

TRINITY COLLEGE DUBLIN
School of Computer Science and Statistics

Extra Questions

ST3009: Statistical Methods for Computer Science

NOTE: There are more example questions at the end of Chapter 3 of the course textbook “A First Course in Probability” by Sheldon Ross.

Question 1. Independent flips of a coin that lands on heads with probability p are made. What is the probability that the first four outcomes are

- (a) H,H,H,H?
- (b) T,H,H,H?

Solution

- p^4
- $p^3(1 - p)$

Question 2. Eight bits are transmitted across a lossy wireless link. Each bit is corrupted independently with probability p .

- (a) What is the probability that no bits are corrupted ?
- (b) What is the probability that 8 bits are corrupted ?
- (c) What is the probability that 3 bits are corrupted ?

Solution

- $(1 - p)^8$
- p^8
- $\binom{8}{3}p^3(1 - p)^5$

Question 3. A bag contains 5 red balls and 10 black balls. Three balls are drawn out independently with replacement (the balls are put back into the bag after being drawn).

- (a) What is the probability that all three balls are red ?
- (b) What is the probability that one red and two black balls are drawn ?
- (c) Suppose now that each ball is not put back in the bag after being drawn i.e. the balls are drawn without replacement. Are the first and second balls drawn still independent ? Explain using the formal definition of independence.

Solution

- $(5/15)^3$
- $\binom{3}{2}(5/15)(10/15)^2$
- Let's look for example at the probability the first ball is red and the second ball is red. Let $R1$ be the event that the first ball is red and $R2$ the event that the second is red. $P(R1) = 5/15$. $P(R2) = P(R2|R1)P(R1) + P(R2|R1^c)P(R1^c) = (4/14)(5/15) + (5/14)(10/15)$. And $P(R1 \cap R2) = P(R2|R1)P(R1) = (4/14)(5/15)$. So $P(R1 \cap R2) \neq P(R1)P(R2)$ and we can conclude that the events are not independent. What's happening is that the chance of drawing a second red ball obviously depends on whether the first ball was red or white.

Question 4. Suppose two 6-sided dice are rolled. Let A be the event that the first die comes up 3 and B that the sum of the dice is 2. Are these events independent? Explain using the formal definition of independence.

Solution $P(A) = 1/6$, $P(B) = 1/36$, $P(A \cap B) = 0 \neq P(A)P(B)$ and so the events are not independent.

Question 5. In a class, there are 4 freshman men, 6 freshman women, and 6 sophister men. How many sophister women must be present if gender and year are to be independent when a student is selected uniformly at random?

Solution Let w be the number of sophister women. Suppose a student is selected at random. Let G be the gender and Y the year. We have $Prob(G = male \cap Y = freshman) = 4/(w + 16)$, $Prob(G = female \cap Y = freshman) = 6/(w + 16)$, $Prob(G = male \cap Y = sophister) = 6/(w + 16)$, $Prob(G = female \cap Y = sophister) = w/(w + 16)$. For independence we require $Prob(G = male \cap Y = freshman) = Prob(G = male)P(Y = freshman)$ etc. Now $Prob(G = male) = 10/(w + 16)$, $Prob(G = female) = (w + 6)/(w + 16)$, $P(Y = freshman) = 10/(w + 16)$, $P(Y = sophister) = (6 + w)/(w + 16)$.

For $Prob(G = male \cap Y = freshman) = 4/(w + 16) = Prob(G = male)P(Y = freshman) = 10/(w + 16) \times 10/(w + 16)$ we require $w = 9$. With this choice, $Prob(G = female \cap Y = freshman) = 6/25 = Prob(G = female)P(Y = freshman) = 15/25 \times 10/25$, $Prob(G = female \cap Y = sophister) = 9/25 = Prob(G = female)P(Y = sophister) = 15/25 \times 15/25$ as required.

Question 6. Suppose events E and F are independent, show that E and F^c are therefore also independent. Hint: start from $P(E) = P(E \cap F) + P(E \cap F^c)$.

Solution $P(E) = P(E \cap F) + P(E \cap F^c) = P(E)P(F) + P(E \cap F^c)$. Therefore $P(E \cap F^c) = P(E) - P(E)P(F) = P(E)(1 - P(F)) = P(E)P(F^c)$.

Question 7. A simplified model for the movement of the price of a stock supposes that on each day the stocks price either moves up 1 unit with probability p or moves down 1 unit with probability $1-p$. The changes on different days are assumed to be independent.

(a) What is the probability that after 2 days the stock will be at its original price?

(b) What is the probability that after 3 days the stocks price will have increased by 1 unit?

(c) Given that after 3 days the stocks price has increased by 1 unit, what is the probability that it went up on the first day?

Solution

- $P(\text{“unchanged”}) = P(\text{“moved up 1 and moved down one unit”}) = P(\text{“moved up 1”})P(\text{“moved down one unit”}) = p(1 - p)$ since independent.
- $P(\text{“increased by 1 unit”}) = \binom{3}{2}p^2(1 - p)$ (has to have increased two days and decreased one, and there are $\binom{3}{2}$ ways in which this might happen).
- $P(\text{“increased first day”} \mid \text{“increased by 1 unit”}) = P(\text{“increased first day”} \cap \text{“increased by 1 unit”}) / P(\text{“increased by 1 unit”})$. If increased on first day and only increased by one unit in total, then on next two days it must have increased by one and decreased by one. There are $\binom{2}{1}$ ways in which that might occur, so $P(\text{“increased first day”} \cap \text{“increased by 1 unit”}) = \binom{2}{1}p^2(1 - p)$ and $P(\text{“increased first day”} \mid \text{“increased by 1 unit”}) = \binom{2}{1}p^2(1 - p) / (\binom{3}{2}p^2(1 - p)) = 2/3$. Alternatively, since we know that it must have increased on two days out of three and these changes are independent we can arrive at $2/3$ directly.

Question 8. A true/false question is to be posed to a husband-and-wife team on a quiz show. Both the husband and the wife will independently give the correct answer with probability p . Which of the following is a better strategy for the couple?

(a) Choose one of them and let that person answer the question.

(b) Have them both consider the question, and then either give the common answer if they agree or, if they disagree, flip a coin to determine which answer to give.

Solution

- If choose one person then the probability of a correct answer is p .
- If both answer, then the two answers are correct (and so the same) with probability p^2 , differ (one correct, one incorrect) with probability $p(1-p)$ and both are incorrect with probability $(1-p)^2$. If they differ, with probability 0.5 the answer selected is correct. So the overall probability of a correct answer is $p^2 + 0.5p(1-p) + 0.5p(1-p) = p^2 + p - p^2 = p$. So the probability of being correct is the same for both strategies.