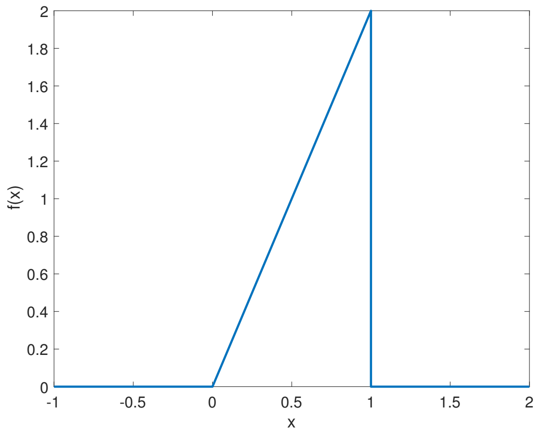

Figure 2: Plot of PDF $f(x)$

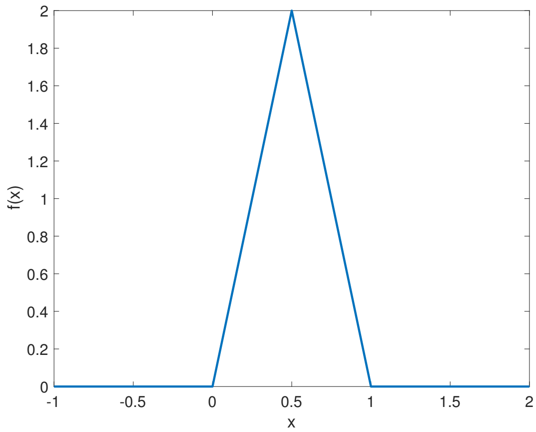

Figure 3: Plot of PDF $f(x)$

Question 4. Suppose a continuous valued random variable X has probability density function:

$$f(x) = \begin{cases} 0 & x \leq 0 \\ 1 & 0 < x \leq 1 \\ 0 & x > 1 \end{cases}$$

(a) Calculate $\int_0^1 x dx$. Hint: recall that this is the area under a triangle with base 1 and height 1.

(b) What is the expected value of X ?

Question 5. The CDF of a continuous valued random variable X is

$$F(x) = \begin{cases} 0 & x \leq 0 \\ x & 0 < x \leq 1 \\ 1 & x > 1 \end{cases}$$

(a) Calculate $P(X \leq 0.25)$
(b) Calculate $P(0 \leq X \leq 0.25)$
(c) Calculate $P(0.5 \leq X \leq 0.75)$
(d) Calculate $P(2 \leq X \leq 3)$
(e) Sketch a graph of the CDF.

Question 6. The CDF of a continuous valued random variable X is

$$F(x) = \begin{cases} 0 & x \leq 0 \\ 1 - e^{-x} & x \geq 0 \end{cases}$$

(a) Calculate $P(X \leq 1)$
(b) Calculate $P(0 \leq X \leq 1)$
(c) Calculate $P(1 \leq X \leq 2)$
(d) Sketch a graph of the CDF.

Question 7. You carry out a poll asking n eskimos selected independently and uniformly at random from the population whether they like warm weather. Let X_i be 1 if eskimo i likes warm weather and 0 otherwise. You calculate the sample average $X = \frac{1}{n}\sum_{i=1}^{n} X_i$ and use this as an estimate of the probability that an eskimo likes warm weather.

(a) State the central limit theorem

(b) How might the central limit theorem be used to obtain a confidence interval for X as an estimate of the probability that an eskimo likes warm weather?

(c) Discuss the assumptions made.

Question 8. In a study on cholestrol levels a sample of 12 men and women was chosen. The plasma cholestrol levels (mmol/L) of the subjects were as follows:

6.0,6.4,7.0,5.8,6.0,5.8,5.9,6.7,6.1,6.5,6.3,5.8

(a) Explain how you estimate the mean of the plasma cholestrol levels, and its 95% confidence interval, using bootstrapping

(b) Write a short matlab program to carry out these estimates. Compare with the confidence interval obtained in last weeks test question 7.

(c) Discuss the assumptions made when using bootstrapping.

(d) How might the central limit theorem be used to obtain a confidence interval ? How do the assumptions differ from when bootstrapping is used.