

REICHENBACH'S E , R AND S IN A FINITE-STATE SETTING

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Abstract

Reichenbach's event, reference and speech times are interpreted semantically by stringing and superposing sets of temporal formulae, structured within regular languages. Notions of continuation branches and of inertia, bound (in a precise sense) by reference time, are developed and applied to the progressive and the perfect.

1 Introduction

The analysis of tense and aspect in terms of an event time E , a reference time R and a speech time S in Reichenbach (1947) counts (arguably) as one of the classic works in formal natural language semantics. In view of the prominence there of the notion of time, it is surprising that Steedman (2000) should claim that

temporal semantics of natural language is not primarily to do with time at all. Instead, the formal devices we need are those related to the representation of causality and goal-directed action.

The present work is an attempt to flesh out the claim above in a finite-state setting that accounts for Reichenbach's basic insights. The main idea is

- (*) to base the account not so much on times but on event-types, construed as regular languages over an alphabet $\text{Pow}(\Phi)$ consisting of subsets of some finite set Φ of formulae.

The formulae in Φ are temporal in that they describe times, including the reference time and speech time. A string $\alpha_1\alpha_2\cdots\alpha_n$ in $\text{Pow}(\Phi)^*$ is to be read as a chronologically ordered sequence of observations, with every formula in α_i understood to hold at the i th point of the sequence. That is, $\alpha_1\alpha_2\cdots\alpha_n$ amounts to a comic strip or movie that begins with the still picture α_1 , followed by $\alpha_2 \dots$ ending with α_n . Strings that are instances of the same event-type are collected in a language $L \subseteq \text{Pow}(\Phi)^*$, taken below to be regular. The finite-state machines accepting the languages provide a vivid image of a causal realm, from which worlds and models arise by executing the machines in (real) time. The divide between extensional and intensional notions comes out as follows: regular languages (machines) are intensions, while screenings of movies (machine runs) in time are extensions. In this sense, the thrust of (*) above is to treat intensions as basic, and extensions as derived.

To ground the discussion in English examples, consider (1).

- (1) a. Pat crossed the road.
 b. Pat was crossing the road.
 c. Pat has crossed the road.

It is commonly held that the simple sentence (1a) is extensional while its progressive correlate (1b) is intensional — at least if, as in Dowty (1979) and Landman (1992), possibly unrealized continuations of the progressive are considered. To keep the semantics of the progressive extensional, Parsons confines himself to realized parts (Parsons 1990). The pressure to devise an extensional account vanishes if (as in the perspective we adopt) extensions are conceived as being *no* more basic than intensions. The issue of realized versus unrealized parts does *not* arise in the case of the perfect; (1c) puts the entire event of Pat crossing the road safely in the past. The challenge of the perfect, however, is how to derive its various readings. For instance, three distinct construction algorithms are provided in Kamp and Reyle (1993) for the perfect: one for (1c) and two for the stative (2), differing on whether or not Pat-live-in-Vienna is asserted to continue into the present.

(2) Pat has lived in Vienna for two years.

Portner (2003) has (among others) argued that a uniform analysis of the perfect should be given. My claim is that the notion of inertia connected to the notorious *frame problem* of McCarthy and Hayes (1969) is the key to a uniform analysis of the perfect.¹ Inertia, as embodied in worlds, is applied in Dowty (1979) to the *imperfective paradox* afflicting the progressive. This affliction is treated below by a finite-state approach to the stages and continuations in Landman (1992), under which worlds are *not* presumed basic, but instead derived by grounding strings in time.

Relating these back now to Reichenbach (1947), the event timed by E is given by the sentence radical to which the aspectual operator applies. For (1) and (2), the radicals are the un-inflected phrases Pat-cross-the-road and Pat-live-in-Vienna-for-two-years, respectively. Our account below proceeds in three steps, detailed in the next three sections. Section 2 turns the event time E into a string or better still: a set of strings — that is, a language. Section 3 brings in the reference time R , and introduces the possibility of branching beyond R . Section 4 imposes inertial laws before factoring in tense via speech time S . With E , R and S in place, section 5 grounds the strings in time, constructing worlds and models. Section 6 concludes.

2 E strung out

Consider the un-inflected phrase (3) on which aspect and tense operate in (2).

(3) Pat-live-in-Vienna for two-years

Let us assume that we have in Φ the formula $\text{live}(p, v)$ for Pat-live-in-Vienna, and let us associate with the phrase two-years a movie of a clock τ marking an interval of two years. At the beginning, the clock τ is at 0; at the end, two years have elapsed. In the middle, the clock ticks; but we will not care how often. That is, we allow for an indeterminate (possibly variable) frequency, reflected in the regular expression (4) by non-zero Kleene iteration \cdot^+ on the empty picture \square . (We are drawing boxes instead of the usual curly braces $\{\cdot\}$ to distinguish sets-as-symbols from say, sets-as-languages, adopting the practice in regular expressions of writing strings for languages.)

