

TRINITY COLLEGE DUBLIN
School of Computer Science and Statistics

Extra Questions

ST3009: Statistical Methods for Computer Science

NOTE: There are many more example questions in Chapters 2 and 3 of the course textbook “A First Course in Probability” by Sheldon Ross.

Question 1. A box contains 3 marbles: 1 red, 1 green, and 1 blue. Consider an experiment that consists of taking 1 marble from the box and then replacing it in the box and drawing a second marble from the box. Describe the sample space. Repeat when the second marble is drawn without replacing the first marble.

Solution

- $S = \{(red, red), (red, green), (red, blue), (green, green), (green, red), (green, blue), (blue, blue), (blue, red), (blue, green)\}$
- $S = \{(red, green), (red, blue), (green, red), (green, blue), (blue, red), (blue, green)\}$

Question 2. Two dice are thrown. Let E be the event that the sum of the dice is odd, let F be the event that at least one of the dice lands on 1, and let G be the event that the sum is 5. Describe the events $E \cap F$, $E \cup F$, $F \cap G$, $E \cap F^c$, and $E \cap F \cap G$.

Solution

- $E \cap F$ is the sum is odd and at least one die lands on 1. So the event is the set $\{(1, 2), (1, 4), (1, 6), (2, 1), (4, 1), (6, 1)\}$
- $E \cup F$ is the sum is odd or at least one die lands on 1.
- $F \cap G$ is at least one die lands on 1 and the dice sum to 5, so the event is $\{(1, 4), (4, 1)\}$.
- $E \cap F^c$ is the sum is odd and no die lands on 1.
- $E \cap F \cap G$ is the sum is odd, at least one die lands on 1 and they the sum is 5, so the event is $\{(1, 4), (4, 1)\}$

Question 3. A committee of 5 is to be selected from a group of 6 men and 9 women. If the selection is made randomly, what is the probability that the committee consists of 3 men and 2 women?

Solution There are $\binom{15}{5}$ possible committees. Of these $\binom{6}{3}\binom{9}{2}$ consist of 3 men and 2 women. Since all committees are equally likely the probability is $\binom{6}{3}\binom{9}{2}/\binom{15}{5}$.

Question 4. An urn contains n balls, one of which is special. If k of these balls are withdrawn one at a time, with each selection being equally likely to be any of the balls that remain at the time, what is the probability that the special ball is chosen?

Solution There are $\binom{n}{k}$ ways to select the k balls. Of these k contain the special ball (namely when the special ball is picked first, picked second, and so on up to the k 'th ball). So the probability is k/n .

Question 5. If a six-sided die is rolled 4 times, what is the probability that 6 comes up at least once?

Solution The trick here is to calculate the probability that a 6 does not come up. There are 6^4 possible outcomes from the 4 rolls. Of these 5^4 contain no six. So the probability that no 6 is rolled is $5^4/6^4 = 0.4823$. Now $\text{Prob}(\text{“a 6”}) = 1 - \text{Prob}(\text{“no 6”}) = 0.5177$.

Alternatively we could have used $\text{Prob}(\text{“no 6”}) = (1 - 1/6)^4 = 0.4823$.

Question 6. A coin is flipped twice. Assuming that all four points in the sample space $S = \{(h, h), (h, t), (t, h), (t, t)\}$ are equally likely, what is the conditional probability that both flips land on heads, given that (a) the first flip lands on heads? (b) at least one flip lands on heads?

Solution

- Given that the first flip lands on heads the possible outcomes are $\{(h, h), (h, t)\}$, both of which are equally likely. The event that both land on heads $\{(h, h)\}$ therefore occurs with probability $1/2$. Alternatively, we can use the definition of conditional probability directly. Let E be the event $\{(h, h)\}$ that both are heads and F the event $\{(h, h), (h, t)\}$ that the first die is heads. Then $P(E|F) = P(E \cap F)/P(F) = P(\{(h, h)\})/P(\{(h, h), (h, t)\}) = (1/4)/(2/4) = 1/2$.
- Given at least one flip lands on heads the possible outcomes are $\{(h, h), (h, t), (t, h)\}$, all of which are equally likely. The event that both land on heads $\{(h, h)\}$ therefore occurs with probability $1/3$.

