

ST3009 Mock Mid-Term Test

Attempt **all** questions. Time: 1 hour 30 mins.

1. (i) Define the terms “sample space”, “event” and “random variable” and give an example of each. [10 points]
(ii) What is an indicator random variable and what is the probability mass function of a discrete random variable? [5 points]
(iii) Define the conditional probability of an event and state Bayes Theorem. [5 points]
(iv) Explain what is meant by “marginalization”. [5 points]
2. Suppose we have two bags, labeled A and B. Bag A contains 3 white balls and 1 black ball, bag B contains 1 white ball and 3 black balls. We toss a fair coin and select bag A if it comes up heads and otherwise bag B. From the selected bag we now draw 5 balls, one after another, replacing each ball in the bag after it has been selected (the bag always contains 4 balls each time a ball is drawn). We observe 4 white balls and 1 black ball. What is the probability that we selected bag A? Hint: use Bayes Rule. [20 points]
3. (i) Define the expected value of a random variable. Give a proof that the expected value is linear i.e. $E[X+Y]=E[X]+E[Y]$ for random variables X and Y. [5 points]
(ii) Define what it means for two random variables to be independent. Give a proof that when two random variables X and Y are independent then $E[XY]=E[X]E[Y]$. [5 points]
(iii) Define the covariance and correlation of two random variables X and Y. [5 points]
4. (i) A bag contains 30 balls, of which 10 are red and the other 20 blue. Suppose you take out 8 balls from this bag, with replacement. What is the probability that among the 8 balls in this sample exactly 3 are red and 5 are blue? [5 points]
(ii) Now suppose that the balls are taken out of the bag without replacement. What is the probability that out of 8 balls exactly 3 are red and 5 are blue? [10 points]