(4) $\boxed{0(\tau)}\square^+\boxed{2\text{years}(\tau)}$

¹Such links with logical AI are advocated broadly in Steedman (2000), and were noted at the inter/multi-sentential level in Dowty (1986).

Next, we form a language (5) for the sentence radical (3) from (4) by superposition $\&$, as defined in (6).

$$(5) \quad \boxed{0(\tau), \text{live}(p, v)} \boxed{\text{live}(p, v)}^+ \boxed{2\text{years}(\tau), \text{live}(p, v)}$$

$$= \boxed{\text{live}(p, v)}^+ \& \boxed{0(\tau)} \boxed{\square}^+ \boxed{2\text{years}(\tau)}$$

$$(6) \quad L\&L' = \bigcup_{n \geq 1} \{(\alpha_1 \cup \alpha'_1) \cdots (\alpha_n \cup \alpha'_n) \mid \alpha_1 \cdots \alpha_n \in L \text{ and } \alpha'_1 \cdots \alpha'_n \in L'\}$$

According to (6), the superposition $L\&L'$ of languages $L, L' \subseteq \text{Pow}(\Phi)^*$ combines strings from L and L' of equal length, forming the componentwise union $\alpha_i \cup \alpha'_i$ of symbols. Each of α_i and α'_i is understood to be a partial snapshot at some i th time, so that the effect of $\&$ is to overlay motion pictures with the same duration. A natural form of conjunction, $\&$ maps regular languages to regular languages (Fernando 2003a). (5) associates with two-years the language (4), and, under pressure from “for”, coerces the formula $\text{live}(p, v)$ for Pat-live-in-Vienna to the language $\boxed{\text{live}(p, v)}^+$. A systematic treatment of temporal “for”/“in” modification is provided in Fernando (2003b), based roughly on a re-analysis of the Vendler classes in Dowty (1979) and Naumann (2001) according to Figure 1 (with \sim as negation).

Vendler	Dowty	Naumann/F
stat[iv]e	$P(\vec{x})$	$\boxed{\varphi}^+$
activity	$\text{Do}[x, P(\vec{x})]$	$\text{Con-BEC}(\varphi) = \boxed{\sim\varphi} \boxed{\varphi}^+$
achievement	$\text{Bec}[P(\vec{x})]$	$\text{Min-BEC}(\varphi) = \boxed{\sim\varphi}^+ \boxed{\varphi}$
accomplishment	$\text{Cause}(\text{Do}[x, P(\vec{x})], \text{Bec}[Q(\vec{y})])$	$\boxed{\sim\varphi, \sim\psi} \boxed{\varphi, \sim\psi}^* \boxed{\varphi, \psi}$

Figure 1

Dowty's decomposition of an accomplishment into an activity and an achievement is re-captured in the third column of Figure 1 as

$$\text{Con-BEC}(\varphi) \& \text{Min-BEC}(\psi) = \boxed{\sim\varphi, \sim\psi} \boxed{\varphi, \sim\psi}^* \boxed{\varphi, \psi}$$

where \cdot^* is Kleene star, $L^+ = L^*L$. A concrete example is provided by the accomplishment Pat-swim-two-miles with ψ as $\text{swim}(\text{pat}, a)$ and φ as $(\exists u \preceq a) \text{swim}(\text{pat}, u)$, where $u \preceq a$ says “ u is a non-null part of a ” and a is a distance of two miles. To record the last fact, we can superpose $\text{Con-BEC}(\varphi) \& \text{Min-BEC}(\psi)$ with the state $\boxed{2\text{miles}(a)}^+$ to get (7).

$$(7) \quad \boxed{\sim\varphi, \sim\psi, 2\text{miles}(a)} \boxed{\varphi, \sim\psi, 2\text{miles}(a)}^* \boxed{\varphi, \psi, 2\text{miles}(a)}$$

The reader concerned about Pat repeatedly swimming two miles might sharpen the formula $\text{swim}(x, y)$ to $\text{swimSince}(x, y, t)$, with a temporal argument t marking the beginning of the swim. Adjusting φ and ψ accordingly, we would then replace (7) by (8), where $\text{time}(t)$ is a formula marking the pictured time as t (Fernando 2003a).

$$(8) \quad \boxed{\text{time}(t), \sim\varphi, \sim\psi, 2\text{miles}(a)} \boxed{\varphi, \sim\psi, 2\text{miles}(a)}^* \boxed{\varphi, \psi, 2\text{miles}(a)}$$

Clearly, our languages may become more complicated than those tabulated in Figure 1.

