INTRODUCTION
BACKGROUND AND MOTIVATION

A brief history of economic contagion
CONTAGION EVENTS

1989 Brazilian crisis
1994 Mexican crisis – Latin American crisis
1998 Southeast Asian crisis
1999 Russian area crisis
2008 subprime mortgage crisis – global financial crisis
2009 European debt crisis
EUROPEAN DEBT CRISIS

- 2009-2012?
- Started in Greece
- Spread to Portugal, Italy, Ireland, and Spain
- PIIGS
- Fear of Spread across Europe
30 YEAR SPREADS
1. Increased interconnectedness through financial globalization and innovation

2. Contagion is an increase in cross-market linkages viewed through increased co-movements immediately following an adverse shock (Forbes and Rigobon 2001, Forbes and Rigobon 2002)

3. The source of contagion is often attributed to one of three channels; a trade channel, credit channel, or a common macroeconomic cause (Hernández and Valdés 2001)
THE FRAMEWORK

Our problem to address
THE MODEL
CONTAGION GAME

• We describe contagion in a game theoretic framework
• We describe a world comprised of players who make strategic decisions according to externalities
• Through extensive endogenization, we explain trade, credit, and common cause channels of contagion
THE PLAYERS & THEIR STRATEGIES

Setting up a game theoretic framework for contagion
There are $n_c$ countries defined by their respective federal governments and their citizens. Countries exclude the role of the central bank.

\[ c = \{c_1, ..., c_{n_c}\} \]

\[ S_c = \{B, D, r, R\} \]
Central Banks

There are $n_e$ domestic and regional central banks. We define central banks as lenders of last resort. E.g. the Federal Reserve as a domestic central bank and the European Central Bank as a regional central bank.

$$e = \{e_1, ..., e_{n_e}\}$$

$$S_e = \{B, D, r, \pi^*\}$$
BANKS
FIRMS

There are \( n_f \) firms who invest, trade, and participate in borrowing and lending activities in order to produce a single representative good

\[
f = \{f_1, \ldots, f_{n_f}\}
\]

\[
S_f = \{I, M, B, D\}
\]
There are $n_h$ households who labor, consume, invest, and participate in borrowing and lending activities.

\[ h = \{ h_1, \ldots, h_{n_h} \} \]

\[ S_h = \{ L, C, I, B, D \} \]
There are $n_q$ financial inter-governmental organizations (FIGOs).

E.g. the World Bank, IMF, as well as regional economic groups and development banks.

\[ q = \{q_1, \ldots, q_{n_q}\} \]

\[ S_q = \{B, D, r\} \]
SIX SETS OF PLAYERS

- **FIGOs**: \( q = \{q_1, \ldots, q_{n_q}\}, S_q = \{B, D, r\} \)
- **Countries**: \( c = \{c_1, \ldots, c_{n_c}\}, S_c = \{B, D, R, r\} \)
- **Central Banks**: \( e = \{e_1, \ldots, e_{n_e}\}, S_e = \{B, D, \pi^*, r\} \)
- **Banks**: \( b = \{b_1, \ldots, b_{n_b}\}, S_b = \{B, D, r\} \)
- **Firms**: \( f = \{f_1, \ldots, f_{n_f}\}, S_f = \{L, I, M, B, D\} \)
- **Households**: \( h = \{h_1, \ldots, h_{n_h}\}, S_h = \{C, I, B, D\} \)
THE MECHANICS

Setting up a default environment
A NOTE ON SETS

Six subsets of players such that \((c, e, b, h, q) \subseteq p\) where

\[ c_n \in c, e_n \in e, b_n \in b, f_n \in f, h_n \in h, q_n \in q \]

\((c, e, b, h, q)\) is collectively exhaustive but not mutually exclusive.

\[ i \cap j = \emptyset \ \forall \ (i, j) \in (e, b, f, h) : i \neq j \]
A NOTE ON SETS

However they are all members of a country such that

\[ b \cap c \neq \emptyset, f \cap c \neq \emptyset, h \cap c \neq \emptyset, e \cap c \neq \emptyset \]

