Towards a many-dimensional modal logic for semantic processing

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Abstract. Notions of context for natural language interpretation are factored in terms of three processes: translation, entailment and attunement. The processes are linked by accessibility relations of the kind studied in many-dimensional modal logic, modulo complications from constraints between translation and entailment (violations in which may trigger re-attunement) and from refinement and underspecification.

1 Introduction

It is familiar practice in semantics to encode context as a sequence, selected components of which can be varied while others are held constant. The indexical *now* was, for instance, brought into temporal logic in [Kam71] via an early form of *multi-dimensional modal logic* (e.g. [MV97]). And to treat presupposition, [Sta78] injected content into context, feeding the development of "dynamic" approaches to semantics such as *Discourse Representation Theory* (DRT). The present work is motivated largely by what is identified in [Kam90] as "the principal challenge to DRT" — viz. "the exact formulation of the construction algorithm" which takes syntactic analyses of English sentences and returns logical forms. Our emphasis, however, is not so much on any particular kind of syntactic analysis or (target) logical form as it is on the incrementality and non-determinism of some such algorithm. That incrementality and non-determinism are analyzed by bringing out contextual parameters that condition processing.

1.1 Background: translation and entailment

To fix notation, let E be the set of inputs to the algorithm and Φ be the set of outputs. Without making any assumptions about E or Φ, let us form the set

$$\mathit{Stage} \quad \subseteq \quad \bigcup_{n \geq 0} (E^n \times \Phi^n)$$

of pairs $(\vec{e}, \vec{\varphi})$ of input and output sequences. These i/o pairs can be construed as stages of a *translation* process insofar as every pair

$$(e_1 e_2 \cdots e_n, \varphi_1 \varphi_2 \cdots \varphi_n) \in \mathit{Stage}$$

can be traced to the initial pair (ϵ, ϵ) (of empty input and output) by the sequence of transitions

$$(\epsilon, \epsilon) \stackrel{e_1}{\to} (e_1, \varphi_1) \stackrel{e_2}{\to} (e_1 e_2, \varphi_1 \varphi_2) \stackrel{e_3}{\to} \cdots \stackrel{e_n}{\to} (e_1 e_2 \cdots e_n, \varphi_1 \varphi_2 \cdots \varphi_n) \quad (1)$$

where for every $e \in E$,

$$\stackrel{e}{\to} \; = \bigcup_{(\vec{e}, \vec{\varphi}) \in \mathit{Stage}} \{((\vec{e}, \vec{\varphi}), (\vec{e}e, \vec{\varphi}\varphi)) \; : \; (\vec{e}e, \vec{\varphi}\varphi) \in \mathit{Stage}\} \; .$$

The existence of the transition sequence (1) can be simplified to the requirement that *Stage* be *prefix-closed* — i.e., whenever $(\vec{e}e, \vec{\varphi}\varphi) \in \mathit{Stage}$, $(\vec{e}, \vec{\varphi}) \in \mathit{Stage}$. The transitions $\stackrel{e}{\to}$ are worth isolating, however, to examine context, the semantic component of which is given by a notion of entailment

$$\vdash \; \subseteq \; \Phi^* \times \Phi$$

(which presumably is the point behind passing from E to Φ). Together, a translation-entailment pair $(\mathit{Stage}, \vdash)$ provides an interpretation of a modal language $\mathcal{L}_\circ$ consisting of formulas A generated according to

$$A \quad ::= \quad \varphi \;\mid\; \langle e \rangle A \;\mid\; \neg A \;\mid\; A \wedge A$$

via a "forcing" predicate $\Vdash_\circ \subseteq \mathit{Stage} \times \mathcal{L}_\circ$ defined by

$$\begin{array}{rcl}
(\vec{e}, \vec{\varphi}) \Vdash_\circ \varphi & \text{iff} & \vec{\varphi} \vdash \varphi \\
(\vec{e}, \vec{\varphi}) \Vdash_\circ \langle e \rangle A & \text{iff} & (\vec{e}e, \vec{\varphi}\varphi) \Vdash_\circ A \text{ for some } \varphi \\
(\vec{e}, \vec{\varphi}) \Vdash_\circ \neg A & \text{iff} & \text{not } (\vec{e}, \vec{\varphi}) \Vdash_\circ A \\
(\vec{e}, \vec{\varphi}) \Vdash_\circ A \wedge B & \text{iff} & (\vec{e}, \vec{\varphi}) \Vdash_\circ A \text{ and } (\vec{e}, \vec{\varphi}) \Vdash_\circ B
\end{array}$$

