

TRINITY COLLEGE DUBLIN
School of Computer Science and Statistics

Extra Questions

ST3009: Statistical Methods for Computer Science

NOTE: There are more example questions in Chapter 4 of the course textbook “A First Course in Probability” by Sheldon Ross.

NOTE 2: It’s a really good idea to write a short matlab simulation which you can use to check whether your answer is reasonable before looking at the solutions. It takes only a few minutes but will give you a lot more confidence in whether your answer is correct or not.

Question 1. Consider an experiment where we toss 3 fair coins. Let random variable Y be the number of heads that appear. The sample space is $\{(h, h, h), (t, h, h), (h, t, h), \dots\}$ and recall that a random event is a subset of the sample space.

- (a) What random event corresponds to $Y = 1$?
- (b) What event corresponds to $Y = 2$?
- (c) What event corresponds to $Y = 3$?
- (d) What are the probabilities of these three events ?

Question 2. Two balls are randomly drawn from an urn containing 3 white, 3 red, and 5 black balls. Suppose that we win €1 for each white ball selected and lose €1 for each red ball selected. Let X denote our total winnings from the experiment.

- (a) What is the random event corresponding to $X = 0$?
- (b) What is the event corresponding to $X = 2$?
- (c) What are the probabilities of these events ?

Question 3. Suppose the cumulative distribution function of random variable X is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ 1/2 & 0 \leq x < 1 \\ 3/5 & 1 \leq x < 2 \\ 4/5 & 2 \leq x < 3 \\ 9/10 & 3 \leq x < 3.5 \\ 1 & x \geq 3.5 \end{cases}$$

Calculate the probability mass function of X .

Question 4. Three balls are drawn independently with replacement from bag contains 3 white and 2 red balls. Let X be the number of red balls drawn. Calculate the PMF and CDF of X

Question 5. Five fair coins are flipped. If the outcomes are assumed independent, find the probability mass function of the number of heads obtained.

Question 6. Suppose that the probability that an item produced by a certain machine will be defective is 0.1. Find the probability that a sample of 10 items will contain at most 1 defective item.

Question 7. It is known that screws produced by a certain company will be defective with probability 0.01, independently of each other. The company sells the screws in packages of 10 and offers a money-back guarantee that at most 1 of the 10 screws is defective. What proportion of packages sold must the company replace?

Question 8. Suppose that a particular trait (such as eye color or left-handedness) of a person is classified on the basis of one pair of genes, and suppose also that d represents a dominant gene and r a recessive gene. Thus, a person with dd genes is purely dominant, one with rr is purely recessive, and one with rd is hybrid. The purely dominant and the hybrid individuals are alike in appearance. Children receive 1 gene from each parent. If, with respect to a particular trait, 2 hybrid parents have a total of 4 children, what is the probability that 3 of the 4 children have the outward appearance of the dominant gene?