Temporal Representations with and without Points

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Abstract Intervals and events are analyzed in terms of strings that represent points as symbols occurring uniquely. Allen interval relations, Dowty's aspect hypothesis and inertia are understood relative to strings, compressed into canonical forms, describable in Monadic Second-Order logic. That understanding is built around a translation of strings replacing stative predicates by their borders, represented in the S-words of Schwer and Durand. Borders point to non-stative predicates, including forces that may compete, succeed to varying degrees, fail and recur.

1 Introduction

To analyze temporal relations between events, James Allen treats intervals as primitive (not unlike [9]), noting

> There seems to be a strong intuition that, given an event, we can always "turn up the magnification" and look at its structure. ... Since the only times we consider will be times of events, it appears that we can always decompose times into subparts. Thus the formal notion of a time point, which would not be decomposable, is not useful. [1, p. 834].

Sidestepping indivisible points, Allen relates intervals a and a' in 13 mutually exclusive ways (reviewed in Sect. 2 below). An example is a *overlaps* a', which can be pictured as the string

$$\boxed{\ }\boxed{a}\boxed{a, a'}\boxed{a'}\boxed{\ } \qquad (1)$$

of length 5,

- starting with an empty box $\boxed{\ }$ for times before a,
- followed by $\boxed{a}$ for times in a but not a',
- followed by $\boxed{a, a'}$ for times in a and a',

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– followed by $\boxed{a'}$ for times in a' but not a,
– followed by $\boxed{}$ for times after a'.[1]

Now, if, in addition, a third interval a'' overlaps both a and a', we can bring a'' into view by turning up the magnification (as Allen puts it) on (1) for the string

$$\boxed{}\boxed{a''}\boxed{a, a''}\boxed{a, a', a''}\boxed{a, a'}\boxed{a'}\boxed{} \tag{2}$$

of length 7,

– splitting the first box $\boxed{}$ and third box $\boxed{a, a'}$ in (1) each into two, $\boxed{}\boxed{a''}$ and $\boxed{a, a', a''}\boxed{a, a'}$, respectively, whilst
– adding a'' to $\boxed{a}$ for $\boxed{a, a''}$.

To understand the change from (1) to (2), it is useful to define for any set A and string $s = \alpha_1 \cdots \alpha_n$ of sets α_i, the *A-reduct of s* to be the intersection of s componentwise with A, written $\rho_A(s)$

$$\rho_A(\alpha_1 \cdots \alpha_n) \ := \ (\alpha_1 \cap A) \cdots (\alpha_n \cap A).$$

For instance, the $\{a, a'\}$-reduct of (2) is

$$\boxed{}\boxed{}\boxed{a}\boxed{a, a'}\boxed{a, a'}\boxed{a'}\boxed{}$$

which we can then compress to (1) by applying a function bc (for *b*lock *c*ompression) that, given a string $\alpha_1 \cdots \alpha_n$, deletes every α_i such that $i < n$ and $\alpha_i = \alpha_{i+1}$

$$bc(\alpha_1 \cdots \alpha_n) \ := \ \begin{cases} \alpha_1 \cdots \alpha_n & \text{if } n < 2 \\ bc(\alpha_2 \cdots \alpha_n) & \text{else if } \alpha_1 = \alpha_2 \\ \alpha_1 \, bc(\alpha_2 \cdots \alpha_n) & \text{otherwise.} \end{cases}$$

Let us agree to call a string $\alpha_1 \cdots \alpha_n$ *stutterless* if $\alpha_i \neq \alpha_{i+1}$ whenever $1 \leq i < n$. Then clearly, $bc(s)$ is stutterless and

$$s \text{ is stutterless} \quad \Longleftrightarrow \quad s = bc(s).$$

The finite-state approach to temporality in [5–7] reduces a string s of subsets of a set A to its stutterless form $bc(s)$, on the assumption that every element $a \in A$ names a stative predicate p_a, understood according to David Dowty's hypothesis that

> the different aspectual properties of the various kinds of verbs can be explained by postulating a single homogeneous class of predicates — *stative predicates* — plus three or four sentential operators or connectives. [3, p. 71].

[1]Boxes are drawn instead of $\emptyset$ and curly braces $\{\cdot\}$ to reduce the risk of confusing, for example, the empty language $\emptyset$ with the string $\Box$ of length one (not to mention the null string of length 0).

A stative predicate here amounts to a set p of intervals such that for all intervals I, J whose union $I \cup J$ is an interval,

$$I \in p \text{ and } J \in p \iff (I \cup J) \in p \tag{3}$$

(with $\Longrightarrow$ making p cumulative, and $\Longleftarrow$ making p divisive). For example, *rain* is stative insofar as it holds of an interval I iff it holds of any pair of intervals whose union is I, illustrated by the equivalence between (a) and (b).

(a) It rained from 8 am to midnight.
(b) It rained from 8 am to noon, and from 10 am to midnight.

For any finite linear order $\prec$, the requirement (3) on a stative predicate p over intervals (relative to $\prec$) is equivalent to reducing p to the set of subintervals of the set $p_{\downarrow}$ of points t for which the interval $\{t\}$ is in p

$$p \ = \{I \subseteq p_{\downarrow} \mid I \text{ is an interval}\} \ \text{ where } \ p_{\downarrow} \ := \{t \mid \{t\} \in p\}.$$

For example, relative to the string

$$\boxed{}\boxed{a}\boxed{a, a'}\boxed{a'}\boxed{},$$

we can interpret a and a' as the subsets

$$U_a = \{2, 3\} \ \text{ and } \ U_{a'} = \{3, 4\}$$

of the set $\{1, 2, 3, 4, 5\}$ of string positions where a and a' (respectively) occur, and then lift U_a and $U_{a'}$ to stative predicates p_a and $p_{a'}$ over intervals, using U_a as $(p_a)_{\downarrow}$

$$p_a = \{I \subseteq U_a \mid I \text{ is an interval}\}$$

and $U_{a'}$ as $(p_{a'})_{\downarrow}$

$$p_{a'} = \{I \subseteq U_{a'} \mid I \text{ is an interval}\}.$$

Over any string, we can repackage any stative predicate as a subset U of string positions.