(with $[e]A$, $A \vee B$, $A \supset B$ and $A \equiv B$ understood as abbreviations of $\neg\langle e \rangle \neg A$, $\neg(\neg A \wedge \neg B)$, $\neg(A \wedge \neg B)$ and $(A \supset B) \wedge (B \supset A)$, respectively).[1] This is just Kripke semantics, with accessibility relations $\stackrel{e}{\to}$ determined by *Stage*, and valuation by $\vdash$. Accordingly, let us henceforth refer to pairs $(\mathit{Stage}, \vdash)$ as $\mathcal{L}_\circ$-*models*. A measure of the ambiguity of e at $(\vec{e}, \vec{\varphi})$ is given by $\mathcal{L}_\circ$-formulas A, if any, such that

$$(\vec{e}, \vec{\varphi}) \quad \Vdash_\circ \quad \langle e \rangle A \;\wedge\; \langle e \rangle \neg A \; ,$$

discerning any difference between logical forms φ and φ' such that $(\vec{e}e, \vec{\varphi}\varphi)$ and $(\vec{e}e, \vec{\varphi}\varphi') \in \mathit{Stage}$. Quantifying over all stages, let us call an $\mathcal{L}_\circ$-formula A $(\mathit{Stage}, \vdash)$-*valid* if for every $(\vec{e}, \vec{\varphi}) \in \mathit{Stage}$, $(\vec{e}, \vec{\varphi}) \Vdash A$. For example, suppose E includes the following English sentences e_1 and e_2.

[1]The multi-modal character of $\mathcal{L}_\circ$ is reminiscent of dynamic logic. For applications to DRT, the difference can be put very briefly as follows: quantified dynamic logic pertains to the model-theoretic interpretation of DRT's logical forms (called DRSs), $\mathcal{L}_\circ$ to the construction algorithm. (More in [Fer99a].)

e_1 : Doris informed Chloe she aced the exam.

e_2 : In fact, she was the only Texan who passed.

Ambiguous though e_1 and e_2 may be, to say that the *she*'s in $e_1 e_2$ co-vary in the $\mathcal{L}_\circ$-model $(\mathit{Stage}, \vdash)$ is to require that the $\mathcal{L}_\circ$-formula

$$[e_1][e_2] \quad (\dot{\forall} x \dot{\in} \text{Texan})(\text{passed}(x) \,\dot{\supset}\, \text{aced}(x))$$

be $(\mathit{Stage}, \vdash)$-valid, where the dotted connectives are connectives in Φ (as opposed to $\mathcal{L}_\circ$).

Before worrying about multiple outputs, we might check that an input e at a stage $(\vec{e}, \vec{\varphi})$ has any output at all, which can be expressed as

$$(\vec{e}, \vec{\varphi}) \;\; ||\!\!-_\circ \;\; \langle e \rangle \top \qquad [\text{ iff } \;(\vec{e}, \vec{\varphi}) \in \mathit{dom}(\stackrel{e}{\rightarrow})\;]$$

where $\top$ is an $\mathcal{L}_\circ$-tautology (valid in every $\mathcal{L}_\circ$-model). In accordance with [Kar74], presupposition can be equated with pre-condition for translation,[2] and presupposition satisfaction in [Hei83] modified slightly to

$$(\vec{e}, \vec{\varphi}) \textit{ satisfies the presuppositions of } e \quad \text{iff} \quad (\vec{e}, \vec{\varphi}) \; ||\!\!-_\circ \langle e \rangle \top \; .$$

Consider, for example,

$\hat{e}$: The king of Buganda is bald.

That "$\hat{e}$ presupposes there is a king of Buganda" can then be stated as

$$\langle \hat{e} \rangle \top \quad \equiv \quad \text{`}\exists\text{KoB'} \tag{2}$$

where '∃KoB' $\in \Phi$ asserts there is a king of Buganda. Moreover, the *negation test* becomes the $\mathcal{L}_\circ$-formula

$$\langle \dot{\neg} e \rangle \top \quad \equiv \quad \langle e \rangle \top \tag{3}$$