Therefore, we say that a player \( i_n \in (b, f, h) \) is a member of country \( c_m \) if and only if

\[ i_n \cap c_m \neq \emptyset \]
A NOTE ON SETS

Furthermore a country $c_m$ has a central bank $e_n$ if and only if

$$c_m \cap e_n \neq \emptyset$$

and uses currency $\eta_n \in \eta$ if and only if

$$c_m \cap \eta_n \neq \emptyset$$

Lastly, FIGOs are independent and therefore exclusive from all other players such that

$$q \cap i = \emptyset \forall i \in (c, e, b, f, h)$$
BORROWING

We define borrowing as both positive and negative to indicate the two possible directions of cash flows.

A player who borrows positively is taking on debt while a player who borrows negatively is taking on credit.
BORROWING, $B_{i_nj_m t}$

\[
B_{i_nj_m t} \overset{\text{def}}{=} \begin{bmatrix}
B^+_{i_nj_m t} \\
-B^-_{i_nj_m t}
\end{bmatrix}
\]

where

$B^+_{i_nj_m t}$ is the amount player $i_n$ borrows positively from player $j_m$ at time $t$, and

$B^-_{i_nj_m t}$ is the amount player $i_n$ borrows negatively from player $j_m$ at time $t$. 
BORROWING COSTS,

\[ r_{injmt} \]

\[ r_{injmt} \overset{\text{def}}{=} \begin{bmatrix} r_{injmt}^+ & r_{injmt}^- \end{bmatrix} \]

where

\[ r_{injmt}^+ \] is the interest rate on \( B_{injmt}^+ \), and

\[ r_{injmt}^- \] the interest rate on \( B_{injmt}^- \).
DEBT, $d_{injmt}$

$$d_{injmt} \overset{\text{def}}{=} \begin{bmatrix} d^+_{injmt} \\ -d^-_{injmt} \end{bmatrix}$$

Traditionally,

$$d_{injmt} = (1 + r_{injmt})d_{injmt-1}$$

However, we will have to account for defaults
Player $i_n$ has the strategic choice of defaulting ($D_{i_n j_m t} = 1$) on positive debt, $d^+_{i_n j_m t}$.

Simple strategic default expression in the style of (Eaton & Gersovitz, 1981)

$$D_{i_n j_m t} = 1 \left( \sum_{\tau=t}^{\infty} \frac{r_{i_n j_m \tau} d^+_{i_n j_m \tau}}{(1 + r_{i_n j_m \tau})^{\tau-t}} > P_{j_m i_n t} \right)$$
ASSUMPTION 1: PENALTY

We claim that this penalty can be assessed as a cost equivalent to a fraction, $\psi_{jm_{int}}$, of the player $i_n$‘s utility at time $t$ such that

$$p_{jm_{int}} = \psi_{jm_{int}} u_{int}(x_{int}, x_{-int})$$
A DEFAULT THRESHOLD

Assume that there is a default threshold at

\[
\sum_{\tau=t}^{\infty} \frac{r_{i,n,j,m,\tau}^+ d_{i,n,j,m,\tau}^+}{(1 + r_{i,n,j,m,\tau}^+)^{\tau-t}} = P_{j,m,n,t}
\]

This threshold is formally defined as

\[
\tilde{a}_{i,n,j,m,t}^+ = \left( \sum_{\tau=t+1}^{\infty} \frac{(1 + r_{i,n,j,m,\tau}^+)^{\tau-t}}{r_{i,n,j,m,\tau}^+ d_{i,n,j,m,\tau}^+} \right) \frac{P_{j,m,n,t}}{r_{i,n,j,m,t}^+}
= \left( \sum_{\tau=t+1}^{\infty} \frac{(1 + r_{i,n,j,m,\tau}^+)^{\tau-t}}{r_{i,n,j,m,\tau}^+ d_{i,n,j,m,\tau}^+} \right) \psi_{i,n,t,U_{i,n,t}}(x_{i,n,t}, x_{-i,n,t}) \frac{\psi_{i,n,t}(x_{i,n,t}, x_{-i,n,t})}{r_{i,n,j,m,t}^+}
\]
ASSUMPTION II: PARTIAL DEFAULT

Players may default on only a portion of their debt. We define the percentage of partial default (haircut) of player $\phi_{injmt}$ such that

$$d_{injmt+1}^+ = B_{injmt}^+ + \left(1 - \phi_{injmt}D_{cnjmt}\right)d_{injmt}^+(1 + r_{injmt}^+)$$
ASSUMPTION II: PARTIAL DEFAULT

Assume players always pay out what they can at default.