3 R and continuation branches

A typical Reichenbachian approach to aspect, relating E to R , is summarized in (9).

- (9) a. **Simple** $E = R$
 b. **Progressive** $R \sqsubseteq E$
 c. **Perfect** $E < R$

Having stepped from event time E to a regular language $L \subseteq \text{Pow}(\Phi)^*$ in the previous section, we can formulate temporal precedence $<$ and containment \sqsubseteq as in (10), construing the reference time R as a formula in Φ that marks a position in a string in L .

- (10) a. $\text{SIMP}(L, R) = L \ \& \ \boxed{\square^* R}$
 b. $\text{PROG}(L, R) = L \ \& \ \boxed{\square^+ R \square^+}$
 c. $\text{PERF}(L, R) = L \ \boxed{\square^* R}$

For the simple aspect, R picks out the end of L (identified with E); for the progressive, it marks an intermediate point; and for the perfect, it marks a point after L . To illustrate, if

$$L = \boxed{0(\tau), \text{live}(p, v)} \boxed{\text{live}(p, v)}^+ \boxed{\text{live}(p, v), 2\text{years}(\tau)}$$

then

$$\begin{aligned} \text{SIMP}(L, R) &= \boxed{0(\tau), \text{live}(p, v)} \boxed{\text{live}(p, v)}^+ \boxed{\text{live}(p, v), 2\text{years}(\tau), R} \\ \text{PROG}(L, R) &= \boxed{0(\tau), \text{live}(p, v)} \boxed{\text{live}(p, v)}^* \boxed{\text{live}(p, v), R} \boxed{\text{live}(p, v)}^* \boxed{\text{live}(p, v), 2\text{years}(\tau)} . \end{aligned}$$

Moving from regular languages to finite automata, the functions in (10) track the computation of an automaton for L : $\text{SIMP}(L, R)$ says that the automaton has reached completion; $\text{PROG}(L, R)$ that it has *not* quite gotten there but is on its way; and $\text{PERF}(L, R)$ that it is history.²

To develop the account further, some notation is useful. Let us call a string $\alpha_1 \cdots \alpha_n \in \text{Pow}(\Phi)^*$ *R-truncated* if for all $i \in \{1, \dots, n\}$,

$$R \in \alpha_i \quad \text{implies} \quad i = n .$$

The *R-truncation* of a string s , denoted s_R , is the largest prefix of s that is R -truncated. An *R-continuation* of s is a string s' with the same R -truncation, $s'_R = s_R$. Now, the idea is to relativize the membership relation $s \in L$ by existentially quantifying over a (contextually given) set $c(s)$ of R -continuations of s

$$s :_c L \quad \text{iff} \quad (\exists s' \in c(s)) s' \in L$$

so as to allow for an R -continuation s' of s different from s . To investigate this notion, let us move the subscript c on $:_c$ over to L , forming the language

$$L/c = \{s \mid c(s) \cap L \neq \emptyset\}$$

(with $s :_c L$ iff $s : L_c$). Let us write $c_R(s)$ for the set of R -continuations of s

$$c_R(s) = \{s' \in \text{Pow}(\Phi)^* \mid s'_R = s_R\}$$

and L_R for the set of R -truncations of strings in L

$$L_R = \{s_R \mid s \in L\} .$$

For the record, we have

Proposition 1. *Let $L \subseteq \text{Pow}(\Phi)^*$ and (for parts (iv) to (vi) below) $c : \text{Pow}(\Phi)^* \rightarrow \text{Pow}(\text{Pow}(\Phi)^*)$.*

²The present account has, in the informal terms just stated, much in common with Narayanan (1997) but differs from it in emphasizing strings/languages, using superposition & to stay regular.

- (i) $L/\{\cdot\} = L$ where $\{\cdot\}$ maps every $s \in \text{Pow}(\Phi)^*$ to $\{s\}$.
- (ii) $L/c_R = L_R \text{Pow}(\Phi - \{R\})^*$.
- (iii) If L is regular, then so is L_R and hence L/c_R .
- (iv) Given $c' : \text{Pow}(\Phi)^* \rightarrow \text{Pow}(\text{Pow}(\Phi))^*$ such that $c(s) \subseteq c'(s)$ for all $s \in \text{Pow}(\Phi)^*$,

$$L/c \subseteq L/c'.$$

(v) $L \subseteq L/c$ provided

(c1) $s \in c(s)$ for every $s \in \text{Pow}(\Phi)^*$.

(vi) $(L/c)_R = L_R$ assuming c satisfies **(c1)** above and

(c2) $c(s) \subseteq c_R(s)$ for every $s \in \text{Pow}(\Phi)^*$.