(where $\dot{\neg}$ is a connective in E, not $\mathcal{L}_\circ$), while the presupposition projection behavior of E-conjunction $\dot{\wedge}$ and E-implication $\dot{\supset}$ (accounted for in [Hei83], going back at least to Karttunen) translate in $\mathcal{L}_\circ$ to the sequentiality conditions

$$\begin{aligned} \langle e \dot{\wedge} e' \rangle \top \quad &\equiv \quad \langle e \rangle \langle e' \rangle \top \\ \langle e \dot{\supset} e' \rangle \top \quad &\equiv \quad \langle e \rangle \langle e' \rangle \top \; . \end{aligned}$$

Turning to the logic of $\mathcal{L}_\circ$, [Fer99a] establishes the following. Call an entailment relation $\vdash$ $\mathcal{L}_\circ$-*transparent* if for all $\varphi_1, \ldots, \varphi_n, \varphi \in \Phi$, we have

[2] The term presupposition has been applied to so many things that any account of it is likely to be contentious. What is at stake in the present paper is the claim that failures in translation are interesting, as are the adjustments (to $\mathcal{L}_\circ$-models) made before, during or after translation. The present shift in presupposition satisfaction from the realm of semantics ([Hei83]) to the construction algorithm is compatible not only with the *presupposition-as-anaphora* view of [San92] but also with the *familiarity condition* of [Hei82].

$\varphi_1 \cdots \varphi_n \mid\!\sim \varphi$ iff the $\mathcal{L}_\circ$-formula $(\varphi_1 \wedge \cdots \wedge \varphi_n) \supset \varphi$ is $(\mathit{Stage}, \mid\!\sim)$-valid, for every $\mathit{Stage} \subseteq \bigcup_{n \geq 0}(E^n \times \Phi^n)$.

Proposition. *$\mid\!\sim$ is $\mathcal{L}_\circ$-transparent iff $\mid\!\sim$ verifies*

$$\text{(Weak)} \quad \frac{\vec{\varphi} \mid\!\sim \varphi}{\psi\vec{\varphi} \mid\!\sim \varphi} \quad \frac{\vec{\varphi} \mid\!\sim \varphi}{\vec{\varphi}\psi \mid\!\sim \varphi} \qquad\qquad \text{(Ref)} \quad \frac{}{\varphi \mid\!\sim \varphi}$$

$$\text{(Cut)} \quad \frac{\vec{\varphi} \mid\!\sim \psi \qquad \psi\vec{\varphi} \mid\!\sim \varphi}{\vec{\varphi} \mid\!\sim \varphi} \; .$$

As formulated above, (Weak), (Ref) and (Cut) imply contraction and exchange

$$\text{(Con)} \quad \frac{\vec{\varphi}\varphi\varphi \mid\!\sim \psi}{\vec{\varphi}\varphi \mid\!\sim \psi} \qquad\qquad \text{(Exc)} \quad \frac{\vec{\varphi}\varphi\psi\vec{\psi} \mid\!\sim \varphi'}{\vec{\varphi}\psi\varphi\vec{\psi} \mid\!\sim \varphi'}$$

(as (Exc) is implicit in doubling (Weak), and (Con) in halving the conclusion of (Cut) from $\vec{\varphi}\,\vec{\varphi} \mid\!\sim \varphi$ to $\vec{\varphi} \mid\!\sim \varphi$). Next, let us add Φ-conjunction and Φ-implication (distinct from $\mathcal{L}_\circ$-connectives) to (Weak), (Ref) and (Cut) in the assumption

(a$\mid\!\sim$) (Weak), (Ref) and (Cut) hold in $\mid\!\sim$, and there are binary connectives $\dot{\wedge}$ and $\dot{\supset}$ in Φ s.t. for all $\varphi, \psi, \varphi' \in \Phi$ and $\vec{\varphi} \in \Phi^*$,

$$\begin{aligned} (\varphi\dot{\wedge}\psi)\vec{\varphi} \mid\!\sim \varphi' \quad &\text{iff} \quad \varphi\psi\vec{\varphi} \mid\!\sim \varphi' \\ \vec{\varphi}\varphi \mid\!\sim \psi \quad &\text{iff} \quad \vec{\varphi} \mid\!\sim \varphi\dot{\supset}\psi \; . \end{aligned}$$

As for *Stage*, let us embed the set Φ of logical forms in the set E of inputs to translation, with the understanding that *Stage* keeps the logical forms fixed, and contains the pair (ϵ, ϵ) — i.e.,