Question 7. Suppose that an urn contains 8 red balls and 4 white balls. We draw 2 balls from the urn without replacement. Let $R1$ and $R2$ denote, respectively, the events that the first and second balls drawn are red. If we assume that at each draw each ball in the urn is equally likely to be chosen, what is the probability $P(R1 \cap R2)$ that both balls drawn are red? Using marginalisation, what is the probability $P(R2)$ that the second ball is red.

Solution

- By the chain rule $P(R1 \cap R2) = P(R1)P(R2|R1) = (8/12)(7/11)$.
- $P(R2) = P(R2|R1)P(R1) + P(R2|W1)P(W1) = (7/11)(8/12) + (8/11)(4/12)$ where $W1$ is the event that the first ball is white.

Question 8. I toss a coin. If it comes up tails I throw a six-sided die and otherwise a 20-sided die. What is the probability that a 5 is thrown? What is the probability that a 15 is thrown? Hint: use marginalisation

Solution

- $\text{Prob}(\text{throw a } 1) = \text{Prob}(\text{throw a } 1\text{—}6\text{-sided die})P(6\text{-sided die}) + \text{Prob}(\text{throw a } 1\text{—}20\text{-sided die})P(20\text{-sided die})$. $\text{Prob}(\text{throw a } 1\text{—}6\text{-sided die})=1/6$ and $\text{Prob}(\text{throw a } 1\text{—}20\text{-sided die})=1/20$. $P(6\text{-sided die})=1/2=P(20\text{-sided die})$. So $\text{Prob}(\text{throw a } 1) = 1/6 \times 1/2 + 1/20 \times 1/2$.
- $\text{Prob}(\text{throw a } 15) = \text{Prob}(\text{throw a } 15\text{—}6\text{-sided die})P(6\text{-sided die}) + \text{Prob}(\text{throw a } 15\text{—}20\text{-sided die})P(20\text{-sided die})$. $\text{Prob}(\text{throw a } 15\text{—}6\text{-sided die})=0$ and $\text{Prob}(\text{throw a } 15\text{—}20\text{-sided die})=1/20$. So $\text{Prob}(\text{throw a } 15) = 1/20 \times 1/2$.

Question 9. An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The companys statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability 0.4, whereas this probability decreases to 0.2 for a person who is not accident prone. If we assume that 30 percent of the population is accident prone, what is the probability that a new policyholder will have an accident within a year of purchasing a policy? Hint: use marginalisation.

Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone? Hint: use Bayes rule.

Solution

- Let $A1$ denote the event that the policy-holder will have an accident within a year of purchasing the policy, and let A denote the event that the policyholder is accident prone. Then $P(A1) = P(A1|A)P(A) + P(A1|A^c)P(A^c) = 0.4 \times 0.3 + 0.2 \times 0.7 = 0.26$
- $P(A|A1) = P(A1|A)P(A)/P(A1) = 0.3 \times 0.4/0.26 = 0.4615$

Question 10. In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let p be the probability that the student knows the answer and $1-p$ be the probability that the student guesses. Assume that a student who guesses at the answer will be correct with probability $1/m$, where m is the number of multiple-choice alternatives. What is the conditional probability that a student knew the answer to a question given that he or she answered it correctly? Hint: use Bayes rule.

Solution Let C and K denote, respectively, the events that the student answers the question correctly and the event that he or she actually knows the answer. Then $P(K|C) = P(C|K)P(K)/P(C)$ with $P(C|K)P(K) = 1 \times p = p$ and $P(C) = P(C|K)P(K) + P(C|K^c)P(K^c) = p + (1/m)(1-p)$.