Consequently, players pay the proportion that they can; the threshold value of debt over the current value of their debt:

$$\phi_{injmt} = \frac{\tilde{d}^+_{injmt}}{d^+_{injmt}}$$
LEMMA 1

Increased interest rate $r^+_{jm}$ with constant penalty, and decreased penalty $P_{jm}$ with constant interest rate, increases the likelihood of default.
LEMMA 1- PROOF

The probability of default for player $i_n$ on debt to player $j_m$ at time $t$ is

$$\Pr(D_{i_nj_m} = 1) = \Pr(r_{i_nj_m}^+ d_{i_nj_m}^+ > P_{j_m i_n}^t)$$
$$= \Pr(r_{i_nj_m}^+ d_{i_nj_m}^+ - P_{j_m i_n}^t > 0)$$

(5)

Given an interest rate $r'_{i_nj_m}^+$ such that $r'_{i_nj_m}^+ > r_{i_nj_m}^+$ and a constant penalty the monotone increasing property of the cumulative distribution function implies that

$$\Pr(r'_{i_nj_m}^+ d_{i_nj_m}^+ - P_{j_m i_n}^t > 0) \geq \Pr(r_{i_nj_m}^+ d_{i_nj_m}^+ - P_{j_m i_n}^t > 0)$$

(6)

Therefore, an increase in the borrowing rate leads to an increase in the probability of default.
STRATEGIC CHOICE VARIABLES

A closer look at the strategic choices that comprise the game
A DEBT MARKET

Borrowing: $B_{injmt} \overset{\text{def}}{=} \begin{bmatrix} B_{injmt}^+ \\ -B_{injmt}^- \end{bmatrix}$

Borrowing costs: $r_{injmt} \overset{\text{def}}{=} \begin{bmatrix} r_{injmt}^+ \\ r_{injmt}^- \end{bmatrix}$

Debt: $d_{injmt} \overset{\text{def}}{=} \begin{bmatrix} d_{injmt}^+ \\ -d_{injmt}^- \end{bmatrix}$

Default: $D_{injmt}, P_{jmint}, \Psi_{jmint}$
THE GOODS MARKET

Household Consumption, $c_{hnt}^m \in \mathbb{R}^+$

Net transfer to government, $R_{c_{nt}} = R_{f_{nt}} + R_{h_{nt}} \in \mathbb{R}$

Imports, $M_{f_{nt}m_{mt}} \in \mathbb{R}^+$
THE LABOR MARKET

Household labor supply: $L_{ht} \in [0,1]$

(Exogenous wages: $w_{ht} \in \mathbb{R}^+$)
ADDITIONAL STRATEGIC VARIABLES

Inflation target: $\pi_{en}^* \in [0,1]$

Private Investment: $I_{in} \in \mathbb{R}^+ \forall i_n \in (h, f)$
PARAMETERS

A description of exogenous variables
\( \bar{Y}_{cn}^t \): GDP potential of country \( c_n \) at time \( t \)

\[ \bar{Y}_{cn}^t \in \mathbb{R}^+, \quad \forall \, c_n \in c \]

\( \bar{Y}_{cn}^u_t \): GDP potential of monetary union \( c_n^u = \{ c_m: c_m \cap e_n \neq \emptyset \} \), where the length \( |c_n^u| \) is the number of countries in central bank monetary union,

\[ \bar{Y}_{cn}^u_t = \sum_{c_m \in c_n^u} \bar{Y}_{cn}^t, \]
\( \pi_{e_n t} \in \mathbb{R} \): Inflation rate of central bank \( e_n \)'s monetary union \( c_n \) at time \( t \)