But now, can we take for granted Dowty's hypothesis that aspect can be based on stative predicates and assume a string representing an event is built solely from stative predicates? This is far from clear. The event nucleus of [14], for instance, postulates not only states but also events that can be extended or atomic, including what Moens and Steedman refer to as "points" (Comrie's semelfactives), which should not be confused with the points that a linear order compares. The present work is concerned with yet another notion of point, defined relative to a string s over the alphabet 2^A. An element $a \in A$ is said to be an *s-point* if it occurs exactly once in s—i.e.,

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$$\rho_{\{a\}}(s) \ \in \ \boxed{\ }^{*}\boxed{a}\boxed{\ }^{*} \tag{4}$$

Just as a string of statives can be compressed by removing stutters through bc, a string s of points can be compressed by deleting all occurrences in s of the empty box $\square$ for $d_{\square}(s)$. More precisely, $d_{\square}(\epsilon) := \epsilon$ (where ϵ is the string of length 0), and

$$d_{\square}(\alpha s) \ := \ \begin{cases} d_{\square}(s) & \text{if } \alpha = \square \\ \alpha\, d_{\square}(s) & \text{otherwise.} \end{cases}$$

Line (4) above simplifies to the equation

$$d_{\square}(\rho_{\{a\}}(s)) \ = \ \boxed{a}.$$

To formulate a corresponding equation for an s-interval a, it is useful to pause and note that in general, a string $s = \alpha_1 \cdots \alpha_n$ of n subsets α_i of a set A specifies for each $a \in A$, a subset of the set

$$[n] := \{1, \ldots, n\}$$

of string positions, namely, the set

$$U_a \ := \ \{i \in [n] \mid a \in \alpha_i\}$$

of positions where a occurs. If we repackage s as the model

$$Mod_A(s) \ := \ \langle [n], S_n, \{U_a\}_{a \in A} \rangle$$

over $[n]$ with successor relation

$$S_n \ := \ \{(i, i+1) \mid i \in [n-1]\}$$

then a theorem due to Büchi, Elgot and Trakhtenbrot says the regular languages over the set 2^A of subsets of A are given by the sentences φ of MSO_A as

$$\{s \in (2^A)^* \mid Mod_A(s) \models \varphi\}$$

where MSO_A is Monadic Second-Order logic over strings with unary predicates labeled by A (e.g., [13]).[2] The Büchi-Elgot-Trakhthenbrot theorem is usually formulated for strings over the alphabet A (as opposed to 2^A above), but there are at least two advantages in using the alphabet 2^A. First, for applications such as (1) and (2), it is convenient to put zero, one or more symbols from A in boxes for a simple temporal construal of succession. The second advantage has to do with restricting

[2] Regularity of languages is interesting here for computational reasons; for instance, since inclusions between regular languages are computable (unlike inclusions between context-free languages), so are entailments in MSO.

an MSO_A-model $M = \langle [n], S_n, \{U_a\}_{a \in A} \rangle$ to a subset A' of A. The *A'-reduct of M* is the $\mathrm{MSO}_{A'}$-model

$$M \upharpoonright A' = \langle [n], S_n, \{U_a\}_{a \in A'} \rangle$$

obtained from M by keeping only the unary predicates U_a with a in the subset A'. As with the componentwise intersection $\rho_{A'}(s)$ of s with A', only elements of A' are observable. The two notions of A'-reduct coincide

$$Mod_A(s) \upharpoonright A' \;=\; Mod_{A'}(\rho_{A'}(s))$$

making the square

$$\begin{array}{ccc} (2^A)^+ & \xrightarrow{Mod_A} & \mathrm{MSO}_A\text{-models} \\ \Big\downarrow \rho_{A'} & & \Big\downarrow \cdot \upharpoonright A' \\ (2^{A'})^+ & \xrightarrow{Mod_{A'}} & \mathrm{MSO}_{A'}\text{-models} \end{array}$$

commute. Notice that a string s fed to the function $\rho_{A'}$ must be formed from sets for $\rho_{A'}$ to carry out intersection (componentwise).

But what if we "turn up the magnification" by allowing inside a box a label for a non-stative predicate? For example, we might expand string (1)

$$\boxed{}\boxed{a}\boxed{a, a'}\boxed{a'}\boxed{}$$

to the string

$$\boxed{l(a)}\boxed{a, l(a')}\boxed{a, a', r(a)}\boxed{a', r(a')}\boxed{}$$

introducing labels

$l(a)$ and $l(a')$ for the left (open) border of a and a' respectively

and

$r(a)$ and $r(a')$ for the right (closed) border of a and a' respectively.

The introduction of borders is made precise in Sect. 2 through a function b on strings, turning the equation

$$d_\square(\rho_{\{a\}}(s)) \;=\; \boxed{a} \qquad \text{for an } s\text{-point } a$$

into the equation

$$d_\square(b(\rho_{\{a\}}(s))) \;=\; \boxed{l(a)}\boxed{r(a)} \qquad \text{for an } s\text{-interval } a$$

(Propositions 1 and 2), and replacing interiors a, a' by borders $l(a), l(a'), r(a), r(a')$ for a picture

$$d_{\Box}(b(\boxed{}\boxed{a}\boxed{a, a'}\boxed{a'}\boxed{})) = \boxed{l(a)}\boxed{l(a')}\boxed{r(a)}\boxed{r(a')}$$

of the ordering of borders characteristic of the Allen relation *a overlaps* a'. In [4], Schwer and Durand call a string s of non-empty sets an *S-word* (S for set), and define for any set A, the *S-projection over A of s* to be $d_{\Box}(\rho_A(s))$, i.e., the A-reduct of s with all occurrences of $\Box$ deleted. Let the *vocabulary* of a string $\alpha_1 \cdots \alpha_n$ of sets α_i be the union

$$voc(\alpha_1 \cdots \alpha_n) := \bigcup_{i=1}^{n} \alpha_i$$

(making $voc(s)$ the $\subseteq$-least set A such that $s \in (2^A)^*$). Let us say *s projects to s'* if s' is the S-projection over $voc(s')$ of s

$$d_{\Box}(\rho_{voc(s')}(s)) = s'.$$

Every subset of $voc(s)$ specifies a potentially different string to which s can project. The problem of satisfying several statements of projection (each statement describing a feature of the same situation) is taken up in the account of superposition in Sect. 3 below. The translation b is inverted in Sect. 4, with an eye to points other than the borders $l(a)$ and $r(a)$. In particular, actions in [2] that give rise to events are described, leading to a formulation of inertia associated with statives. That said, special attention is paid in Sects. 2 and 3 to Allen interval relations and the transitivity table in [1] enumerating the Allen relations that can hold between three intervals.

The present work steps beyond the previous work [5–7] in exploring non-stative predicates given by the border translation b and actions over and above borders of statives. Dowty's aspect hypothesis is tested with and without points, understood in different ways, one of which is indivisibility at a fixed granularity. (More in the Conclusion.)

2 Points and the Border Translation

Given a string s of subsets of A, an *s-point* is an element a of A that occurs exactly once in s. This condition is expressed in MSO through a unary predicate symbol P_a labeled by a (interpreted U_a by $Mod_A(s)$) as the MSO$_{\{a\}}$-sentence

$$(\exists x)(\forall y)(P_a(y) \equiv x = y)$$

(with biconditional $\equiv$) stating there is a position x where a occurs and nowhere else.

Proposition 1 *For any $a \in A$ and $s \in (2^A)^*$, the following are equivalent*

(i) $\rho_{\{a\}}(s) \in \boxed{}^* \boxed{a} \boxed{}^*$
(ii) $Mod(s) \models (\exists x)(\forall y)(P_a(y) \equiv x = y)$
(iii) *s projects to* $\boxed{a}$.