(a$_\circ$) $\Phi \subset E$, $(\epsilon, \epsilon) \in \mathit{Stage}$ and for every $(e_1 \cdots e_n, \varphi_1 \cdots \varphi_n) \in \mathit{Stage}$,

$$\begin{aligned} (\forall i \in \{1, \ldots, n\}) \qquad & \text{if } e_i \in \Phi \text{ then } e_i = \varphi_i \\ (\forall \varphi \in \Phi) \qquad & (e_1 \cdots e_n\varphi, \varphi_1 \cdots \varphi_n\varphi) \in \mathit{Stage} \; . \end{aligned}$$

The assumption (a$_\circ$) says that every logical form φ in Φ can be obtained unambiguously and without risk of presupposition failure by choosing an appropriate input, conveniently named φ. This re-use of φ is perhaps confusing, but otherwise harmless. Now, applying $\dot{\wedge}$ to combine any finite number of logical forms entailed at a certain stage into one, and $\dot{\supset}$ to anticipate logical forms entailed at a next stage, we get

Theorem. *The set of $\mathcal{L}_\circ$-formulas valid in all $\mathcal{L}_\circ$-models $(\mathit{Stage}, \mid\!\sim)$ satisfying (a$\mid\!\sim$) and (a$_\circ$) is decidable, and consists of the theorems of normal modal logic, supplemented with the axiom schemes*

(A1)	$\varphi\dot{\supset}\varphi$	*(A4)*	$\varphi \supset [e]\varphi$
(A2)	$\varphi \wedge \psi \;\equiv\; \varphi\dot{\wedge}\psi$	*(A5)*	$\langle\varphi\rangle\psi \;\equiv\; \varphi\dot{\supset}\psi$
(A3)	$\varphi\dot{\supset}\psi \;\supset\; \varphi \supset \psi$	*(A6)*	$\langle\varphi\rangle A \supset [\varphi]A$

For a fixed entailment relation $\vdash$ satisfying (a$\vdash$), it suffices to strengthen (A1)-(A3) to the scheme

$$\varphi_1 \wedge \cdots \wedge \varphi_n \quad \supset \quad \varphi$$

for all $\varphi_1, \ldots, \varphi_n, \varphi$ such that $\varphi_1 \cdots \varphi_n \vdash \varphi$.

Observe that adding $E = \Phi$ to ($a_\circ$) and (a$\vdash$) would validate the commuting squares

$$\langle e\rangle\langle e'\rangle A \quad \equiv \quad \langle e'\rangle\langle e\rangle A$$

in [MV97]. Our analysis of ambiguity and presupposition, however, depends on inputs E to *Stage* going beyond the outputs Φ — or more to the point: on *Stage* being unrulier than a function (let alone a bijection).

1.2 Plan of present paper

Identifying the presupposition of e with $\langle e\rangle\top$ raises the question: what if $\langle e\rangle\top$ fails? Take, for instance, $\hat{e}$ above, uttered say, in isolation — i.e. at the initial stage (ϵ, ϵ) of some $\mathcal{L}_\circ$-model $(\mathit{Stage}, \vdash)$. If $\epsilon \vdash$ '∃KoB' then $\hat{e}$'s presupposition is met. But if not — and note that we can hardly expect before hearing $\hat{e}$ to have chosen $\vdash$ such that $\epsilon \vdash$ '∃KoB' — then according to (2), $\hat{e} \notin \mathit{dom}(\mathit{Stage})$. How then can we translate $\hat{e}$? If interpretation is to proceed (and many have argued it may), we have no choice but to leap out of $(\mathit{Stage}, \vdash)$, and into some other $\mathcal{L}_\circ$-model $(\mathit{Stage}', \vdash')$ where '∃KoB' is taken for granted:

$$\epsilon \vdash' \text{'∃KoB'} \quad \text{whence (2) implies} \quad \hat{e} \in \mathit{dom}(\mathit{Stage}').$$

