

TRINITY COLLEGE DUBLIN
School of Computer Science and Statistics

Extra Questions

ST3009: Statistical Methods for Computer Science

Question 1. Suppose two continuous valued random variables X and Y have the following joint PDF

$$f_{XY}(x, y) = \begin{cases} 0 & x < 0, y < 0 \\ 1 & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & x > 1, y > 1 \end{cases}$$

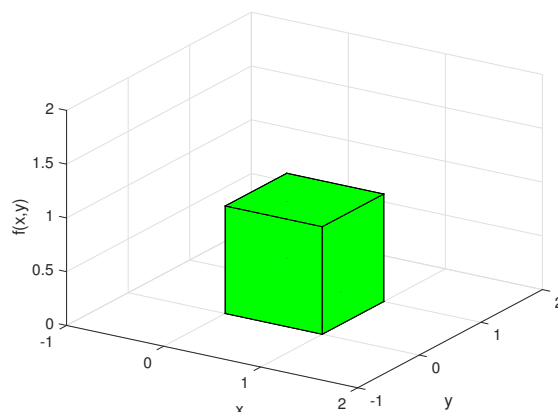


Figure 1: Plot of PDF $f_{XY}(x, y)$

- (a) Calculate $P(0 \leq X \leq 0.5 \text{ and } 0 \leq Y \leq 0.5)$?
- (b) Calculate $P(0 \leq X \leq 2 \text{ and } 0 \leq Y \leq 0.5)$?

Question 2. Suppose two continuous valued random variables X and Y have the following joint CDF

$$F(x, y) = \begin{cases} 0 & x < 0, y < 0 \\ xy & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ y, x > 1, 0 \leq y \leq 1 \\ x, y > 1, 0 \leq x \leq 1 \\ 1 & x > 1, y > 1 \end{cases}$$

- (a) Sketch the graph of this CDF.
- (b) Calculate $P(X \leq 0.5 \text{ and } Y \leq 0.5)$
- (c) Calculate $P(0.1 \leq X \leq 0.5 \text{ and } 0.1 \leq Y \leq 0.5)$?
- (d) Calculate $P(0 \leq X \leq 2 \text{ and } 0 \leq Y \leq 0.5)$?
- (e) Are X and Y independent ? Hint: recall $P(X \leq x) = F(x, \infty)$.

Question 3. Suppose two continuous valued random variables X and Y have the following joint CDF

$$F(x, y) = \begin{cases} 0 & x < 0, y < 0 \\ x^2y/2 + xy^3/2 & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ x^2/2 + x/2 & 0 \leq 0 \leq x \leq 1, y \geq 1 \\ y/2 + y^3/2, x > 1, 0 \leq y \leq 1 \\ 1 & x > 1, y > 1 \end{cases}$$

- (a) Calculate $P(X \leq 0.5 \text{ and } Y \leq 0.5)$
- (b) Calculate $P(0.1 \leq X \leq 0.5 \text{ and } 0.1 \leq Y \leq 0.5)$?
- (c) Calculate $P(0 \leq X \leq 2 \text{ and } 0 \leq Y \leq 0.5)$?

Question 4. Suppose random variables X and Y have PDFs $f_X(x) = e^{-x}$, $f_Y(y) = 0.5e^{-0.5y}$. Suppose also the X and Y are independent.

- (a) What is their joint PDF ?
- (b) Sketch a graph of this PDF.

Question 5. Suppose random variable X had PDF

$$f_X(x) = \begin{cases} 0 & x \leq 0 \\ 1 & 0 < x \leq 1 \\ 0 & x > 1 \end{cases}$$

and random variable Y has PDF

$$f_Y(y) = \begin{cases} 0 & y \leq 0.5 \\ 1 & 0.5 < y \leq 1.5 \\ 0 & y > 1.5 \end{cases}$$

Suppose also the X and Y are independent.

- (a) What is their joint PDF ?
- (b) Sketch a graph of this PDF.