Points marking the borders of an interval are made explicit by a string function b mentioned in the introduction, to which we turn next. Let l and r be two 1-1 functions with domain A such that the three sets

$$A, \ \ \{l(a) \mid a \in A\} \ \text{ and } \{r(a) \mid a \in A\}$$

are pairwise disjoint. It is useful to think of $l(a)$ and $r(a)$ as syntactic terms (rather than say, numbers), and to collect these in

$$A_\bullet \ := \ \{l(a) \mid a \in A\} \cup \{r(a) \mid a \in A\}.$$

Now, let the function

$$b_A : (2^A)^* \to (2^{A_\bullet})^*$$

map a string $\alpha_1 \cdots \alpha_n$ of subsets α_i of A to a string $\beta_1 \cdots \beta_n$ of subsets β_i of $A_\bullet$ with

$$\begin{aligned} \beta_i \ &:= \{l(a) \mid a \in \alpha_{i+1} - \alpha_i\} \ \cup \ \{r(a) \mid a \in \alpha_i - \alpha_{i+1}\} \quad \text{for } i < n \\ \beta_n \ &:= \{r(a) \mid a \in \alpha_n\}. \end{aligned}$$

For example,

$$b_{\{a,a'\}}(\boxed{}\boxed{a}\boxed{a, a'}\boxed{a'}\boxed{}) \ = \ \boxed{l(a)}\boxed{l(a')}\boxed{r(a)}\boxed{r(a')}\boxed{}$$

and in general, for $A' \subseteq A$,

$$\begin{array}{ccc} (2^A)^* & \xrightarrow{b_A} & (2^{A_\bullet})^* \\ \downarrow{\scriptstyle \rho_{A'}} & & \downarrow{\scriptstyle \rho_{A'_\bullet}} \\ (2^{A'})^* & \xrightarrow{b_{A'}} & (2^{A'_\bullet})^* \end{array}$$

commutes. To simplify notation, we will often drop the subscript A on b_A. The idea behind b is to describe a half-open interval a as $(l(a), r(a)]$ with open left border $l(a)$ and closed right border $r(a)$. For an interval analog of Proposition 1, let $\text{bounded}_a(x, y)$ be the $\text{MSO}_{\{a\}}$-formula

$$\text{bounded}_a(x, y) \ := \ (\forall z)(P_a(z) \equiv (x < z \wedge z \leq y))$$

saying a picks out (via P_a) string positions after x but before or equal to y, and observe that for any string s of subsets of A,

$$b(\rho_{\{a\}}(s)) \;=\; \rho_{\{l(a),r(a)\}}(b(s)).$$

Proposition 2 *For any $a \in A$ and $s \in (2^A)^*$, the following are equivalent*

(i) $\rho_{\{a\}}(s) \in \boxed{\,}^+\boxed{a}^+\boxed{\,}^*$

(ii) $Mod(s) \models (\exists x)(\exists y)(x < y \wedge bounded_a(x, y))$

(iii) $b(\rho_{\{a\}}(s)) \in \boxed{\,}^*\boxed{l(a)}\boxed{\,}^*\boxed{r(a)}\boxed{\,}^*$

(iv) $b(s)$ *projects to* $\boxed{l(a)}\boxed{r(a)}$.

Let us define an *s-interval* to be an element a that satisfies any (equivalently, all) of (i)–(iv) in Proposition 2. To the list (i)–(iv), we can add

(v) $\mathit{bc}(\rho_{\{a\}}(s)) = \boxed{\,}\boxed{a}$ or $\mathit{bc}(\rho_{\{a\}}(s)) = \boxed{\,}\boxed{a}\boxed{\,}$.

The case of

$$\mathit{bc}(\rho_{\{a\}}(s)) = \boxed{\,}\boxed{a}\boxed{\,}$$

in (v) is that of a *period a* in [2]. Alternatively, we can relax any assumption of boundeness by dropping $\boxed{\,}$ on either side of $\boxed{a}$, expanding (v) to

$$\mathit{bc}(\rho_{\{a\}}(s)) \in \{\boxed{a},\ \boxed{a}\boxed{\,},\ \boxed{\,}\boxed{a},\ \boxed{\,}\boxed{a}\boxed{\,}\}.$$

It is convenient for what follows to work with the more restrictive notion described by Proposition 2. When considering strings s over the alphabet $2^{A_\bullet}$ (as opposed to 2^A), we overload the definition of an *s-interval* to apply to a when s projects to

$$\boxed{l(a)}\boxed{r(a)}.$$

We say s *demarcates* A if each $a \in A$ is an s-interval. For any finite set A, we collect the strings of non-empty subsets of A that demarcate A in the language

$$\mathcal{L}_\bullet(A) \;:=\; \{s \in (2^{A_\bullet} - \{\Box\})^* \mid \text{every } a \in A \text{ is an } s\text{-interval}\}.$$

For example,

$$\mathcal{L}_\bullet(\{a\}) \;=\; \{\boxed{l(a)}\boxed{r(a)}\}$$

and for syntactically distinct a, a',

$$\mathcal{L}_\bullet(\{a, a'\}) \;=\; \{\mathfrak{s}_R(a, a') \mid R \in \mathcal{AR}\}$$

where $\mathcal{AR}$ is the set

$$\mathcal{AR} \;:=\; \{<, >, \text{d}, \text{di}, \text{f}, \text{fi}, \text{m}, \text{mi}, \text{o}, \text{oi}, \text{s}, \text{si}, =\}$$

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Table 1 Allen interval relations as strings of points, after [4]

R	aRa'	$\mathfrak{s}_R(a, a')$	R^{-1}	$\mathfrak{s}_{R^{-1}}(a, a')$
$<$	a before a'	$\boxed{l(a)}\boxed{r(a)}\boxed{l(a')}\boxed{r(a')}$	$>$	$\boxed{l(a')}\boxed{r(a')}\boxed{l(a)}\boxed{r(a)}$
m	a meets a'	$\boxed{l(a)}\boxed{r(a), l(a')}\boxed{r(a')}$	mi	$\boxed{l(a')}\boxed{r(a'), l(a)}\boxed{r(a)}$
o	a overlaps a'	$\boxed{l(a)}\boxed{l(a')}\boxed{r(a)}\boxed{r(a')}$	oi	$\boxed{l(a')}\boxed{l(a)}\boxed{r(a')}\boxed{r(a)}$
s	a starts a'	$\boxed{l(a), l(a')}\boxed{r(a)}\boxed{r(a')}$	si	$\boxed{l(a), l(a')}\boxed{r(a')}\boxed{r(a)}$
d	a during a'	$\boxed{l(a')}\boxed{l(a)}\boxed{r(a)}\boxed{r(a')}$	di	$\boxed{l(a)}\boxed{l(a')}\boxed{r(a')}\boxed{r(a)}$
f	a finishes a'	$\boxed{l(a')}\boxed{l(a)}\boxed{r(a), r(a')}$	fi	$\boxed{l(a)}\boxed{l(a')}\boxed{r(a), r(a')}$
=	a equal a'	$\boxed{l(a), l(a')}\boxed{r(a), r(a')}$	=	

of 13 interval relations R in [1], pictured in Table 1 (from [4]) by a string $\mathfrak{s}_R(a, a')$ with vocabulary

$$\{a, a'\}_\bullet = \{l(a), r(a), l(a'), r(a')\}$$

such that for $s \in (2^A)^*$,

$$aRa' holds in s \iff b(s) \text{ projects to } \mathfrak{s}_R(a, a').$$

Note that aRa' is said to hold in a string s of subsets of A, rather than $A_\bullet$.[3]

