TRINITY COLLGE DUBLIN

School of Computer Science and Statistics

Extra Questions ST3009: Statistical Methods for Computer Science

NOTE: There are many more example questions in Chapter 4 of the course textbook "A First Course in Probability" by Sheldon Ross, and also some in Chapters 6 and 7 (but ignore the questions involving continuous random variables).

NOTE 2: It's a really good idea to write a short matlab simulation which you can use to check whether your answer is reasonable before looking at the solutions. It takes only a few minutes but will give you a lot more confidence in whether your answer is correct or not.

Question 1. Let X denote a random variable that takes on any of the values -1, 0, and 1 with respective probabilities $P(X = -1) = 0.2$, $P(X = 0) = 0.5$, $P(X = 1) = 0.3$. Compute $E[X]$ and $E[X^2]$.

Question 2. There are 6 cards in a hat: $1\heartsuit$, $1\spadesuit$, $1\diamondsuit$, $2\heartsuit$, $2\spadesuit$, $2\clubsuit$. You draw one card uniformly at random. If its suit is $\diamondsuit$ then you draw one more card, otherwise you stop. Let X be the sum of the ranks on the one or two cards drawn.

(a) What is the probability that one card is drawn ?

(b) What is the probability that two cards are drawn ?

(c) Find the PMF of X and $E[X]$.

Question 3. Consider the following game. A player bets on one of the numbers 1 through 6. Three dice are then rolled, and if the number bet by the player appears i times, $i = 1, 2, 3$, then the player wins i euros; if the number bet by the player does not appear on any of the dice, then the player loses €1. What is the expected win/loss for the player ?

Question 4. Find the expected total number of successes that result from n trials when trial i is a success with probability p_i, $i = 1, \dots, n$.

Question 5. 100 snails are competing in a relay race. Each snail takes on average 10 minutes to complete the race course. What is the expected time for the race to complete (i.e. for all 100 snails to complete the course, one after another) ? Explain your reasoning.

Question 6. A bag contains 100 chips, 75 chips with 1 marked on them and 25 with -1 marked on them. Pick a chip from the bag and let random variable X be the value of the chip.

(a) What is $E[X]$?

(b) What is $Var(X)$?

(c) Suppose now that you draw 50 chips from the bag, with replacement. Let Y be the sum of values of the chips drawn. What is $E[Y]$? Hint: use linearity of the expectation.

(d) What is $Var(Y)$? Hint: use the fact that the chips are drawn independently.

(e) What is $E[\frac{1}{50}Y]$ and $Var(\frac{1}{50}Y]$? Hint: linearity of the expectation again, and that $Var(aX) = a^2Var(X)$.

Question 7. You select one student uniformly at random and let random variable $X = 1$ if they have brown hair and $X = 0$ otherwise. Suppose there are 100 students of whom 40 have brown hair.

(a) What is $Prob(X = 1)$?

(b) What is $E[X]$? Are these the same or different, and why ?

You now carry out a poll of students in the class to try to estimate the number with brown hair. You do this by selecting n students uniformly at random and checking their hair colour, letting $X_i = 1$ if it is brown for student i. Let $Y = \sum_{i=1}^{n} X_i$.

(c) What is $E[Y]$? Is it the same as $E[X]$ or different ?

(d) What is $E[\frac{1}{n}Y]$?

(e) What is the variance of $\frac{1}{n}Y$ (express in terms of $Var(X)$) ?

Hints: use linearity of the expectation and the fact that students are sampled independently.

Question 8. Compute the variance of a binomial random variable X with parameters n and p.

Question 9. Suppose that the son of a man of height x (in inches) attains a height that randomly distributed with mean $x + 10$ and variance 4. What is the expected height at full growth of the son of a man who is 170cm tall?

Question 10. Suppose that N people throw their hats into the center of a room. The hats are mixed up, and each person randomly selects one. Find the expected number of people that select their own hat.

Question 11. I toss a coin. If it comes up heads I then throw a six-sided die and win the amount that comes up in euros. If tails then I pay €1.

(a) If I play the game once what is my expected win/loss ?

(b) I repeat the game 10 times. What is my expected win/loss ?

Question 12. When I travel to work by bike it takes on average 20 minutes. When I take the bus it takes on average 45 minutes. I take the bus when it rains, and on any given day suppose it rains with probability 0.25. What is my overall expected travel time to work ? Hint: use marginalisation of conditional expectation.

Question 13. A product that is sold seasonally yields a net profit of 5 dollars for each unit sold and a net loss of 1 dollar for each unit left unsold when the season ends. The number of units of the product that could be sold at a specific department store during any season is a random variable having probability mass function $p(i)$, $i = 0, 1, 2, \ldots$. If the store must stock this product in advance, what is the expected profit (express in terms of $p(i)$ and the number n of units stocked) ? Suppose $p(i) = 1/10$ for $i = 1, \ldots, 10$ and $p(i) = 0$ for $i > 10$. Using matlab plot the expected profit vs n and determine the number of units n the store should stock so as to maximize its expected profit.

Question 14. I am listening to new songs on spotify. Suppose the probability that I don't like each suggested song is 0.75, independently for each song. On average how many songs do I have to listen to before I find one that I like.

Question 15. Suppose that the number of people entering a department store on a given day is a random variable with mean 50. Suppose further that the amounts of money spent by these customers are independent random variables having a common mean of €9. Finally, suppose also that the amount of money spent by a customer is also independent of the total number of customers who enter the store. What is the expected amount of money spent in the store on a given day?

Question 16. Suppose X and Y are random variables with the following joint pmf. Are X and Y independent?

X/Y	1	2	3
1	1/18	1/9	1/6
2	1/9	1/6	1/18
3	1/6	1/18	1/9

Question 17. Let I_A and I_B be indicator variables for the events A and B. Express $Cov(I_A, I_B)$ in terms of $P(A)$, $P(B)$ and $P(A \cap B)$.

Question 18. Suppose X and Y are random variables with $P(X = -1) = 1/2$, $P(X = 1) = 1/2$, $P(Y = -1) = 1/2$, $P(Y = 1) = 1/2$. Let $c = P(X = 1 \text{ and } Y = 1)$.

(a) Determine the joint PMF of X and Y.
(b) Also $Cov(X, Y)$ and $Corr(X, Y)$.
(c) For what values of c are X and Y independent ?
(d) For what values of c are X and Y 100% correlated ?

Question 19. A computer game draws a rectangle with random length X and breadth Y, with $E[X] = 1$ and $E[Y] = 2$. The length and breadth are selected independently. The area of the rectangle is XY. What is the expected area of the rectangle ?