\( r^*_{e_n t} \in [0,1] \): Assumed equilibrium real interest rate of central bank \( e_n \) at time \( t \)

\( \mu_t \in \mathbb{R} \): Market risk (systematic) at time \( t \)
$\alpha_{c_n} \in [0,1]$: Elasticity parameter for country $c_n$’s utility

$\varphi_{c_n} \in \mathbb{R}^+: \text{Parameter weighing production } Y_{c_n t} \text{ against borrowing costs and revenues } 1/\Phi_{c_n t} \text{ in country } c_n \text{’s utility } U_{c_n t}$

$\gamma_{c_n} \in \mathbb{R}^+: \text{Debt seeking parameter of country } c_n$

$\zeta_{c_n} \in \mathbb{R}^+: \text{Debt aversion parameter of country } c_n$
\( a_\pi > 0 \): Taylor rule weight on inflation

\( a_Y > 0 \): Taylor rule weight on GDP

\( \beta_{e_n} \in [0,1] \): Relative weight of central bank \( e_n \)'s monetary union’s utilities against profit and loss from banking activities

\( \omega_{q_n} \in [0,1] \): Weight given to the collective benefit of the countries to which FIGO \( q_n \) lends against its own costs of borrowing
DEPENDENT VARIABLES

A description of endogenous variables
$d_{injm_t}$: Debt of player $i_n$ to player $j_m$ at time $t$, $(i_n,j_m) \in p$

$$d_{injm_t} = \left[ (1 - D_{injm_t}) \left( B_{injm_t}^+ + Q^+ \right) \right]$$

$$\left( D_{jm in t} - 1 \right) \left[ B_{injm_t}^- + Q^- \right]$$

$$Q^+ = \sum_{\tau=1}^{t-1} \left\{ \begin{aligned} B_{injm_{\tau}}^+ & \prod_{k=\tau}^{t-1} (1 - D_{injm_{\tau}})(1 + r_{injm_{\tau}}) \end{aligned} \right\}$$

$$Q^- = \sum_{\tau=1}^{t-1} \left\{ \begin{aligned} B_{injm_{\tau}}^- & \prod_{k=\tau}^{t-1} (1 - D_{jm in k})(1 + r_{jm in k}) \end{aligned} \right\}$$
$x_{i_nt} \in S_{i_n}$: Strategy profile of player $i_n$ at time $t$, $i_n \in p$

$x_{-i_nt} \in S_{i_n}$: Strategy profile of all players not including player $i_n$ at time $t$, $-i_n \in p$

$U_{i_nt}(x_{i_nt}, x_{-i_nt}) \in \mathbb{R}$: Utility of player $i_n$ at time $t$, $i_n \in p$
$M_{cnt}$: Imports to country $c_n$ from all other countries at time $t$

$$M_{cnt} = \sum_{c_m \in c, \atop c_m \neq c_n} M_{c_mcmt}, (c_n, c_m) \in c$$

$X_{cnt}$: Exports from country $c_n$ to all other countries at time $t$

$$X_{cnt} = \sum_{c_m \in c, \atop c_m \neq c_n} M_{c_mcnt}, (c_n, c_m) \in c$$
$Y_{c_n t} \in \mathbb{R}^+ \text{: GDP of country } c_n \text{ at time } t, c_n \in c$

$Y_{c_n^u t} \text{: GDP of monetary union } c_n^u \text{ at time } t, c_n^u \in c^u$

$$Y_{c_n^u t} = \sum_{c_n \in c_n^u} Y_{c_n t} \in \mathbb{R}^+$$
$\lambda_{j_m t} \in \mathbb{R}$: Idiosyncratic risk (unsystematic) on negative borrowing to player $j_m$ at time $t$, $j_m \in p$

$$\lambda_{j_m t} = \sup_{i_n \in p} \left( \phi_{j_m i_n t} \Pr \left( D_{j_m i_n t} = 1 \right) \right)$$

$s \eta_k \eta_{l t} \in \mathbb{R}^+$: Exchange rate as spot price (rate): value of currency $\eta_k$ in terms of currency $\eta_l$, $k \neq l, k \neq l$
EXCHANGE RATE MODEL

An uncovered interest parity model of foreign exchange in a default environment
UNCOVERED INTEREST PARITY

- Exchange rates are determined using uncovered interest parity (UIP).

