The value of information in some stopping problems

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Introduction

• We consider some stopping problems, and study the implications of the properties of the value of perfect and imperfect information.

• Stopping problems represent a general set of problems where candidates are made sequentially available and the decision maker has to decide whether to stop and pick the candidate or continue.

• Applications of Stopping Problems:

• What if we do not observe the candidate perfectly? What would be the value of information?
Stopping Problems

The basic problem:
- Candidates whose values are continuous independent random variables $X_1, X_2, \ldots, X_N$
- Fixed cost $c \geq 0$ for surveying
- Objective is to maximize the expected value to the decision maker

Special cases:
- Identical distribution
- Normal distribution
- Dependent candidates

More variations:
- With or without recall
- Exit allowed or disallowed
- Finite or infinite horizon
- Distribution parameters known or learned over time
- Risk aversion

Our Base Set Up
- Independent candidates
- Known Parameters
- Finite Horizon
- No recall
- Exit disallowed
- Risk neutrality
Information Gathering in Stopping Problems

• **Completely Observable Stopping problem (COS)**
  The basic problem usually assumes that candidates are *completely observable upon being surveyed*  
  [MacQueen and Miller 1960, Chow and Robbins 1961, Sakaguchi 1961, etc.]

• **Partially Observable Stopping problem (POS)**
  The decision maker observes imperfect information about the value of a candidate when surveyed
  ▪ We observe Y rather than X and need a model for likelihood of Y|X
  ▪ Linked to some literature on VoI computations [Monahan 1980]

• **Not Observable Stopping problem (NOS)**
  The decision maker does not get to observe any further information about the candidate, even when surveyed
  ▪ Clueless decision maker: Diamond, Work of art, Start-up
Not Observable Stopping Problem

• Last candidate surveyed

\[ g_N^N = \mu_N \]

• \( n^{th} \) candidate surveyed with \( n < N \)

\[ g_n^N = \max(\mu_n, g_{n+1}^N - c) \]

\& \[ G_{NOS}^N = \max(0, g_1^N) \]

Optimal policy when all candidates have the same distribution with mean \( \mu \):

(i) Pick the first candidate if \( \mu > c \)
(ii) Don’t enter the process otherwise
Partially Observable Stopping Problem

- $\mu_n(y) = E[X_n | y]$

- General recursion for $n < N$

  $$\gamma_n^N(y) = \max(\mu_n(y), \tau_n^N - c),$$

- Where

  $$\tau_n^N = \int_{Y_{n+1}} \max(\mu_{n+1}(u), \tau_{n+1}^N - c) \cdot f_{Y_{n+1}}(u) \, du$$

- Optimal policy based on thresholds

  $$G_{POS}^N = \max(0, \tau_0^N - c),$$
POS with Independent Normally Distributed Candidates

- When $X \sim N(\mu, \sigma^2)$
- Assume $Y$ is univariate normal $Y \sim N(0,1)$
- Let $\rho$ denote the correlation coefficient between $Y$ and $X$

Then

$$\mu(y) = \mu + \sigma \rho y$$

$$\tau_n^N = \mu + (\tau_{n+1}^N - c - \mu) \cdot \Phi\left(\frac{\tau_{n+1}^N - c - \mu}{\sigma \rho}\right) + \sigma \rho \cdot \Phi\left(\frac{\tau_{n+1}^N - c - \mu}{\sigma \rho}\right)$$
Value of Information in NOS problem

- Assumptions
  - Timing:
    - Decision about getting an information source is upfront
    - Information received upon candidate being surveyed
  - Risk neutrality

\[ V_{OII} = g_{POS}^N(\rho) - g_{NOS}^N \]

- Findings

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<table>
<thead>
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<tr>
<td>( \mu )</td>
<td>Independent when exit is not allowed and ( \mu \geq c )</td>
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<tr>
<td>( \sigma )</td>
<td>Variability increases the value of information</td>
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<tr>
<td>( \rho )</td>
<td>As expected</td>
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Insight

The effect of N on $RVOI = \frac{VOII}{VOPI}$

- When exit is allowed and when $c = 0$, $RVOI$ tends towards $\rho$.
- When exit is disallowed and when $c = 0$, $RVOI = \rho$ for all $N \geq 2$.

When surveying candidates is almost free of cost,

$$VOII = \rho \times VOPI$$

can be a useful guideline for valuing imperfect information in the NOSI-N problem, whether exit is allowed or disallowed.
Used Car Reports: Bargain or Cheat?

- In the United States, several companies offer information about used cars to prospective buyers.
  - CARFAX: 1 report for $40, 5 for $50 and unlimited for $55,
  - Autocheck: 1 report for $30 and unlimited for $45

Buying a used car? Just say - Show me the CARFAX!

Don't run the risk of buying used cars with costly hidden problems. Get a detailed vehicle history report from our nationwide database within seconds.

Is it worth it?
Modeling Set Up

- Buyer is interested in a limited set of similar cars
  - One distribution for the value of the car

- Parameters estimated using data collected from Kelley Blue Book (worth) and craigslist (selling prices).
  - “2007 to 2009 Honda Civic, Toyota Corolla or Nissan Altima four-door sedan.
  - $\mu = 870$ and $\sigma = 2350$

- Buyer will look at about a dozen car
  - Finite horizon with $N=12$

  - 1/2 day car hunting trip yields 3-5 candidates
  - $c = 20$/car surveyed

- Correlation coefficient $\rho$ between signal and true value
  - Reports only provide partial (reported) information about car history so $\rho << 1$
Results

VOPI: $3000

VOII: $234 for $\rho = 0.1$

VOII: $50$ for $\rho = 0.035$
Conclusion

Contributions:

• Extended the work on COS to practical problems such as NOS and POS.
• Explored the properties of VOPI, VOII, RVOI in some stopping problems, with respect to the problem parameters, and presented several general results
• Method to determine whether to buy vehicle history reports

Future research:

• Explore the case of information on some, not all, candidates
• Extend our work to treating POS as the base problem
• There are several potential extensions: With recall, risk-aversion, bounds, etc.