Interval networks based on Allen relations treat a set A of interval names as the set of vertices (nodes) of a graph with edges (arcs) labeled by the set of Allen relations understood to be possible between the vertices. The obvious question is: given a specification $f : (A \times A) \to 2^{\mathcal{AR}}$ of sets $f(a, a')$ of Allen relations possible for pairs (a, a') from A, is there a string s that meets that specification in the sense of (5) below?

$$\text{for all } a, a' \in A, \text{ there exists } R \in f(a, a') \text{ such that } aRa' \text{ holds in } s \tag{5}$$

This question is approached in [1] through a transitivity table $T : (\mathcal{AR} \times \mathcal{AR}) \to 2^{\mathcal{AR}}$ mapping a pair (R, R') from $\mathcal{AR}$ to the set $T(R, R')$ of relations $R'' \in \mathcal{AR}$ such that for some intervals X, Y and Z,

[3]The strings $\mathfrak{s}_R(a, a')$ can be derived from strings $\mathfrak{s}^\circ_R(a, a')$ over the alphabet $\{a, a'\}$ by the equation

$$\mathfrak{s}_R(a, a') = b(\Box\mathfrak{s}^\circ_R(a, a')).$$

For example,

$$\mathfrak{s}^\circ_<(a, a') = \boxed{a}\boxed{}\boxed{a'} \quad\text{and}\quad \mathfrak{s}^\circ_{\mathrm{m}}(a, a') = \boxed{a}\boxed{a'}.$$

A full list of $\mathfrak{s}^\circ_R(a, a')$, for every Allen relation R, can be found in Table 7.1 in [5, p. 223].

$$\text{X } R \text{ Y and Y } R' \text{ Z and X } R'' \text{ Z}.$$

For example, $T(<, <) = \{<\}$ since $<$ is transitive

$$\text{if X} < \text{Y and Y} < \text{Z then X} < \text{Z}$$

while $T(\mathrm{o}, \mathrm{d}) = \{\mathrm{o}, \mathrm{d}, \mathrm{s}\}$ since any one of the three possible conclusions in the implication

if XoY and YdZ then (either XoZ or XdZ or XsZ)

can be realized. We can describe the function T in terms of the language $\mathcal{L}_\bullet(\{1, 2, 3\})$, using 1, 2, 3 as names of the intervals X, Y, Z, respectively. For all $R, R' \in \mathcal{AR}$, $T(R, R')$ is the set of $R'' \in \mathcal{AR}$ such that some string in $\mathcal{L}_\bullet(\{1, 2, 3\})$ projects to each of

$$\mathfrak{s}_R(1, 2),\ \mathfrak{s}_{R'}(2, 3) \text{ and } \mathfrak{s}_{R''}(1, 3).$$

Next, a function $f : (A \times A) \to 2^{\mathcal{AR}}$ labeling each pair $(a, a') \in A \times A$ with a set $f(a, a') \subseteq \mathcal{AR}$ of Allen relations is *T-consistent* if for all $a, a', a'' \in A$,

$$f(a, a'') \subseteq \bigcup_{R \in f(a,a')} \bigcup_{R' \in f(a',a'')} T(R, R').$$

T-consistency falls short of true consistency; Fig. 5 in [1, p. 838] provides a T-consistent labeling f of a set A of 4 intervals for which there is *no* string s of subsets of A satisfying (5) above. But for A of 3 or fewer intervals, every T-consistent labeling of A has a string s validating (5). Moreover, if we require that $f(a, a')$ always be a singleton, T-consistency suffices.

More precisely, let us say a function $g : A \times A \to \mathcal{AR}$ is *3-consistent* if for all $a, a', a'' \in A$,

$$g(a, a'') \in T(g(a, a'), g(a', a'')).$$

Every string $s \in \mathcal{L}_\bullet(A)$ demarcating A defines a function $\mathcal{AR}_s : A \times A \to \mathcal{AR}$ given by

$$\mathcal{AR}_s(a, a') \;=\; \text{unique } R \in \mathcal{AR} \text{ s.t. } s \text{ projects to } \mathfrak{s}_R(a, a').$$

For example, if $A = \{1, 2, 3\}$ and s is

$$\boxed{l(1), l(3)}\boxed{r(1), l(2)}\boxed{r(2)}\boxed{r(3)} \;=\; b(\boxed{}\boxed{1,3}\boxed{2,3}\boxed{3})$$

then $\mathcal{AR}_s(1, 2) = \mathrm{m}$ (meet), $\mathcal{AR}_s(1, 3) = \mathrm{s}$ (start) and $\mathcal{AR}_s(2, 3) = \mathrm{d}$ (during). For any $s \in \mathcal{L}_\bullet(A)$, $\mathcal{AR}_s$ is manifestly 3-consistent, and, in fact, every 3-consistent function g from $A \times A$ to $\mathcal{AR}$ can be obtained in this way from some A-demarcation in $\mathcal{L}_\bullet(A)$. As shown next, this can be seen through the pointwise ordering implicit in g.

Proposition 3 *For any finite set A and function $g : A \times A \to \mathcal{AR}$,*

$$g \text{ is 3-consistent} \iff g = \mathcal{AR}_s \text{ for some } s \in \mathcal{L}_\bullet(A).$$

Proof. The non-trivial direction $\Longrightarrow$ is proved by induction on the cardinality n of A. For $n = 1$, the required A-demarcation is

$$\boxed{l(a)}\boxed{r(a)}$$

where $A = \{a\}$. For $A = A' \cup \{a\}$ with $a \notin A'$, apply the induction hypothesis to A' for an A'-demarcation s (noting that the restriction of g to $A' \times A'$ is 3-consistent). We incorporate a into s by inserting the borders $l(a)$ and $r(a)$ of a one at a time, working with relations $<$ and $=$ on string positions (in s and the extension of s that we are seeking), not to be confused with the Allen interval relations in Table 1. To insert $l(a)$ in s, we consult g to determine for each $a' \in A'$, which disjuncts in

$$\begin{array}{l} l(a') < l(a) \text{ or } l(a') = l(a) \text{ or } l(a) < l(a') \\ r(a') < l(a) \text{ or } r(a') = l(a) \text{ or } l(a) < r(a') \end{array}$$

hold under g, fixing a single position for $l(a)$ that is acceptable to all $a' \in A'$, by virtue of the 3-consistency of g. More specifically, for $\gamma_1, \gamma_2 \in \{l, r\}$, we define

$$\gamma_1(a_1) =_g \gamma_2(a_2) \iff \mathfrak{s}_{g(a_1,a_2)}(a_1, a_2) \text{ projects to } \boxed{\gamma_1(a_1), \gamma_2(a_2)}$$

and

$$\gamma_1(a_1) <_g \gamma_2(a_2) \iff \mathfrak{s}_{g(a_1,a_2)}(a_1, a_2) \text{ projects to } \boxed{\gamma_1(a_1)}\boxed{\gamma_2(a_2)}$$

and argue along 3 cases.

CASE 1: $l(a) <_g$ every A'-border in s. Put the new box $\boxed{l(a)}$ before (left of) s.

CASE 2: $l(a) =_g$ some A'-border. Then put $l(a)$ into the same box of s as that border, appealing to the 3-consistency of g for the uniqueness of that box.

CASE 3: otherwise, $l(a) >_g$ some A'-border. Put the new box $\boxed{l(a)}$ just after the last position in s with such an A'-border.