- A simple model of uncovered interest parity states that

\[ r_{Ft} + E_t \ln(s_{t+1}) - \ln(s_t) - r_{Dt} = 0 \]

- where \( s_t \) is the exchange rate, and \( i_{Dt} \) and \( i_{Ft} \) are the domestic and foreign interest rates
REALITY CHECK

\[ r_{Ft} + E_t \ln(s_{t+1}) - \ln(s_t) - r_{Dt} = 0 \]

This equation will not hold in reality. Due to default and market risks,

\[ r_{Ft} + E_t \ln(s_{t+1}) - \ln(s_t) - r_{Dt} \geq 0 \]
UIP WITH DEFAULT

We have default probability: \( \Pr(D_{cnimt} = 1) \)

with haircut: \( \phi_{cnit} \)

and risk free rate: \( rf_{cnt} \).

Therefore \( r_{cnimt}^- = rf_{cnt} - \sup_{im \in p} \left( \Pr(D_{cnimt} = 1) \right) \)
UIP WITH DEFAULT

Expected rate of depreciation for the foreign currency

\[ \hat{\delta}_{\eta|mt+1}^{ND}, \quad \hat{\delta}_{\eta|mt+1}^{D} \]

Thus, the unconditional expected rate of depreciation of the foreign currency is:

\[ \hat{\delta}_{\eta|mt+1} = \Pr(D_{cni|mt} = 0) \hat{\delta}_{\eta|mt+1}^{ND} + \Pr(D_{cni|mt} = 1) \hat{\delta}_{\eta|mt+1}^{D} t \]

Then the effective interest rate is

\[ r_{cni|mt}^* = r_{f|nt} + \hat{\delta}_{\eta|mt+1} + \Pr(D_{cni|nt} = 1) \phi_{cni|nt} \]
UIP WITH DEFAULT

\[ r_{cm}^{*} - r_{cn}^{*} + \mu_t = \ln \left( E_t \left[ s \eta_k \eta_{l+1} \right] \right) - \ln \left( s \eta_k \eta_l \right) \]

such that \( c_m \cap \eta_k \neq \emptyset, c_n \cap \eta_l \neq \emptyset \).

Thus

\[ \ln \left( E_{t-1} \left[ s \eta_k \eta_l \right] \right) = r_{cm}^{*} - r_{cn}^{*} + \mu_{t-1} + \ln \left( s \eta_k \eta_{l-1} \right) \]

\[ \implies \hat{s} \eta_k \eta_l = s \eta_k \eta_{l-1} \exp \left( r_{cm}^{*} - r_{cn}^{*} + \mu_{t-1} \right) \]
EQUATIONS FOR UTILITY

Description of decision making equations
Countries have GDP production function $Y_{cnt}$

$$Y_{cnt} = A_{cnt}K_{cnt}^\alpha (1 - L)_{cnt}^{1-\alpha}$$

where

$$K_{cnt} = \sum_{h_m \in c_n} I_{ht} + \sum_{f_t \in c_n} I_{ft}, \quad L_{cnt} = \sum_{h_m \in c_n} L_{ht} + \sum_{f_t \in c_n} L_{ft}$$
Countries have the objective to maximize social welfare.

\[
U(x_{c_n t}, x_{-c_n t}) = \sum_{t}^{T} \beta^{t} \left[ \sum_{h_m \in c_n} U(x_{h_m t}, x_{-h_m t}) \right]
\]