Proof. All of the assertions are straightforward, except perhaps for (iii). A finite automaton for L is turned into one for L_R as follows. For every transition $\xrightarrow{\alpha} q$ with $R \in \alpha$, replace its target state q by a new state that is added to the set of final/accepting states. Since such states are new, no transition may come out of them. \dashv

Conditions **(c1)** and **(c2)** from Proposition 1 allow for any number of choices of $c(s)$ between $\{s\}$ and $c_R(s)$ that can be applied to the imperfective paradox. That is,

$$s \in \text{PROG}(L, R)/c \quad \text{need not imply} \quad s \in \text{PROG}(L, R).$$

The converse, however, *does* hold, according to part (v) of Proposition 1, supporting the insight in Landman (1992) that

if an accomplishment manages to get completed, it is unproblematic to assume (in retrospect) that the progressive is true during the development stage ... even if the event gets completed against all odds.

The progressive aside, the exact choice of c satisfying **(c1)** and **(c2)** need *not* matter for the aspectual functions in (10). To be more precise, let us call a language L *R-truncated* if $L_R = L$. Examples include $\text{SIMP}(L', R)$ and $\text{PERF}(L', R)$ for languages L' that are R -free in that

$$\text{for every string } \alpha_1 \cdots \alpha_n \in L', \quad R \notin \bigcup_{i=1}^n \alpha_i.$$

For R -truncated L , it follows from Proposition 1 (vi) that $(L/c)_R = L$. As far as R -truncations are concerned, the construction \cdot/c is innocuous.

4 S after inertia

A Reichenbachian approach to tense relates R to S as in (11)

- (11) a. **Past** $R < S$
- b. **Present** $R = S$
- c. **Future** $R > S$

To translate (11) in finite-state terms, it is useful to define the language $1(R)$ of strings in which R occurs at most once

$$1(R) = \text{Pow}(\Phi - \{R\})^* \text{Pow}(\Phi) \text{Pow}(\Phi - \{R\})^* .$$

Intersection with $1(R)$ is one of the regular operations, with which we can formalize the tenses.

- (12) a. $\text{PAST}(L, S) = (L \square^* \& \square^* \boxed{R} \square^* \boxed{S} \square^*) \cap 1(R)$
 b. $\text{PRES}(L, S) = (L \& \square^* \boxed{R, S} \square^*) \cap 1(R)$
 c. $\text{FUTU}(L, S) = (\square^* L \& \square^* \boxed{S} \square^* \boxed{R} \square^*) \cap 1(R)$

An instructive example is provided by the sentence ‘‘Pat has left Vienna,’’ worked out in (13).³

- (13) a. Pat-leave-Vienna
 $\boxed{\text{in}(p, v)} \boxed{\text{leave}(p, v)} \boxed{\sim \text{in}(p, v)}$
 b. Perfect(Pat-leave-Vienna)
 $\boxed{\text{in}(p, v)} \boxed{\text{leave}(p, v)} \boxed{\sim \text{in}(p, v)} \square^* \boxed{R}$
 c. Present(Perfect(Pat-leave-Vienna))
 $\boxed{\text{in}(p, v)} \boxed{\text{leave}(p, v)} \boxed{\sim \text{in}(p, v)} \square^* \boxed{R, S}$
 d. Resultative reading of ‘‘Pat has left Vienna’’
 $\boxed{\text{in}(p, v)} \boxed{\text{leave}(p, v)} \boxed{\sim \text{in}(p, v)} \boxed{\sim \text{in}(p, v)}^* \boxed{\sim \text{in}(p, v), R, S}$

The obvious problem is how to bridge the gap between (13c) and (13d). Evidently, the formula $\sim \text{in}(p, v)$ in (13c) must spill over onto R . With this in mind, let us introduce a set $\text{Inr} \subseteq \Phi$ of inertial formulae, forming $\text{PERF}_{\text{Inr}}(L, R)$ with inertial formulae at the end of L persisting

$$\text{PERF}_{\text{Inr}}(L, R) = \{ \alpha_1 \cdots \alpha_k \theta^n (\theta \cup \boxed{R}) \mid \alpha_1 \cdots \alpha_k \in L, \theta = \alpha_k \cap \text{Inr} \text{ and } n \geq 0 \} .$$

Assuming $\sim \text{in}(p, v) \in \text{Inr}$ but $\text{leave}(p, v) \notin \text{Inr}$,

$$\text{PERF}_{\text{Inr}}((13a), R) = \boxed{\text{in}(p, v)} \boxed{\text{leave}(p, v)} \boxed{\sim \text{in}(p, v)} \boxed{\sim \text{in}(p, v)}^* \boxed{\sim \text{in}(p, v), R}$$

from which (13d) results after applying PRES.