Such leaps, called accommodation in the presupposition literature, can be quite tricky, subject, as they are, to various constraints such as consistency and plausibility which may or may not be amenable to formalization. Without settling which leaps are legitimate, we can nonetheless survey the possibilities by expanding $\mathcal{L}_\circ$ to a language $\mathcal{L}$, the models of which are formed from families of $\mathcal{L}_\circ$-models ([Fer99b]). Indexing a family of $\mathcal{L}_\circ$-models by a set I, the idea is to expose the heretofore implicit contextual parameter $\alpha \in I$ determining an $\mathcal{L}_\circ$-model $(\mathit{Stage}_\alpha, \vdash_\alpha)$. Passing from $\mathcal{L}_\circ$-stages $(\vec{e}, \vec{\varphi})$ to $\mathcal{L}$-stages $(\vec{e}, \vec{\varphi})_\alpha$, where α keeps track of the $\mathcal{L}_\circ$-model to which $(\vec{e}, \vec{\varphi})$ is understood to belong, the present paper explores the problem of factoring an $\mathcal{L}$-transition

$$(\vec{e}, \vec{\varphi})_\alpha \quad \overset{e}{\rightsquigarrow} \quad (\vec{e}e, \vec{\psi}\psi)_\beta \tag{4}$$

that changes the index α and previous translations $\vec{\varphi}$ (to β and $\vec{\psi}$, respectively) in the course of translating e to ψ. An example of (4) is considered in section

2, motivating investigations in sections 3 and 4 of the question: can we treat the three changes (to α and to $\vec{\varphi}$, plus the translation of e) separately? Section 3 provides a partial positive reply, centering around merged translation sets. Complications involving entailment are taken up in section 4, refining (as well as underspecifying) the notion of logical form.

2 An example: *attunement*

Recalling §1.1 above, consider the negation $\dot{\neg}\hat{e}$ of $\hat{e}$

$\dot{\neg}\hat{e}$: The king of Buganda is not bald.

Combining the negation test (3) with (2) yields

$$\langle\dot{\neg}\hat{e}\rangle\top \quad\equiv\quad \text{`}\exists\text{KoB'} \ . \tag{5}$$

Assuming (for the sake of the argument) that $\dot{\neg}\hat{e}$ is uttered at stage (ϵ,ϵ) of an $\mathcal{L}_\circ$-model $(\mathit{Stage}_0, \vdash_0)$ for which $\epsilon \not\vdash_0$ '∃KoB' (and $\dot{\neg}\hat{e} \notin \mathit{dom}(\mathit{Stage}_0)$), a minimal adjustment of background assumptions leads to $(\mathit{Stage}_1, \vdash_1)$ where

$$\begin{array}{rcl} (\vec{e},\vec{\varphi}) \in \mathit{Stage}_1 & \text{iff} & (\text{`}\exists\text{KoB'}\vec{e}, \text{`}\exists\text{KoB'}\vec{\varphi}) \in \mathit{Stage}_0 \\ \vec{\varphi} \vdash_1 \varphi & \text{iff} & \text{`}\exists\text{KoB'}\vec{\varphi} \vdash_0 \varphi \end{array}$$

for all $\vec{\varphi} \in \Phi^*$, $\varphi \in \Phi$ and $\vec{e} \in E^*$. The adjustment would be no different were $\hat{e}$ uttered instead of $\dot{\neg}\hat{e}$. But suppose we were to follow $\dot{\neg}\hat{e}$ by

$\dot{\neg}$'∃KoB' : Buganda has no king.

Or, for that matter, what if $\epsilon \vdash_0 \dot{\neg}$'∃KoB'? Under such circumstances, uttering $\dot{\neg}\hat{e}$ at $(\epsilon,\epsilon)_0$ might be odd, but surely not outright contradictory the way $\hat{e}$ would? Perhaps (5) is too strong and ought to be weakened to

$$\neg\dot{\neg}\text{`}\exists\text{KoB'} \quad\supset\quad (\langle\dot{\neg}\hat{e}\rangle\top \ \equiv\ \text{`}\exists\text{KoB'}) \tag{6}$$

where the $\mathcal{L}_\circ$-formula $\neg\dot{\neg}$'∃KoB' expresses the (syntactic) consistency of '∃KoB'. Then, assuming $\epsilon \vdash_0 \dot{\neg}$'∃KoB', $\dot{\neg}\hat{e}$ need not trigger a leap between $\mathcal{L}_\circ$-models. But if (as is eminently plausible) neither $\epsilon \vdash_0$ '∃KoB' nor $\epsilon \vdash_0 \dot{\neg}$'∃KoB', then a leap to $(\mathit{Stage}_1, \vdash_1)$ would run afoul of a subsequent utterance of $\dot{\neg}$'∃KoB'. For (6), we could leap instead to an $\mathcal{L}_\circ$-model $(\mathit{Stage}_2, \vdash_2)$ that rejects '∃KoB'

$$\begin{array}{rcl} (\vec{e},\vec{\varphi}) \in \mathit{Stage}_2 & \text{iff} & (\dot{\neg}\text{`}\exists\text{KoB'}\vec{e}, \dot{\neg}\text{`}\exists\text{KoB'}\vec{\varphi}) \in \mathit{Stage}_0 \\ \vec{\varphi} \vdash_2 \varphi & \text{iff} & \dot{\neg}\text{`}\exists\text{KoB'}\vec{\varphi} \vdash_0 \varphi \ . \end{array}$$

