Analyzing Decision Situations with Decision Circuits

Debarun Bhattacharjya (IBM T. J. Watson Research Center)
Ross Shachter (Stanford University)
Outline

- Appraisal in Decision Analysis
- Decision Circuits
- Analyzing Decision Situations
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- Appraisal in Decision Analysis
  - Decision Circuits
  - Analyzing Decision Situations
The Role of Appraisal

- **Formulation**
- **Evaluation**
- **Appraisal**

Decide

Uncertainties

- Decisions
- Preferences
Some Influence Diagram Examples

**Example I:**

The decision maker owns an oil field. Utility (U) is a function of the Drilling strategy (D) and the Amount of oil (A).

**Example II:**

The decision maker can conduct a seismic Test (T), resulting in a Report (R). Her information gathering decision and the report will be known to her when she decides her drilling strategy.
A Medical Decision Making Example

CPCS belief network, 422 nodes  [Pradhan et al, 1994]

Slide from Breese and Koller, AAAI tutorial, 1997.
Why Analyze?

- Parameter Refinement
- Defending a Strategy
- Model Robustness
- Creative Alternatives
- Auxiliary Decisions
Outline

- Appraisal in Decision Analysis

  ▪ Decision Circuits

- Analyzing Decision Situations
### Parameters for Dynamic Programming Functions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Node</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lambdas:</strong></td>
<td><strong>Uncertainty</strong></td>
<td>There is a mask ( \lambda_x ) for each possible state ( x ) of each uncertainty ( X ). The mask will be 0 when the observed evidence is inconsistent with ( x ) and 1 o/w.</td>
</tr>
<tr>
<td><strong>Masks (Indicators)</strong></td>
<td><strong>Decision</strong></td>
<td>There is a mask ( \lambda_d ) for each possible alternative ( d ) of each decision ( D ).</td>
</tr>
<tr>
<td><strong>Utility</strong></td>
<td>There is a mask for the two outcomes of ( U ) (best and worst prospects).</td>
<td></td>
</tr>
<tr>
<td><strong>Thetas:</strong></td>
<td><strong>Uncertainty</strong></td>
<td>There is a probability ( \theta_{x</td>
</tr>
<tr>
<td><strong>Probabilities/ Feasibility Parameters/ Utilities</strong></td>
<td><strong>Decision</strong></td>
<td>There is a feasibility parameter ( \theta_{d</td>
</tr>
<tr>
<td><strong>Utility</strong></td>
<td>There is a utility ( \theta_{u</td>
<td>\text{pa}(U)} ) for the two states of ( U ) (best and worst prospects) and each scenario ( \text{pa}(U) ) of its parents ( \text{Pa}(U) ).</td>
</tr>
</tbody>
</table>

#### Dynamic programming function:

\[
\max_d \left[ \lambda_d \theta_d \left( \sum_a \lambda_a \theta_a \left( \sum_u \lambda_u \theta_{u|ad} \right) \right) \right] 
\]
An Example Decision Circuit

A decision circuit is a rooted, directed acyclic graph whose leaf nodes are labeled with constants or variables and whose other nodes are either summation, multiplication, or maximization.

Dynamic programming function:

$$\max_d \left[ \lambda_d \theta_d \left( \sum_a \lambda_a \theta_a \left( \sum_u \lambda_u \theta_u|ad \right) \right) \right]$$
Sweeping through Decision Circuits

<table>
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<tr>
<th>Amount of oil</th>
<th>Prob.</th>
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<tr>
<td>high</td>
<td>0.35</td>
</tr>
<tr>
<td>low</td>
<td>0.65</td>
</tr>
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\[ u(.) - \text{Exponential} \]
\[ \gamma = 0.002 \]
Sweeping through Decision Circuits

Root: \( g(e' | s^*) \)

Certain Equivalent (CE): = 3.17$M

- The optimal strategy \( s^* \) is computed on the upward sweep
- Partial derivatives are computed on a downward sweep

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</table>

<table>
<thead>
<tr>
<th>U</th>
<th>Value (M $)</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>yes high</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>yes low</td>
<td>-50</td>
</tr>
<tr>
<td></td>
<td>no high</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>no low</td>
<td>0</td>
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\( u(.) - \text{Exponential} \)

\( \gamma = 0.002 \)
Outline

- Appraisal in Decision Analysis
- Decision Circuits

Analyzing Decision Situations
There is a wealth of partial derivative information available from decision circuits, which can be used for efficiently analyzing the decision maker’s decision situation.
Types of Analyses

- One-way sensitivity analysis plots
- Sensitivity to risk aversion
- Bounds on admissible intervals
- Comparing strategies
- Decision circuit
- Value of information
- Value of perfect hedging
Comparing Strategies: Main Result

Theorem (Comparing Strategies):
If we change the current optimal policy for decision D from $d^*(\text{pa}(D))$ to $d'(\text{pa}(D))$, keeping all other policies fixed at the current optimal strategy $s^*$, the CE for the new strategy $s'$ is:

$$CE(s') = u^{-1} \left( \frac{1}{g(e | s^*)} \left[ \sum_{\text{pa}(D)} \frac{\partial g(e' | s^*)}{\partial \theta_{d''|\text{pa}(D)}} \right] \right)$$