Proposition 2. *If L is regular, so is $\text{PERF}_{\text{Inr}}(L, R)$.*

Proof. Map a finite automaton $\langle Q, F, \rightarrow, q_0 \rangle$ for L to the automaton $\langle Q, \{f\}, \rightarrow', q_0 \rangle$ for $\text{PERF}_{\text{Inr}}(L, R)$ where the set Q' of states is

$$Q' = Q \cup \{f\} \cup \text{Pow}(\text{Inr})$$

with the sets $Q, \{f\}$ and $\text{Pow}(\text{Inr})$ assumed to be mutually disjoint, and transitions \rightarrow' equal to

$$\rightarrow \cup \{ (q, \alpha, \alpha \cap \text{Inr}) \mid q \in Q \text{ and } (\exists q' \in F) q \xrightarrow{\alpha} q' \} \cup \{ (\theta, \theta, \theta) \mid \theta \subseteq \text{Inr} \} \cup \{ (\theta, \theta \cup \boxed{R}, f) \mid \theta \subseteq \text{Inr} \}$$

That is, whenever $q \xrightarrow{\alpha} q' \in F$, we add arcs $q \xrightarrow{\alpha} \theta$, $\theta \xrightarrow{\theta} \theta$, $\theta \xrightarrow{\theta \cup \boxed{R}} f$, where $\theta = \alpha \cap \text{Inr}$ and $\theta \cup \boxed{R} = \theta \cup \boxed{R}$. \dashv

³The point to be made presently applies to other choices for (13a) such as $\boxed{\text{in}(p, v)} \boxed{\text{leave}(p, v), \sim \text{in}(p, v)}$.

We can recover $\text{PERF}(L, R)$ in (10) by setting Inr to \emptyset

$$\text{PERF}(L, R) = \text{PERF}_{\emptyset}(L, R).$$

Playing with different choices of Inr , we can account for various readings of the perfect. For example, consider again the un-inflected phrase Pat-live-in-Vienna-for-two-years. Building on our analysis in (5), we get

$$\text{PERF}((5), R) = \boxed{0(\tau), \text{live}(p, v) \mid \text{live}(p, v)}^+ \boxed{\text{live}(p, v), 2\text{years}(\tau)} \square^* R.$$

If $\text{live}(p, v) \in \text{Inr}$ but $2\text{years}(\tau) \notin \text{Inr}$, then

$$\text{PERF}_{\text{Inr}}((5), R) = \boxed{0(\tau), \text{live}(p, v) \mid \text{live}(p, v)}^+ \boxed{\text{live}(p, v), 2\text{years}(\tau) \mid \text{live}(p, v)}^* \boxed{\text{live}(p, v), R}$$

and $\text{PRES}(\text{PERF}_{\text{Inr}}((5), R), S)$ is

$$\boxed{0(\tau), \text{live}(p, v) \mid \text{live}(p, v)}^+ \boxed{\text{live}(p, v), 2\text{years}(\tau) \mid \text{live}(p, v)}^* \boxed{\text{live}(p, v), R, S}.$$

The last language expresses a continuative reading of (2), Pat has lived in Vienna for two years. What about an existential one that is silent on Pat's current domicile? The simplest way to block inertial flow is to restrict Inr , declaring that $\text{live}(p, v) \notin \text{Inr}$. But how do we justify this move?

One approach is to link existential readings with (possibly implicit) questions or topics that override default settings for Inr . An existential reading of (2) is, for instance, licensed by

Has Pat *ever* lived in Vienna for two years?

A further test is (14).

(14) I have lost my key but have found it.

Out of the blue, (14) is odd; but it is a fine reply to

Have you *ever* lost your key?

Similarly for an existential reading of "Pat has left Vienna."

An alternative account of existential readings can be based on the observation that $\text{PAST}(L, S)$ does not provide for inertial flow beyond R . That is, S is a barrier to inertial flow when R temporally precedes S . Inferences from the past to the present fail, such as

Pat was happy $\not\models$ Pat is happy

under (15).

(15) a. Pat-be-happy

$$\boxed{\text{happy}(p)}^+$$

b. Simple(Pat-be-happy)

$$\boxed{\text{happy}(p)}^* \boxed{\text{happy}(p), R}$$

c. Pat was happy.

$$\boxed{\text{happy}(p)}^* \boxed{\text{happy}(p), R} \square^* S \square^*$$

d. Pat is happy.

$$\boxed{\text{happy}(p)}^* \boxed{\text{happy}(p), R, S}$$

The blockage here of inertial flow suggests an alternative mechanism for deriving present existential readings: rather than manipulating Inr , an *ever* question might shift temporal perspective, introducing a displaced speech time $S' < S$ that yields in the case (for example) of “Pat has lived in Vienna for two years” the language

$$\boxed{0(\tau), \text{live}(p, v) \mid \text{live}(p, v)}^+ \boxed{\text{live}(p, v), 2\text{years}(\tau) \mid \text{live}(p, v)}^* \boxed{\text{live}(p, v), R, S'} \square^* \boxed{S}.$$

This analysis would seem to be compatible with an “extended now,” discussed for example in Portner (2003). It is not clear to me if it is superior to the previous (Inr -revising) approach.