Question 11. A bin contains 3 different types of disposable flashlights. The probability that a type 1 flashlight will give over 100 hours of use is 0.7, with the corresponding probabilities for type 2 and type 3 flashlights being 0.4 and 0.3, respectively. Suppose that 20 percent of the flashlights in the bin are type 1, 30 percent are type 2, and 50 percent are type 3

What is the probability that a randomly chosen flashlight will give more than 100 hours of use?

Solution Let A denote the event that the flashlight chosen will give over 100 hours of use, and let F_j be the event that a type j flashlight is chosen, $j = 1, 2, 3$. To compute $P(A)$, we condition on the type of the flashlight and use marginalisation to obtain $P(A) = P(A|F_1)P(F_1) + P(A|F_2)P(F_2) + P(A|F_3)P(F_3) = 0.7 \times 0.2 + 0.4 \times 0.3 + 0.3 \times 0.5 = 0.41$.

Question 12. There are 8 cards in a hat: $1\heartsuit, 1\spadesuit, 1\diamondsuit, 1\clubsuit, 2\heartsuit, 2\spadesuit, 2\diamondsuit, 2\clubsuit$. You draw one card uniformly at random. If its rank is 1 you stop otherwise you draw one more card. Let X be the sum of the ranks on the one or two cards drawn. What is the probability that one card is drawn? What is the probability that two cards are drawn? Find $Prob(X = 1)$, $Prob(X = 3)$, $Prob(X = 4)$.

Solution

- Probability of one card is probability that draw $1\heartsuit, 1\spadesuit, 1\diamondsuit$ or $1\clubsuit$. There are four possible cards out of eight so the probability is $4/8$.
- Probability of two cards is probability that draw $2\heartsuit, 2\spadesuit, 2\diamondsuit, 2\clubsuit$. Again there are four possible cards out of eight so the probability is $4/8$.
- When one card is drawn $X = 1$, so $X = 1$ with probability $4/8$. When two cards are drawn then $X = 2 + \text{rank of second card}$. The probability the second card is rank 1 is $4/7$, the probability that it is rank 2 is $3/7$. So $X = 3$ with probability $4/8 \times 4/7$ and $X = 4$ with probability $4/8 \times 3/7$.

Question 13. There are four dice in a drawer, one with 4 sides, one with 6 sides and two with 8 sides. You pick one uniformly at random. Let S be the number of sides of the chosen die.

What is the probability that $S = 4, S = 6, S = 8$?

Without looking at the die to see what type it is, suppose you throw it and observe a 3. Using Bayes Rule calculate the probability that the die was 4 sided and the probability

that it was 6-sided. Which is the most likely die if a 6 is observed ? And if a 7 is observed ?

Solution

- $P(S = 4) = 1/4, P(S = 6) = 1/4, P(S = 8) = 2/4$
- Let random variable R be the value observed. We want $P(S = 4|R = 3)$. By Bayes, $P(S = 4|R = 3) = P(R = 3|S = 4)P(S = 4)/P(R = 3)$. $P(R = 3|S = 4) = 1/4, P(S = 4) = 1/4$ so we just need $P(R = 3)$. Now $P(R = 3) = P(R = 3|S = 4)P(S = 4) + P(R = 3|S = 6)P(S = 6) + P(R = 3|S = 8)P(S = 8) = 1/4 \times 1/4 + 1/6 \times 1/4 + 1/8 \times 2/4 = 1/6$. So $P(S = 4|R = 3) = 3/8$.
- $P(S = 6|R = 3) = P(R = 3|S = 6)P(S = 6)/P(R = 3) = (1/6 \times 1/4)/(1/6)$.
- If $R = 6$ then it must be either the 6 or the 8 sided die. $P(S = 6) = 1/4, P(S = 8) = 2/4$ so the 8-sided die is more likely.
- If $R = 7$ then it must be the 8-sided die.