Having positioned $l(a)$ in s, we then insert $r(a)$ to the right of $l(a)$, taking into account which disjuncts in

$$\begin{array}{l} l(a') <_g r(a) \text{ or } l(a') =_g r(a) \text{ or } r(a) <_g l(a') \\ r(a') <_g r(a) \text{ or } r(a') =_g r(a) \text{ or } r(a) <_g r(a') \end{array}$$

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hold. The argument breaks up into 3 cases similar to that above for inserting $l(a)$; for example,

$$\boxed{r(a)} \text{ is put right after } \boxed{l(a)}$$

in case $r(a)$ is $<_g$ every A'-border $>_g l(a)$ (the analog of Case 1 above). □

Not only do A-demarcations in $\mathcal{L}_\bullet(A)$ represent all 3-consistent functions from $A \times A$ to $\mathcal{AR}$ (by Proposition 3), they do so canonically; any difference between two strings s and s' in $\mathcal{L}_\bullet(A)$ is directly reflected in the assignments $\mathcal{AR}_s$ and $\mathcal{AR}_{s'}$ of Allen relations.[4] The remainder of the present paper works with string representations.

3 Superposition Respecting Projection

In this section, we show, amongst other things, how to generate the set $\mathcal{L}_\bullet(A)$ bottom-up, for any finite set A. The crucial tool is superposition, the most basic form of which is the componentwise union $\&_\circ$ of two strings $\alpha_1 \cdots \alpha_k$ and $\alpha'_1 \cdots \alpha'_k$ of sets with the same length k

$$(\alpha_1 \cdots \alpha_k) \;\&_\circ\; (\alpha'_1 \cdots \alpha'_k) \;:=\; (\alpha_1 \cup \alpha'_1) \cdots (\alpha_k \cup \alpha'_k).$$

Complications calling for additional care arise from

(i) projections formed not simply out of reducts (but by compressing, $d_\Box$), and
(ii) the requirement that outputs project to the inputs.

To address these complications, it is useful to work with inductive rules such as (s0) and (s1) below

$$\frac{}{\&(\epsilon, \epsilon, \epsilon)}\ (\text{s0}) \qquad\qquad \frac{\&(s, s', s'')}{\&(\alpha s, \alpha' s', (\alpha \cup \alpha')s'')}\ (\text{s1})$$

which generate $\&_\circ$ in that

$$s \&_\circ s' = s'' \iff \&(s, s', s'') \text{ is derivable from (s0), (s1).}$$

The componentwise intersection described by (s1) superposes in lockstep. We can relax this by interleaving/shuffling under the rules (d1) and (d2)

$$\frac{\&(s, s', s'')}{\&(\alpha s, s', \alpha s'')}\ (\text{d1}) \qquad\qquad \frac{\&(s, s', s'')}{\&(s, \alpha' s', \alpha' s'')}\ (\text{d2}).$$

---

[4] To see this, consider the first position where s and s' differ. Let α and α' be the symbols there of s and s', respectively. Then $(\alpha \cup \alpha') - (\alpha \cap \alpha')$ is non-empty. Let $\gamma(a)$ be an element of that set, and $\gamma'(a')$ belong in $(\alpha \cup \alpha') - \{\gamma(a)\}$. Notice that $\mathcal{AR}_s(a, a') \neq \mathcal{AR}_{s'}(a, a')$.

Proposition 4 *For any finite set Σ and $s, s', s'' \in (2^{\Sigma} - \{\square\})^*$, the following are equivalent*

(i) $\&(s, s', s'')$ *is derivable from* (s0), (s1), (d1) *and* (d2)
(ii) *for some $r \in d_{\square}^{-1}s$ and some $r' \in d_{\square}^{-1}s'$, $r\&_{\circ}r' = s''$.*

Let us collect the triples described by Proposition 4 in

$$\&^{\mathrm{d}} := \{(s, s', s'') \in (2^{\Sigma} - \{\square\})^* \times (2^{\Sigma} - \{\square\})^* \times (2^{\Sigma} - \{\square\})^* \mid \&(s, s', s'') \text{ is derivable from (s0), (s1), (d1) and (d2)}\}$$

and pick out the part of $\&^{\mathrm{d}}$ preserving projections in

$$\&_{\mathrm{d}} := \{(s, s', s'') \in \&^{\mathrm{d}} \mid s'' \text{ projects to } s \text{ and } s'\}.$$

To generate $\&_{\mathrm{d}}$, we introduce subscripts Σ, Σ' to constrain the rules (s1)

$$\frac{\&(s, s', s'') \qquad \alpha \cap \Sigma' \subseteq \alpha' \qquad \alpha' \cap \Sigma \subseteq \alpha}{\&(\alpha s, \alpha' s', (\alpha \cup \alpha')s'')}\ (\mathrm{s1})_{\Sigma,\Sigma'}$$

and (d1) and (d2)

$$\frac{\&(s, s', s'') \qquad \alpha \cap \Sigma' = \emptyset}{\&(\alpha s, s', \alpha s'')}\ (\mathrm{d1})_{\Sigma'} \qquad\qquad \frac{\&(s, s', s'') \qquad \alpha' \cap \Sigma = \emptyset}{\&(s, \alpha' s', \alpha' s'')}\ (\mathrm{d2})_{\Sigma}.$$

Proposition 5 *For any finite set Σ and $s, s', s'' \in (2^{\Sigma} - \{\square\})^*$,*

$$\&_{\mathrm{d}}(s, s', s'') \iff \&(s, s', s'') \textit{ is derivable from (s0), (s1)}_{voc(s),voc(s')}, \textit{(d1)}_{voc(s')} \textit{ and (d2)}_{voc(s)}.$$

On pairs s, s' with disjoint vocabularies, $\&_{\mathrm{d}}$ is no different from $\&^{\mathrm{d}}$

$$\&^{\mathrm{d}}(s, s', s'') \iff \&_{\mathrm{d}}(s, s', s'') \qquad \text{whenever } voc(s) \cap voc(s') = \emptyset.$$

For any finite set A, we can generate $\mathcal{L}_{\bullet}(A)$ bottom-up through a binary operation $\&_{\square}$ on languages L, L' given by $\&^{\mathrm{d}}$ according to

$$L \&_{\square} L' := \{s'' \in (2^{A} - \{\square\})^* \mid (\exists s \in L)(\exists s' \in L') \&_{\mathrm{d}}(s, s', s'')\}\ .$$

Then for any finite string $a_1 \cdots a_n$, we define a language $\mathcal{L}_{a_1 \cdots a_n}$ over the alphabet $2^{A_\bullet} - \{\square\}$ by induction on n, setting

$$\mathcal{L}_{\epsilon} := \epsilon$$

and

$$\mathcal{L}_{sa} \;:=\; \mathcal{L}_s \,\&_{\Box}\, \boxed{l(a)}\boxed{r(a)}\,.$$