The remainder of this section aims to isolate a general principle underlying PERF_{Inr} . Let us introduce a further parameter $\Sigma \subseteq \text{Pow}(\Phi)$ specifying a notion of “legal” stills/snaps that is \subseteq -closed

$$(\forall \alpha \in \Sigma)(\forall \alpha' \subseteq \alpha) \quad \alpha' \in \Sigma$$

and free of \sim -pairs

$$(\forall \varphi \in \Phi) \quad \boxed{\varphi, \sim \varphi} \notin \Sigma.$$

Leaving the exact choice of Σ open, Σ regulates inertial flow according to the rules

$$\frac{s\alpha\alpha's'}{s\alpha(\alpha' \cup \boxed{\varphi})s'} \quad \varphi \in \alpha \cap \text{Inr} \text{ and } \alpha' \cup \boxed{\varphi} \in \Sigma$$

and (reversing the flow of time)

$$\frac{s\alpha\alpha's'}{s(\alpha \cup \boxed{\varphi})\alpha's'} \quad \varphi \in \alpha' \cap \text{Inr} \text{ and } \alpha \cup \boxed{\varphi} \in \Sigma.$$

These rules induce the operator Γ that maps a language L to the language

$$\begin{aligned} \Gamma(L) = & \{s\alpha(\alpha' \cup \boxed{\varphi})s' \mid s\alpha\alpha's' \in L, \varphi \in \alpha \cap \text{Inr} \text{ and } \alpha' \cup \boxed{\varphi} \in \Sigma\} \cup \\ & \{s(\alpha \cup \boxed{\varphi})\alpha's' \mid s\alpha\alpha's' \in L, \varphi \in \alpha' \cap \text{Inr} \text{ and } \alpha \cup \boxed{\varphi} \in \Sigma\} \end{aligned}$$

(suppressing the subscripts Inr and Σ to simplify notation). Let us define L to be (Inr, Σ) -full if $L \subseteq \Sigma^*$ and for all $s \in L$, $\Gamma(\{s\}) \subseteq \{s\}$. Iterating Γ over the natural numbers, let

$$\begin{aligned} \Gamma^0(L) &= L \\ \Gamma^{n+1}(L) &= \Gamma(\Gamma^n(L)) \\ \Gamma_\infty(L) &= \bigcup_{n \geq 0} \Gamma^n(L). \end{aligned}$$

Proposition 3. $\text{PERF}_{\text{Inr}}(L, R)$ is the (Inr, Σ) -full fragment of $\Gamma_\infty(\text{PERF}(L, R))$

$$\text{PERF}_{\text{Inr}}(L, R) = \{s \in \Gamma_\infty(\text{PERF}(L, R)) \mid \Gamma(\{s\}) \subseteq \{s\}\}$$

assuming L is (Inr, Σ) -full, $R \notin \text{Inr}$ and for all $\alpha \subseteq \Phi - \{R\}$,

$$\alpha \in \Sigma \quad \text{iff} \quad \alpha \cup \boxed{R} \in \Sigma.$$

Inertia has, under the same assumptions as in Proposition 3, no effect on the progressive or the simple of a language L

$$\begin{aligned} \text{SIMP}(L, R) &= \{s \in \Gamma_\infty(\text{SIMP}(L, R)) \mid \Gamma(\{s\}) \subseteq \{s\}\} \\ \text{PROG}(L, R) &= \{s \in \Gamma_\infty(\text{PROG}(L, R)) \mid \Gamma(\{s\}) \subseteq \{s\}\}. \end{aligned}$$

5 Background extensions: worlds and models

To interpret the strings in $\text{Pow}(\Phi)^*$ model-theoretically, the basic idea is to apply the formulae to times for a notion of truth (Fernando 2003a, Fernando 2003b). In this section, we shall take a piecewise approach in terms of relations $p \subseteq \text{Ti} \times \Phi$ between a set Ti of times and Φ , reading

$$p(t, \varphi) \quad \text{as} \quad \varphi \text{ holds at } t, \text{ according to } p.$$

Given a sequence $t_1 \cdots t_n \in \text{Ti}$, p induces the string

$$\text{str}(p, t_1 \cdots t_n) = \boxed{\varphi \mid p(t_1, \varphi)} \cdots \boxed{\varphi \mid p(t_n, \varphi)}.$$