Implications

- The CE for the new strategy $s'$ (as defined above) is a function of partial derivatives w.r.t the old strategy $s^*$.
- We can efficiently find the CE of a new strategy, given that all earlier decisions remain the same and all future decisions are re-optimized.
- The order of complexity of this method is $O(|\text{pa}(D)|)$. 
Comparing Strategies: Example
Comparing Strategies: Example

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Change from s*</th>
<th>CE (M $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test and drill if “positive”, don’t drill if “negative” (s*)</td>
<td>No change</td>
<td>5.21</td>
</tr>
<tr>
<td>Don’t test, drill</td>
<td>Change Test policy to: Don’t test</td>
<td>3.17</td>
</tr>
<tr>
<td>Test and drill regardless of the seismic report</td>
<td>Change Drill policy to: Always drill, regardless of the seismic report</td>
<td>-16.83</td>
</tr>
</tbody>
</table>

APPLICATIONS OF STRATEGY COMPARISONS:

1. DM may be at that decision epoch and therefore already made the earlier decisions.
2. There are practical conditions (like organizational reasons) which restrict the earlier decisions.
Sensitivity to Risk Aversion: Main Result

**Theorem (Sensitivity to Risk Aversion):**
For a decision maker with an exponential utility function, $CE(s^*)$ can be written as a function of the risk aversion coefficient $\gamma$:

$$CE\left(s^*\right) = \frac{1}{\gamma} \ln \left[ \sum_{pa(U)} \frac{\partial g\left(e^i | s^*\right)}{\partial \theta_{i|pa(U)}} \times e^{-\gamma*V\left(pa(U)\right)} \right]$$

**Implications**
- There is a closed-form, non-linear relationship between $CE(s^*)$ and the risk aversion coefficient, and we can plot this graph using the original partial derivatives.
- The order of complexity of this method is $O(R * pa(U))$, where $R$ is the number of points over which the plot is drawn.
- The caveat: this plot is obtained at a particular strategy.
Sensitivity to Risk Aversion: Example

Current risk aversion = 0.002

Current optimal strategy
Drill without testing
Neither test nor drill
### Sensitivity to Risk Aversion: Example

<table>
<thead>
<tr>
<th>Strategy</th>
<th>CE at $\gamma = 0.002$ (in M $)</th>
<th>CE at $\gamma = 0.01$ (in M $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test and drill if “positive”, don’t drill if “negative” (s*)</td>
<td>5.21</td>
<td>-5</td>
</tr>
<tr>
<td>Don’t test, don’t drill</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Drill without testing</td>
<td>3</td>
<td>-17</td>
</tr>
</tbody>
</table>

#### APPLICATION OF SENSITIVITY TO RISK AVERSION:

Previous research models a more **flexible** plan, i.e. one that can handle unanticipated or unmodeled uncertainty, as equivalent to one that corresponds to a higher risk aversion. In this example, the price of flexibility = $ (5.21 – 0) M = $ 5.21 M.
The VoPH is defined as the most that the decision maker (DM) should be willing to pay for the best possible deal $Y$ that is determined solely by $X$ (i.e. $Y$ is a deterministic function of $X$) such that $E[Y] = 0$.

**QUESTIONS:**
1. What is your CE for deal #1?
2. Is your CE for deal #2 the same as deal #1?
3. What is your CE for deal #2 if you already own deal #1?

**Some properties of VoPH**
- A DM should not pay more than $E[Y] + \text{VoPH}(X)$ for any deal $Y$ determined solely by $X$.
- VoPH is non-negative and zero for a risk averse and risk neutral DM respectively.
- VoPH is zero when the uncertainty is not relevant to the value.
Theorem (VoPH – a general result):
For a decision maker with an exponential utility function, the perfect hedge Y (determined solely by X) at s* is:

\[
y(x) = E[CE(s^* | X)] - CE(s^* | x)
\]

The VOPH at s* is:

\[
VoPH(X | e, s^*) = E[CE(s^* | X)] - CE(s^*)
\]

Theorem (VoPH – partial derivatives):

\[
CE(s^* | x) = u^{-1} \left( \frac{\partial g(e' | s^*)}{\partial \lambda_x} / \frac{\partial g(e | s^*)}{\partial \lambda_x} \right)
\]

\[
E[CE(s^* | X)] = \frac{1}{g(e | s^*)} \sum_x \frac{\partial g(e | s^*)}{\partial \lambda_x} CE(s^* | x)
\]
Value of Perfect Hedging: Some Intuition

**Deal Before Perfect Hedge**
- $CE(s^*|x_1)$
- $CE(s^*|x_2)$
- $CE(s^*|x_3)$

**Deal After Perfect Hedge**
- $E[CE(s^*|X)]$

$CE$ of this deal = $CE(s^*)$

$CE$ of this deal = $E[CE(s^*|X)]$

Value of Perfect Hedging: Some Intuition
Using Decision Circuits

- Real-time decision making
- Decision systems
- Time-critical applications

Computational representation that provides users with insights into their decision situation
References on Decision Circuits