Proposition 6 *For any finite string $a_1 \cdots a_n$,*

$$\mathcal{L}_{a_1 \cdots a_n} = \mathcal{L}_{\bullet}(\{a_1, \ldots, a_n\}).$$

The case $n = 2$ leads to the set $\mathcal{AR}$ of Allen relations,

$$\begin{aligned}\mathcal{L}_{aa'} &= \boxed{l(a)}\boxed{r(a)} \,\&_{\Box}\, \boxed{l(a')}\boxed{r(a')}\\ &= \mathcal{L}_{\bullet}(\{a, a'\})\\ &= \{\mathfrak{s}_R(a, a') \mid R \in \mathcal{AR}\}.\end{aligned}$$

Next, the case $n = 3$ encodes the transitivity table T

$$\begin{aligned}T(R, R') \;=\; \{R'' \in \mathcal{AR} \mid (\exists s \in \mathcal{L}_{a_1 a_2 a_3})\ s \text{ projects to}\\ \mathfrak{s}_R(a_1, a_2),\ \mathfrak{s}_{R'}(a_2, a_3) \text{ and } \mathfrak{s}_{R''}(a_1, a_3)\}\,.\end{aligned} \tag{6}$$

Implicit in (6) is a generate-and-test approach, which we can improve by the constrained superposition $\mathfrak{s}_R(a_1, a_2) \,\&_{\Box}\, \mathfrak{s}_{R'}(a_2, a_3)$

$$\begin{aligned}T(R, R') \;=\; \{R'' \in \mathcal{AR} \mid \text{some string in } \mathfrak{s}_R(a_1, a_2) \,\&_{\Box}\, \mathfrak{s}_{R'}(a_2, a_3)\\ \text{projects to } \mathfrak{s}_{R''}(a_1, a_3)\}\end{aligned}$$

sidesteping the full set $\mathcal{L}_{a_1,a_2,a_3}$, most strings in which do *not* project to both $\mathfrak{s}_R(a_1, a_2)$ and $\mathfrak{s}_{R'}(a_2, a_3)$.

4 More on the Border: Expansions, Inertia and Events

The requirement that $l(a)$ and $r(a)$ mark the left and right borders of a can be expressed with the help of certain $\text{MSO}_{\{a\}}$-formulas over a free variable x. Let $\chi_{l(a)}(x)$ say P_a holds right after x but not at x

$$\chi_{l(a)}(x) \;:=\; (\exists y)(xSy \wedge P_a(y)) \wedge \neg P_a(x)$$

and $\chi_{r(a)}(x)$ say P_a holds at x but not right after

$$\chi_{r(a)}(x) \;:=\; P_a(x) \wedge \neg(\exists y)(xSy \wedge P_a(y)).$$

We can then interpret $P_{l(a)}$ and $P_{r(a)}$ in terms of P_a according to the set

$$\Phi(A_\bullet) \;:=\; \{(\forall x)(P_t(x) \equiv \chi_t(x)) \mid t \in A_\bullet\}$$

of $\mathrm{MSO}_{A\cup A_\bullet}$-sentences equating $P_t(x)$ with $\chi_t(x)$. Given a string s of subsets of $A_\bullet$, the strings over the alphabet 2^A that b maps to s can be collected in the set

$$b^{-1}s \;=\; \{\rho_A(s') \mid s' \in s\&(2^A)^* \text{ and } (\forall\varphi \in \Phi(A_\bullet))\; Mod(s') \models \varphi\}$$

in 3 steps

STEP 1: expand with labels from A, superposing s with $(2^A)^*$
STEP 2: constrain by $\Phi(A_\bullet)$
STEP 3: reduce by ρ_A (for A-reduct).

To compute b from Steps 1–3 above, it suffices to replace A by $A_\bullet$ in steps 1 and 3. The difference between b and b^{-1} comes down to the subalphabet added in Step 1 and preserved in Step 3. $\Phi(A_\bullet)$ is, however, arguably more in sync with b than with b^{-1}, grounding, as it does, $P_{l(a)}$ and $P_{r(a)}$ in P_a. The inverse b^{-1} invites us to consider the reverse:

(Q) how do we interpret P_a, given interpretations of $P_{l(a)}$ and $P_{r(a)}$?

Answering (Q) is an instructive exercise, pointing to actions (or forces) and events.

Our answer to (Q) comes in two parts, assuming $P_{l(a)}$ and $P_{r(a)}$ are interpreted as subsets $U_{l(a)}$ and $U_{r(a)}$ (respectively) of the set $[n]$ of positions of a string of length n. The first part is an inductive construction

$$U_a = \bigcup_{i\geq 0} U_{a,i} \tag{7}$$

of the interpretation U_a of P_a according to the definitions

$$\begin{aligned} U_{a,0} &:= U_{r(a)} \\ U_{a,i+1} &:= U_{a,i} \cup \{k \in [n-1] \mid k+1 \in U_{a,i} \text{ and } k \notin U_{l(a)}\} \end{aligned}$$

suggested by the implications

$$\begin{aligned} (\forall x)(P_{r(a)}(x) \supset P_a(x)) \\ (\forall x)(\forall y)((xSy \wedge P_a(y) \wedge \neg P_{l(a)}(x)) \supset P_a(x)) \end{aligned} \tag{8}$$

from $\Phi(A_\bullet)$. The second part of our answer consists of two conditions

$$U_{l(a)} \cap U_a = \emptyset \tag{9}$$

$$\{i+1 \mid i \in U_{l(a)}\} \subseteq U_a \tag{10}$$

expressed by the implications

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$$(\forall x)(P_{l(a)}(x) \supset \neg P_a(x))$$
$$(\forall x)(P_{l(a)}(x) \supset (\exists y)(xSy \wedge P_a(y)))$$

implicit in $\Phi(A_\bullet)$. (9) and (10) hold precisely if $l(a)$ and $r(a)$ are positioned properly under $U_{l(a)}$ and $U_{r(a)}$ — i.e., there is a string in

$$(\epsilon + \boxed{r(a)})(\boxed{l(a)}\boxed{r(a)})^*$$

to which the string s corresponding to the $\text{MSO}_{A_\bullet}$-model $\langle [n], S_n, \{U_{t \in A_\bullet}\} \rangle$ projects

$$d_\square(\rho_{\{l(a),r(a)\}}(s)) \in (\epsilon + \boxed{r(a)})(\boxed{l(a)}\boxed{r(a)})^*. \tag{11}$$

Proposition 7 *For every* $s \in (2^{A_\bullet})^*$,

$$b^{-1}s \neq \emptyset \iff (11) \textit{ holds for every } a \in A.$$

Moreover, if $Mod(s) = \langle [n], S_n, \{U_t\}_{t \in A_\bullet} \rangle$ *then for every* $s' \in (2^A)^*$ *such that* $b(s') = s$,

$$Mod(s') = \langle [n], S_n, \{U_a\}_{a \in A} \rangle$$

where U_a *is given by (7) above from the sets* $U_{l(a)}$ *and* $U_{r(a)}$ *in* $Mod(s)$*. That is, under* b*,* P_a *is definable from* $P_{l(a)}$ *and* $P_{r(a)}$ *according to*

$$P_a(x) \ \equiv (\exists X)(X(x) \wedge a\text{-}path(X))$$

where a-$path(X)$ *is the conjunction*

$$\forall x(X(x) \supset (P_{r(a)}(x) \vee \exists y(xSy \wedge X(y)))$$
$$\wedge \neg \exists x(X(x) \wedge P_{l(a)}(x))$$

saying X *is an* S*-path to* $P_{r(a)}$ *that avoids* $P_{l(a)}$.