Conversely, a time-stamping $t_1 \cdots t_n$ on a string $\alpha_1 \cdots \alpha_n \in \text{Pow}(\Phi)^*$ determines a relation p such that for all $t \in \text{Ti}$ and $\varphi \in \Phi$,

$$p(t, \varphi) \quad \text{iff} \quad (\exists i \in \{1, \dots, n\}) t = t_i \text{ and } \varphi \in \alpha_i.$$

Along with Ti , let us fix a binary relation $\text{succ} \subseteq \text{Ti} \times \text{Ti}$ such that its transitive closure succ^+ is irreflexive, writing $\text{ch}(\text{succ})$ for the set of finite succ -chains

$$\text{ch}(\text{succ}) = \{t_1 \cdots t_n \in \text{Ti}^+ \mid \text{succ}(t_i, t_{i+1}) \text{ for } 1 \leq i < n\}.$$

(As will become clear shortly, the intuition is that succ represents a level of granularity for taking snapshots.) To step from explicit to implicit information, let us assume a background $\text{P} \subseteq \text{Pow}(\text{Ti} \times \Phi)$ of “possible” pieces, defining the *forcing relation* \Vdash_{P} with domain P so that for $p \in \text{P}$,

$$p \Vdash_{\text{P}} L, t \quad \text{iff} \quad p(t, R) \text{ and } (\exists \mathbf{t} \in \text{ch}(\text{succ})) \text{str}(p, \mathbf{t}) :_c L$$

for some function $c : \text{Pow}(\Phi)^* \rightarrow \text{Pow}(\text{Pow}(\Phi)^*)$. The force of P (as background) comes out in interpreting negation \neg universally relative to the restriction \supseteq_{P} of \supseteq to P

$$p \Vdash_{\text{P}} \neg A \quad \text{iff} \quad \text{not } (\exists p' \supseteq_{\text{P}} p) p' \Vdash_{\text{P}} A.$$

Applying negation \neg twice, we get our satisfaction relation \models_{P}

$$\begin{aligned} p \models_{\text{P}} A & \quad \text{iff} \quad p \Vdash_{\text{P}} \neg \neg A \\ & \quad \text{iff} \quad (\forall p' \supseteq_{\text{P}} p) (\exists p'' \supseteq_{\text{P}} p') p'' \Vdash_{\text{P}} A. \end{aligned}$$

Worlds for \models_{P} are derived from selections in P that cover all of time and are, at each time, maximal. More precisely, relative to a set Ψ of formulas that may occur to the right of \Vdash_{P} (picking out a subset of Ti , named in Ψ), we define a subset G of P to be P -generic if

- (i) for all $p \in G$ and $p' \in \text{P}$ such that $p' \subseteq p$, $p' \in G$
- (ii) for all $p, p' \in G$, there exists $p'' \in G$ such that $p'' \supseteq p \cup p'$
- (iii) for all $A \in \Psi$, there is a $p \in G$ such that either $p \Vdash_{\text{P}} A$ or $p \Vdash_{\text{P}} \neg A$.

Leaving open exactly what other connectives are available within Ψ , we may expect to extract a model $M[G]$ from a P -generic G such that

$$M[G] \models A \quad \text{iff} \quad (\exists p \in G) p \Vdash_{\text{P}} A$$

and for $\mathcal{G}(\mathbb{P}, p) = \{G \subseteq \mathbb{P} \mid G \text{ is } \mathbb{P}\text{-generic and } p \in G\}$,

$$p \Vdash_{\mathbb{P}} A \quad \text{iff} \quad (\forall G \in \mathcal{G}(\mathbb{P}, p)) \ M[G] \models A$$

provided we have suitable $\Vdash_{\mathbb{P}}$ -clauses for all $A \in \Psi$ guaranteeing persistence

$$p' \supseteq_{\mathbb{P}} p \text{ and } p \Vdash_{\mathbb{P}} A \quad \text{imply} \quad p' \Vdash_{\mathbb{P}} A$$

(e.g. (Keisler 1973)). It is natural to identify a generic set with a world, and, assuming without loss of generality that $\emptyset \in \mathbb{P}$, to associate the common ground $\mathcal{G}(\mathbb{P}, \emptyset)$ with \mathbb{P} , which a formula A in Ψ updates to $\mathcal{G}(\mathbb{P}_A, \emptyset)$, where

$$\mathbb{P}_A = \{p \in \mathbb{P} \mid p' \Vdash_{\mathbb{P}} A \text{ for some } p' \in \mathbb{P} \text{ such that } p \subseteq p'\}.$$