However, if after $\dot{\neg}\hat{e}$, we are told that the king of Buganda has curls (rather than $\dot{\neg}$'∃KoB'), then we get a contradiction in $(\mathit{Stage}_2, \vdash_2)$, from which we would have been immune in $(\mathit{Stage}_1, \vdash_1)$. Inasmuch as there is no telling what may come after the input $\dot{\neg}\hat{e}$, no correct choice can be made at $(\epsilon,\epsilon)_0$ between $(\mathit{Stage}_1, \vdash_1)$ and $(\mathit{Stage}_2, \vdash_2)$.[3]

[3]This is not to deny that there might be a preference (which can be expressed through modal operators; e.g. [Fer99b]) for the presupposition '∃KoB', but only to affirm that, as a default, it is subject to retraction (through another leap).

2.1 Leaps triggered by pre- and post-conditions

Notice that if one of the logical forms in Φ is contradictory, call it $\bot$, then assumption ($a_\circ$) from §1.1 above provides stages forcing $\bot$ — e.g. $(\bot, \bot)$. Garbage in, garbage out. That said, we can, with $\mathcal{L}_\circ$, discriminate between stages that force $\bot$ and those that do not (i.e. those forcing $\neg\bot$). These consistency considerations can trigger leaps between $\mathcal{L}_\circ$-models before or after logical forms are constructed. It is noteworthy that [Gaz79] lumps presupposition with Gricean implicature, conceived as a pragmatic adjustment made after logical forms have been constructed. Whatever picture of presupposition one adopts, there are many reasons (including implicatures) for adding background information α to the explicit discourse context $(\vec{e}, \vec{\varphi})$. The generality of logical forms (e.g. [MB97]) is brought out by keeping α separate from the translation $\vec{\varphi}$ of an input $\vec{e}$ (rather than packing it into $\vec{\varphi^\alpha}$) and then applying $\vec{\varphi}$ to different α's. The process of choosing α I propose to label *attunement*. It is clearly related to the two other processes of entailment and translation, both of which take on a monotonic character in §1.1. The expansion from $\mathcal{L}_\circ$ to $\mathcal{L}$ allows non-monotonicity to enter through transitions

$$(\vec{e}, \vec{\varphi})_\alpha \stackrel{-}{\rightarrow} (\vec{e}, \vec{\varphi})_\beta \quad \text{and} \quad (\vec{e}, \vec{\varphi})_\alpha \stackrel{\Diamond}{\rightarrow} (\vec{e}, \vec{\psi})_\alpha$$

that change the indices and translations respectively. Section 3 explores how to factor $\mathcal{L}$-transitions $\stackrel{e}{\rightsquigarrow}$ in (4) between $\stackrel{-}{\rightarrow}$, $\stackrel{\Diamond}{\rightarrow}$ and $\stackrel{e}{\rightarrow}$.

2.2 From denials to underspecification

Rather than leaping to conclusions that are defeasible, we might try to avoid non-monotonicity by minimizing any choices made, postponing these until the necessary contextual elements become available. Returning to $\hat{e}$, for example, [San91] argues that $\dot{\neg}\hat{e}$ can be used not only as an assertion, in which case (5) holds, but also as a *denial*, in which case it is not an assertion and not subject to (5). Assuming this argument were correct, the challenge becomes: what if we are told "the king of Buganda is not bald" but are unable to tell if the utterance is an assertion or a denial? Matters would be simple were we to restrict our inputs to representations for which the question "am I an assertion or a denial?" is answered. The problem is that inputs often come in the form of *underspecified representations*, and just how much of the underspecification we should try to resolve is tricky. Whether or not we should resolve only what would support *monotonic semantic interpretation* ([AC92]), it is natural to leave some underspecification, suggesting transitions

$$(\vec{e}, \vec{\varphi})_\alpha \rightsquigarrow (\vec{\varphi}, \vec{\psi})_\beta \tag{7}$$

that feed outputs $\vec{\varphi}$ of α as inputs to β (to resolve some underspecification left over after α). While (7) drops the label e on $\stackrel{e}{\rightsquigarrow}$ in (4), it also changes $\vec{e}$, making it no more obvious to factor than (4). We shall see in section 4.