A crucial ingredient of the analysis of b^{-1} described by Proposition 7 is the implication (8) underlying the inductive step $U_{a,i+1}$. That step lets a spread to the neighboring left box unless $l(a)$ is in that box. This property of $l(a)$ can be associated with a label $f(a)$ by the implication

$$(\forall x)(\neg P_a(x) \wedge (\exists y)(xSy \wedge P_a(y)) \ \supset \ P_{f(a)}(x)) \tag{12}$$

which (without the converse of (12)) falls short of reducing $P_{f(a)}$ to $P_{l(a)}$. For the sake of symmetry, we also introduce labels $\overline{a}$ and $P_{f(\overline{a})}$ subject to

$$(\forall x)(P_{\overline{a}}(x) \equiv \neg P_a(x))$$

making $\overline{a}$ the negation of a, and the implication

$$(\forall x)(P_a(x) \wedge (\exists y)(xSy \wedge \neg P_a(y)) \supset P_{f(\overline{a})}(x)). \tag{13}$$

tracking, with (12), any changes in $a/\overline{a}$. The intuition is that $f(a)$ and $f(\overline{a})$ mark the applications of forces for and against a (respectively). The slogan behind (12) and (13) is

no change unless forced

or, in one word, *inertia*. Equating syntactically $f(a)$ with $l(a)$ and $f(\overline{a})$ with $r(a)$ ensure (12) and (13) hold, but let us be careful to resist these identifications and allow $P_{f(a)}$ and $P_{f(\overline{a})}$ to diverge from $P_{l(a)}$ and $P_{r(a)}$ outside (12) and (13), saving the idea of inertia from vacuity. Whereas under (11), $l(a)$ and $r(a)$ cannot occur in the same box, there is nothing a priori illegitimate about a box containing both $f(a)$ and $f(\overline{a})$. Clashing forces are commonplace and merit logical scrutiny (rather than dismissal). Over any given stretch of time, any number of forces can be at play, some of which may be neutralized by competition. A force in isolation may have very different effects with company. That said, there is no denying that we detect and evaluate forces by the state changes they effect.

Stative predicates labeled by $a \in A$ differ significantly from non-stative predicates labeled by $l(a)$, $r(a)$, $f(a)$ and $f(\overline{a})$ in how strings built from them compress to canonical forms. Recall from the Introduction the link between homogeneity and block compression bc, deleting stutters

$$\mathit{bc}^{-1}\mathit{bc}(s) = \alpha_1^+ \cdots \alpha_n^+ \quad \text{if } \mathit{bc}(s) = \alpha_1 \cdots \alpha_n$$

just as $d_\Box$ deletes $\Box$

$$d_\Box^{-1} d_\Box(s) = \Box^*\alpha_1\Box^* \cdots \Box^*\alpha_n\Box^* \quad \text{if } d_\Box(s) = \alpha_1 \cdots \alpha_n.$$

Next, let us collect the image of $(2^\Sigma)^*$ under bc in

$$\mathcal{L}_{bc}(\Sigma) \ := \ \{\mathit{bc}(s) \mid s \in (2^\Sigma)^*\}$$

and its image under $d_\Box$ in

$$\mathcal{L}_d(\Sigma) \ := \ \{d_\Box(s) \mid s \in (2^\Sigma)^*\}$$

and note that the square

$$\begin{array}{ccc} (2^A)^* & \xrightarrow{\ \mathit{bc}\ } & \mathcal{L}_{bc}(A) \\ \downarrow b & & \downarrow b \\ (2^{A_\bullet})^* & \xrightarrow{\ d_\Box\ } & \mathcal{L}_d(A_\bullet) \end{array}$$

Table 2 ARKV as reconstructed in [14], annotated with strings

	atomic	extended
+conseq	culmination $\boxed{\overline{a}}\boxed{a}$	culminated process $\boxed{\overline{a},\ \mathrm{ap}(f)}\boxed{\overline{a},\ \mathrm{ap}(f),\mathrm{ef}(f)}\boxed{\mathrm{ef}(f),\ a}$
−conseq	point $\boxed{\mathrm{ap}(f)}\boxed{\mathrm{ef}(f)}$	process $\boxed{\mathrm{ap}(f)}\boxed{\mathrm{ap}(f),\ \mathrm{ef}(f)}\boxed{\mathrm{ef}(f)}$

does not quite commute as

$$b(\mathit{bc}(\boxed{a}\boxed{})) \;=\; \boxed{r(a)}\boxed{} \;\neq\; d_{\Box}(b(\boxed{a}\boxed{})) \;=\; \boxed{r(a)}.$$

But this is easily repaired by appending a non-empty set to $(2^A)^*$, replacing $(2^\Sigma)^*$ by

$$\mathcal{L}_+(\Sigma) \;:=\; \{s\alpha \mid s \in (2^\Sigma)^* \text{ and } \alpha \in 2^\Sigma - \{\Box\}\}$$

for

$$\begin{array}{ccc} \mathcal{L}_+(A) & \xrightarrow{\mathit{bc}} & \mathcal{L}_{\mathit{bc}}(A) \\ \downarrow b & & \downarrow b \\ \mathcal{L}_+(A_\bullet) & \xrightarrow{d_\Box} & \mathcal{L}_d(A_\bullet) \end{array}$$

which does commute.

Proposition 8 *For every $s \in \mathcal{L}_+(A)$,*

$$b(\mathit{bc}(s)) \;=\; d_\Box(b(s))$$

and $\mathit{bc}(s)$ is the unique string in the set $b^{-1}d_\Box(b(s))$.

Underlying both notions of compression, $d_\Box$ and bc, is the slogan

no time without change.

But while bc represents that change in terms of decomposable intervals/statives, $d_\Box$ employs non-decomposable points/borders (not to mention forces). The function $d_\Box$ is simpler than bc, and provides a pointed twist on the interval-based approach in [5] to Dowty's aspect hypothesis.

An important test for Dowty's aspect hypothesis is the Aristotle-Ryle-Kenny-Vendler verb classification (ARKV), a version of which, due to [14], is annotated with strings in Table 2. In Table 2 the symbol a is understood to represent the consequent state, to which $+$ and $-$ are attached in the first column. By contrast, f is a force, the application of which is marked as ap(f), and effect as ef(f). That effect need not preclude the repeated application of f, as is clear from the rightmost (extended/process) column of the table, featuring ap(f) in adjacent boxes. The effect may, for example, be an incremental change along some scale (for instance, temperature in the case of the verb *cool*). The contrast between ap(f)/ef(f) in the −conseq row and the statives $\overline{a}/a$ in the +conseq row is compatible with *Manner Result complementarity*

> the proposal that verbs lexicalize either manner or result meaning components, but not both [12].