What about the function $c : \text{Pow}(\Phi)^* \rightarrow \text{Pow}(\text{Pow}(\Phi)^*)$ invoked to determine whether or not $p \Vdash_{\mathbb{P}} L, t$? For R -truncated L , the exact choice of c satisfying **(c1)** and **(c2)** is immaterial to $\Vdash_{\mathbb{P}}$, and can be assumed to be the singleton map $\{\cdot\}$ (so that $L/c = L$). But for languages that are not R -truncated, it is natural to investigate c through other approaches to $\Vdash_{\mathbb{P}}$ such as

$$\begin{aligned} p \Vdash_{\mathbb{P}} L, t & \quad \text{iff} \quad (\exists p' \subseteq p) \ p'(t, R) \text{ and } p' :_{\text{cb}} L, t \\ p :_{\text{cb}} L, t & \quad \text{iff} \quad (\exists p' \in \text{cb}(p, t)) (\exists t_1 \cdots t_n \in \text{ch}(\text{succ})) \\ & \quad \quad \quad \{t_1, \dots, t_n\} \supseteq \text{domain}(p') \text{ and } \text{str}(p', t_1 \cdots t_n) \in L \end{aligned}$$

for some function $\text{cb} : (\text{Pow}(\text{Ti} \times \Phi) \times \text{Ti}) \rightarrow \text{Pow}(\text{Pow}(\text{Ti} \times \Phi))$ satisfying

- (a) $p \in \text{cb}(p, t)$
- (b) for all $p' \in \text{cb}(p, t)$, $\varphi \in \Phi$ and $t' \preceq t$, $p'(t', \varphi)$ iff $p'(t', \varphi)$

where \preceq is the reflexive transitive closure of succ . The “continuation branch” function cb is similar to the modal base function Condoravdi (2002) applies for a temporal interpretation of might, with pieces $p \subseteq \text{Ti} \times \Phi$ in place of worlds (Fernando 2003c). Conditions (a) and (b) correspond to **(c1)** and **(c2)** respectively, condition (b) building in *historical necessity* (Thomason 1984).⁴ That said, there is a gap between $c(s) \subseteq \text{Pow}(\Phi)^*$ and $\text{cb}(p, t) \subseteq \text{Pow}(\text{Ti} \times \Phi)$ to be bridged. The idea is that for all $t_1 \cdots t_n \in \text{ch}(\text{succ})$ and $p \subseteq \text{Ti} \times \Phi$ such that

$$\{t_1, \dots, t_n\} \supseteq \text{domain}(p) \quad \text{and} \quad \text{for some } i \in \{1, \dots, n\}, \ p(t_i, R),$$

we can identify $c(\text{str}(p, t_1 \cdots t_n))$ with the language

$$\begin{aligned} \{\text{str}(p', t'_1 \cdots t'_k) \mid p' \in \text{cb}(p, t_i), \ t'_1 \cdots t'_k \in \text{ch}(\text{succ}), \\ \{t'_1, \dots, t'_k\} \supseteq \text{domain}(p') \text{ and } t_1 \cdots t_i \text{ is a prefix of } t'_1 \cdots t'_k\}. \end{aligned}$$

As different choices of p and $t_1 \cdots t_n$ may converge on the same string $s = \text{str}(p, t_1 \cdots t_n)$, we must be prepared to consider different functions $c : \text{Pow}(\Phi)^* \rightarrow \text{Pow}(\text{Pow}(\Phi)^*)$ within $\Vdash_{\mathbb{P}}$. Reference to any *one* function c is less than optimal in exposing the factors the come into the choice of continuation branches — but is irresistible when abstracting away these factors (or so section 3 above suggests).

⁴The candidate $\text{cb}(p, t) = \{p\}$ is Ockhamist, as opposed to Peircean (in the sense of Prior; page 143 of Thomason (1984)).

6 Conclusion

The account above refines traditional Reichenbachian E , R , S analyses of tense and aspect in at least three ways.

1. Rather than taking event structures (Kamp and Reyle 1993) for granted, certain temporal formulae are strung together and superposed to form event-types, instantiations of which may have time E .⁵
2. The stative/non-stative contrast is traced to a distinction between inertial and non-inertial formulae, with ramifications for the perfect.
3. R is construed as a non-inertial formula, marking out a point in an intensional model, up to which inertia flows and beyond which there is branching.

The intensionality described 3 is effected above *not* by relativizing extensional notions to worlds, but by working with notions (such as event-type) that are meaningful prior to their extensional grounding. Worlds are not presumed to be primitive, but rather constructed (via standard techniques recalled in section 5) from runs of machines that may or may not get interrupted. What emerges is a finite-state alternative to (Priorian) tense logic, with conjunctive operations $\&$, \cap in place of modal operators P, F, G, H (Fernando 2003a). Relations between E , R and S are kept simple, with much of the complexities swept over to the context dependence of inertia and continuation branches. Different choices of inertial formulae induce different readings of the perfect; the choice of continuations c satisfying (c1) and (c2) shapes the progressive.

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⁵Instantiations of the event-types above may well have more complicated location times than the single intervals assigned by event structures in Kamp and Reyle (1993).

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