Subject to a budget constraint

\[
\sum_{h_m \in c_n} R_{h_m t} + \sum_{f_i \in c_n} R_{f_i t} + \sum_{j_m \in p} \left[ B_{c_n j_m t}^+ + (1 - \phi_{j_m c_n t} D_{j_m c_n t})r_{c_n j_m t-1}^- a_{c_n j_m t-1}^- + P_{c_n j_m t} D_{j_m c_n t} \right] - \sum_{j_m \in p} \left[ B_{c_n j_m t}^- + (1 - \phi_{c_n j_m t} D_{c_n j_m t})r_{c_n j_m t-1}^+ d_{c_n j_m t-1}^+ + P_{j_m c_n t} D_{c_n j_m t} \right] + \sum_{q_m \in q} E_{c_n q_m t}
\]
Central Banks set lending rates according to the Taylor Rule:

\[
\tilde{r}_{e_{nt}} = \pi_{e_{nt}} + r_{e_{nt}}^* + a_{\pi}(\pi_{e_{nt}} - \pi_{e_{nt}}^*) + a_Y \left( \log \left( \frac{Y_{C_{nt}}}{Y_{C_{nt}}^*} \right) \right)
\]

And weigh objectives of maximizing the utility of its member countries as well as its balance sheets

\[
U_{e_{nt}}(x_{e_{nt}}, x_{-e_{nt}}) = \sum_{\tau=t}^{T} \beta^\tau \left[ \omega_e \bar{U}_{C_{nt}\tau}(x_{e_{nt}}, x_{-e_{nt}}) - (1 - \omega_e) \Pi_{e_{nt}} \right]
\]
UTILITY OF A CENTRAL BANK

\[ \Pi_{e nt} = \sum_{j_m \in p} r_{enj_m t}d_{enj_m t} + \left( \sum_{e_m \in e} r_{emt} s_{emt} \chi_{emt} - \sum_{e_n \in e} r_{ent} s_{ent} \chi_{ent} \right) \]
The inclusion of gain and loss on currency exchange in the central banks utility function reflects the sensitivity of demand for currency exchange to changes in interest rates.

A positive interest rate change thus has the effect of increasing currency demand while a negative change decreases demand. The value of exchange rates are therefore determined by the strategic choices of each central bank.
AGGREGATE UTILITY OF A MONETARY UNION

\[ \bar{U}_{c_{nt}}(x_{ent}, x_{-ent}) = \sum_{c_n \in c_{nt}} U_{c_nt}(x_{c_nt}, x_{-c_nt}) \]
BANK LENDING RATES

Banks borrow and set interest rates in order to make profits.

They set interests according to

\[ r_{b_n b_m t}^- = r_{b_n e_k t}^+ + \mu_t \forall (b_n, b_m) \in b, e_k \in e \]

We assume that \( r_{b_n b_m t}^- = r_{b_i b_j t}^- \forall (b_n, b_m, b_i, b_j) \in b \)

\[ r_{h_m b_n t}^- = r_{b_n b_k t}^+ + \lambda_{h_m t} \forall (b_n, b_k) \in b, h_m \in h \]
**BANK UTILITY**

Banks are strictly profit seeking

\[
U_{bnt}(x_{bnt}, x_{-bnt}) = -\sum_{t=T}^{\tau} \beta^\tau \left( \sum_{j_m \in p} \left[ r_{bntj}d_{bnjnt} + P_{jm}b_{nt}D_{bnjnt} - P_{bntj}D_{jmbnt} 
+ B_{bntj}^-D_{bnjnt} - B_{jm}^-b_{nt}D_{jmbnt} \right] + R_{bnct} \right)
\]
Firm Utility