Levin and Rappaport Hovav are careful to distinguish *lexicalized meaning* from "additional facets of meaning that can be inferred from a particular use of that verb in context and from the choice of noun phrases serving as arguments of the verb." If we enlarge the vocabulary of a string from statives in A to borders in $A_\bullet$, note that the transition

$$\boxed{\overline{a}}\boxed{a} \text{ expands to } \boxed{\overline{a}, l(a)}\boxed{a}$$

with $l(a)$ acting as a force $f(a)$ implicated by the inertial principle (12)

$$(\forall x)(\neg P_a(x) \wedge (\exists y)(xSy \wedge P_a(y)) \supset P_{f(a)}(x)) . \tag{12}$$

By resisting to identify $f(a)$ syntactically with $l(a)$, we are acknowledging a notion of a force (represented by f) over and above borders of statives. As a scheme for lexicalized meaning, Table 2 describes a force f in isolation from other forces that may, in a particular context of use, interfere with f. Any such interference is abstracted away from the input/output pairs constituting the meaning of a program in *Dynamic Logic* [10] (applied in [15, 16] for an account of ARKV). It is tempting to liken the formal difference in Dynamic Logic between programs (interpreted as binary relations on states) and formulas (interpreted as subsets of states) to the basic aspectual distinction between non-stative and stative predicates (drawn in [11] and many other works). Under the present account, however, stative and non-stative predicates alike can be formulated as symbols interpreted as unary predicates, and the line between statives and non-statives is to be discerned not so much within a model (which in MSO is a string) as between models related by projections. In particular, the question of whether or not ap(f) is stative comes to whether or not reducing the string

$$\boxed{\text{ap}(f)}\boxed{\text{ap}(f)} \text{ to } \boxed{\text{ap}(f)}$$

by block compression bc is appropriate. (It is if ap(f) is homogeneous, as statives in [3] are understood to be.) The link forged above between ap(f) and the left border $l(a)$ (with a as ef(f)) suggests otherwise, even if, according to the last column of Table 2, ap(f) may recur in a string.

5 Conclusion

The main contribution of the present work is the introduction of non-stative predicates through a border translation b, which is then applied to Allen intervals and events. For convenience, let us collect labels of statives in a set D (for Dowty), and restrict stutter equivalence to subsets of D

$$s\alpha\alpha s' \approx s\alpha s' \qquad \text{for } \alpha \subseteq D \tag{14}$$

whilst disregarding the empty box $\Box$ for strings of sets disjoint from D

$$s\Box s' \approx ss' \qquad \text{for } voc(ss') \cap D = \emptyset. \tag{15}$$

(14) attends to statives, (15) to non-statives. The shortest $\approx$-representative is obtained by block compression bc in the case of (14), and depadding $d_\Box$ in the case of (15). For $s \in (2^A)^+$, we can devise an MSO$_A$-formula stutter$_A(x, y)$ to pick out stutters as adjacent string positions x, y with the same A-labels

$$\text{stutter}_A(x, y) \;:=\; xSy \wedge \bigwedge_{a\in A}(P_a(x) \equiv P_a(y))$$

and characterize stutterless strings by ruling stutter$_A(x, y)$ out

$$bc(s) = s \iff Mod_A(s) \models \neg\exists x\exists y \;\text{stutter}_A(x, y)$$

and similarly for $\Box$-removal $d_\Box$

$$d_\Box(s) = s \iff Mod_A(s) \models \neg\exists x \;\text{empty}_A(x)$$

where empty$_A(x)$ picks out string positions x at which nothing from A occurs

$$\text{empty}_A(x) \;:=\; \bigwedge_{a\in A} \neg P_a(x).$$

The subscript A on stutter$_A(x, y)$ and empty$_A(x)$ is an indispensible ingredient representing granularity, commonly consigned to the background. A is foregrounded as a *signature* in *institutions* [8], where A is attached as a subscript on $\models$ along with the condition

$$M \models_A \varphi \iff M \upharpoonright A' \models_{A'} \varphi$$

for all MSO$_A$-models M, subsets A' of A, and MSO$_{A'}$-sentences φ. It is through A that we can turn up the magnification, except that it is magnification turned *down* which is technically more convenient to analyze, using A'-reducts $\rho_{A'}(s)$ where $A' \subseteq A$. The difference between statives and non-statives is revealed by the choice of compression to apply to A'-reducts for A'-projections —

$$\rho_{A'};\, bc \quad \text{in the case of statives}$$

and

$$\rho_{A'};\, d_{\Box} \quad \text{in the case of non-statives.}$$

But what does all this have to do with points?

Various notions of point are relevant here. One is the indivisibility Allen associates with points, a reason (he argues) for preferring intervals. Our reconstruction of interval relations and transitivities in Sects. 2 and 3 is based on finite strings $\alpha_1 \cdots \alpha_n$ with finitely many string positions $[n] = \{1, \ldots, n\}$, each one an indivisible point in the MSO$_A$-model $Mod_A(\alpha_1 \cdots \alpha_n)$. But, enlarge A and we can split a box with the apparatus of projections above, as illustrated by the example of strings

$$\fbox{}\fbox{a}\fbox{a, a'}\fbox{a'}\fbox{} \tag{1}$$

and

$$\fbox{}\fbox{a''}\fbox{a, a''}\fbox{a, a', a''}\fbox{a, a'}\fbox{a'}\fbox{} \tag{2}$$

from the Introduction. That is, a box is an indivisible point only if we keep A fixed. This notion of point is somewhat slippery inasmuch as granularity can be varied. For temporal representations with points, fix A; for temporal representations without points, vary A.

A different notion of point is brought out by the border translation b. The use of borders in Sects. 2 and 3 trades the box-splitting of (1) in (2) for the introduction of an empty box that $d_{\Box}$ can remove or superposition with another string can fill (Sect. 3). We move from the interior described by a stative a to the border described by $l(a)$ or $r(a)$. An essential feature of the border $l(a)$ spelled out by the MSO-equivalence

$$P_{l(a)}(x) \;\equiv\; (\exists y)(xSy \wedge P_a(y)) \wedge \neg P_a(x)$$

from Sect. 4 is that $l(a)$ depends on two adjacent positions that differ on a. (The case of $r(a)$ is complicated by occurrences at the end of the string.) This explains why in the ARKV table, Table 2, the atomic column is filled by strings of length 2. Inasmuch as the strings in the domain of b are built from statives, our use of b is arguably compatible with Dowty's hypothesis that a basis of statives suffices for aspect. However, the $-$conseq, atomic entry in Table 2 involves a force f over and above statives a. Weakening the border $l(a)$ to a force $f(a)$ implicit in the idea of inertia and explicit in

$$\neg P_a(x) \wedge (\exists y)(xSy \wedge P_a(y)) \;\supset\; P_{f(a)}(x) \tag{12}$$

hints at the existence of forces beyond the borders introduced by the border translation b. There is an opening here to break Dowty's aspect hypothesis apart, deriving statives from forces, some guidance on which is sought in Sect. 4 from the inverse b^{-1} of b.

Yet another notion of point is that of an s-point a occurring uniquely in s (Sect. 2). This unique occurrence requirement should be understood as applying to particulars (singulars, tokens) rather than universals (plurals, types). To see this, note from Proposition 7 that $l(a)$ and $r(a)$ may occur more than once in a string as long as a does not describe a single interval. While particulars are singularly helpful as labels for forming linear orders, there is nothing wrong with a stative scattered over many intervals in a string. Likewise, a string built with a label representing many borders. It is noteworthy that superposition in Sect. 3 applies as much to universals (e.g., event types) as to particulars (e.g., event tokens).

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