TRINITY COLLGE DUBLIN

School of Computer Science and Statistics

Extra Questions ST3009: Statistical Methods for Computer Science

Question 1. Suppose two continuous valued random variables X and Y have the following joint PDF

$$f_{XY}(x,y) = \begin{cases} 0 & x < 0, y < 0 \\ 1 & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & x > 1, y > 1 \end{cases}$$

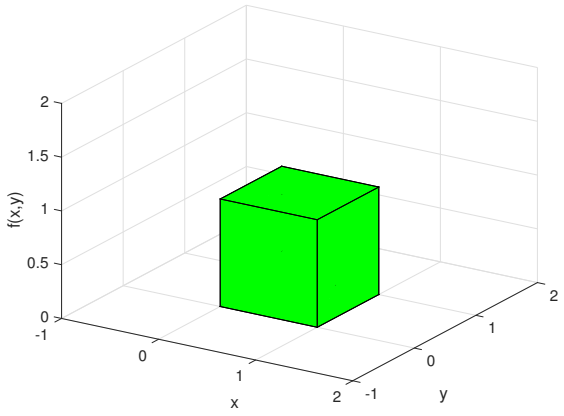

Figure 1: Plot of PDF $f_{XY}(x,y)$

(a) Calculate $P(0 \leq X \leq 0.5 \text{ and } 0 \leq Y \leq 0.5)$?
(b) Calculate $P(0 \leq X \leq 2 \text{ and } 0 \leq Y \leq 0.5)$?

Question 2. Suppose two continuous valued random variables X and Y have the following joint CDF

$$F(x,y) = \begin{cases} 0 & x < 0, y < 0 \\ xy & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ y, x > 1, 0 \leq y \leq 1 & \\ x, y > 1, 0 \leq x \leq 1 & \\ 1 & x > 1, y > 1 \end{cases}$$

(a) Sketch the graph of this CDF.
(b) Calculate $P(X \leq 0.5 \text{ and } Y \leq 0.5)$
(c) Calculate $P(0.1 \leq X \leq 0.5 \text{ and } 0.1 \leq Y \leq 0.5)$?
(d) Calculate $P(0 \leq X \leq 2 \text{ and } 0 \leq Y \leq 0.5)$?
(e) Are X and Y independent ? Hint: recall $P(X \leq x) = F(x, \infty)$.

Question 3. Suppose two continuous valued random variables X and Y have the following joint CDF

$$F(x,y) = \begin{cases} 0 & x < 0, y < 0 \\ x^2y/2 + xy^3/2 & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ x^2/2 + x/2 & 0 \leq 0 \leq x \leq 1, y \geq 1 \\ y/2 + y^3/2, x > 1, 0 \leq y \leq 1 & \\ 1 & x > 1, y > 1 \end{cases}$$

(a) Calculate $P(X \leq 0.5 \text{ and } Y \leq 0.5)$
(b) Calculate $P(0.1 \leq X \leq 0.5 \text{ and } 0.1 \leq Y \leq 0.5)$?
(c) Calculate $P(0 \leq X \leq 2 \text{ and } 0 \leq Y \leq 0.5)$?

Question 4. Suppose random variables X and Y have PDFs $f_X(x) = e^{-x}$, $f_Y(y) = 0.5e^{-0.5y}$. Suppose also the X and Y are independent.
(a) What is their joint PDF ?
(b) Sketch a graph of this PDF.

Question 5. Suppose random variable X had PDF

$$f_X(x) = \begin{cases} 0 & x \leq 0 \\ 1 & 0 < x \leq 1 \\ 0 & x > 1 \end{cases}$$

and random variable Y has PDF

$$f_Y(y) = \begin{cases} 0 & y \leq 0.5 \\ 1 & 0.5 < y \leq 1.5 \\ 0 & y > 1.5 \end{cases}$$

Suppose also the X and Y are independent.
(a) What is their joint PDF ?
(b) Sketch a graph of this PDF.

Question 6. Suppose two random variables X and Y have PDFs $f_X(x) = e^{-x}$, $f_Y(y) = 0.5e^{-0.5y}$ and conditional PDF $f_{Y|X}(y|x) = e^{-|x-y|}$. Using Bayes Rule for PDFs write an expression for $f_{X|Y}(x|y)$.

Question 7. (a) Give Bayes rule for PDFs
(b) Explain the difference between the maximum likelihood and the MAP estimate of a random variable
(c) Suppose after observing data the likelihood of parameter θ is $L(\theta) = e^{-(\theta-1)^2}$. What is the maximum likelihood estimate of θ ?

Question 8. Suppose and urn contains balls and that fraction θ of the balls are white and the rest are red. I draw n balls, with replacement, from the urn and let X be the number of white balls observed.
(a) Give an expression for the likelihood $P(X = x|\theta)$
(b) Suppose $n = 100$ and I observe 25 white balls. What is the maximum likelihood estimate for θ (use matlab to plot the value of $P(X = x|\theta)$ for a range of values of θ).
(c) Suppose now that before drawing the balls my prior probability was $P(\theta) = \frac{1}{20\pi}e^{-100(\theta-0.5)^2}$ and for simplicity assume that $P(X = 25) = 1$ (since it just scales the posterior). Give an expression for the posterior $P(\theta|X = x)$ (use Bayes rule).
(d) What is the MAP estimate for θ (use matlab to plot the value of $P(\theta|X = x)$ for a range of values of θ). Discuss why it differs from the maximum likelihood estimate.