Firms borrow, invest, produce, import, export, pay taxes, pay wages, and are able to default. Given their profit seeking nature, the utility function of the firm is

\[ U(x_{ft}, x_{-ft}) = \sum_{t=1}^{T} (\beta_{ft})^t \Pi_{ft} \]
\[ \Pi_{fn} = Y_{fn} + B_{fn} + \delta I_{fn-1} - I_{fn} - \sum(w_{hi}L_{hi}) - (1+r)d_{hi-1} \\
+ \left( M_{fn} - X_{fn} \right) - R_{fn}c_{mt} \\
+ \sum_{j_m \in p} \left[ B^+_{fnj} r^-_{fnj} - d^+_{fnj} - d^-_{fnj} - 1 + P_{fnj}D_{fnj} \right] \\
- \sum_{j_m \in p} \left[ B^-_{fnj} r^+_{fnj} - d^+_{fnj} - d^-_{fnj} - 1 + P_{fnj}D_{fnj} \right] \\
\]

\[ M_{ft} = \sum_{c_j \in c} \eta_{nc} \eta_{mt} M_{fcj} \] , \[ X_{ft} = \sum_{c_j \in c} X_{fcj} \]
THE HOUSEHOLD

The HH utility function is Cobb-Douglass and balances consumption and labor

\[ U(x_{ht}, x_{-ht}) = \sum_{\tau=t}^{T} (\beta_{ht})^{\tau} C_{ht}^{\alpha} L_{ht}^{1-\alpha} \]
HOUSEHOLD BUDGET CONSTRAINT

\[ \begin{align*}
B_{hn,t} + I_{hn,t} + C_{hn,t} + R_{hn,cmt} \\
+ \sum_{j_m \in p} \left[ B^{-}_{fnjm_t} + (1 - \phi_{fnjm_t} D_{fnjm_t}) r^+_{fnjm_{t-1}} d^+_{fnjm_{t-1}} + P_{jmfn_t} D_{fnjm_t} \right] \\
= w_{hn_t} L_{hn_t} + \delta I_{hn_{t-1}} \\
+ \sum_{j_m \in p} \left[ B^+_{fnjm_t} + (1 - \phi_{jmfn_t} D_{jmfn_t}) r^-_{fnjm_{t-1}} d^-_{fnjm_{t-1}} + P_{fnjm_t} D_{jmfn_t} \right]
\end{align*} \]
FIGOs receive money in the form of donations from various players and lend money to selected countries assessing market risk $\mu_t$ and idiosyncratic risk $\lambda_{j_m t}$.

They lend according to

$$r_{q_n j_{m t}}^- = \pi_{j_{m t}} \forall j_{m} \in p$$
FIGO UTILITY

The FIGO weighs the collective benefit of the countries to which it lends against its own costs of borrowing as follows:

\[
U_{q_n T}(x_{q_n T}, x_{m-q_n T}) = \sum_{\tau=t}^{T} \left( \beta_{q_n} \right)^{T} \omega_{q_n} \left( \sum_{c_m \in c} \frac{U_{c_m \tau}(x_{c_n \tau}, x_{m-c_n \tau})}{U_{c_m \tau-1}(x_{c_n \tau-1}, x_{m-c_n \tau-1})} \right)
- \left( 1 - \omega_{q_n} \right) \left( \sum_{j_m \in p} r_{q_n j_m \tau} d_{q_n j_m \tau} - \sum_{c_m \in c} E_{c_m q_n \tau} \right)
\]
CONDITIONS FOR NASH EQUILIBRIUM
We determine a pure-strategy Nash Equilibrium where all players maximize their utility functions.

We define an equilibrium as a set of strategies $(x_{c_{nt}}^*, x_{e_{nt}}^*, x_{b_{nt}}^*, x_{h_{nt}}^*, x_{q_{nt}}^*)$ such that

$$U_{int}(x_{int}^*, x_{-int}^*) \geq U_{int}(x_{int}, x_{-int}) \quad \forall \, i \in p, \, x_{int} \in S_i, \, i \in p, \, x_{int} \neq x_{int}^*$$
STYLIZED SIMULATIONS

Simulations of several examples
ONE COUNTRY EXAMPLE

Stylized Simulation for one country
TWO COUNTRY EXAMPLE
Stylized Simulation for two players
TWO COUNTRY & ONE BANK EXAMPLE

Stylized Simulation for three players
NEXT STEPS
TO BE ADDED TO THIS PAPER

• Additional Simulations…
TO BE Addressed IN FUTURE PAPERS

• Stochastic interest rates and stochastic shocks
• Monte Carlo simulations
THANK YOU