3 $\mathcal{L}$-models and merged translations

Consider a family of $\mathcal{L}_\circ$-models $(Stage_\alpha, \vdash_\alpha)_{\alpha \in I}$ indexed by some set I. To allow the notion of logical form to vary with α, we will *not* assume that there is a single set Φ of logical forms on which every entailment relation $\vdash_\alpha$ is defined. However, let us assume (as in (a$_\circ$) in §1.1) that all logical forms belong to E,[4] and combine the different $\mathcal{L}_\circ$-stages in a set

$$\bigcup_{\alpha \in I} \{(\vec{e}, \vec{\varphi})_\alpha : (\vec{e}, \vec{\varphi}) \in Stage_\alpha\} \;\subseteq\; \bigcup_{n \geq 0} (E^n \times E^n \times I)$$

where we write $(\vec{e}, \vec{\varphi})_\alpha$ instead of $(\vec{e}, \vec{\varphi}, \alpha)$. So, without mentioning $\mathcal{L}_\circ$, we can define an $\mathcal{L}$-*model* to be a pair $(STAGE, \{\vdash_\alpha\}_{\alpha \in I})$ such that

$$STAGE \;\subseteq\; \bigcup_{n \geq 0} (E^n \times E^n \times I) \quad \text{and for each } \alpha \in I,\;\; \vdash_\alpha \,\subseteq\, E^* \times E\ .$$

Having obliterated in $\mathcal{L}$ the distinction (in $\mathcal{L}_\circ$) between E and Φ, we will, nevertheless, continue to write $(\vec{e}, \vec{\varphi})$ instead of $(\vec{e}, \vec{e'})$, and sometimes φ instead of e. For $\mathcal{L}$ to extend $\mathcal{L}_\circ$ conservatively, let us agree that

$$\begin{array}{rcl} (\vec{e}, \vec{\varphi})_\alpha \models \varphi & \text{iff} & \vec{\varphi} \vdash_\alpha \varphi \\ (\vec{e}, \vec{\varphi})_\alpha \models \langle e \rangle A & \text{iff} & (\vec{e}e, \vec{\varphi}\varphi)_\alpha \models A \text{ for some } \varphi \\ (\vec{e}, \vec{\varphi})_\alpha \models \neg A & \text{iff} & \text{not } (\vec{e}, \vec{\varphi})_\alpha \models A \\ (\vec{e}, \vec{\varphi})_\alpha \models A \wedge B & \text{iff} & (\vec{e}, \vec{\varphi})_\alpha \models A \text{ and } (\vec{e}, \vec{\varphi})_\alpha \models B \end{array}$$

for all $(\vec{e}, \vec{\varphi})_\alpha \in STAGE$ and $\varphi, e \in E$. Now, the point in expanding $\mathcal{L}_\circ$ to $\mathcal{L}$ is to allow changes in $\alpha \in I$, the simplest of which is given by

$$(\vec{e}, \vec{\varphi})_\alpha \models \langle - \rangle A \qquad \text{iff} \qquad (\vec{e}, \vec{\varphi})_\beta \models A \text{ for some } \beta$$

Such a modality is, along with $\Diamond$ where

$$(\vec{e}, \vec{\varphi})_\alpha \models \Diamond A \qquad \text{iff} \qquad (\vec{e}, \vec{\psi})_\alpha \models A \text{ for some } \vec{\psi}\ ,$$

familiar from multi-dimensional modal logic, with accessibility relations $\xrightarrow{-}$, $\xrightarrow{\Diamond}$ $\subseteq$ $STAGE \times STAGE$ that restrict changes to the index and translations respectively

$$\begin{array}{rcl} (\vec{e}, \vec{\varphi})_\alpha \xrightarrow{-} (\vec{e'}, \vec{\varphi'})_{\alpha'} & \text{iff} & \vec{e} = \vec{e'} \text{ and } \vec{\varphi} = \vec{\varphi'} \\ (\vec{e}, \vec{\varphi})_\alpha \xrightarrow{\Diamond} (\vec{e'}, \vec{\varphi'})_{\alpha'} & \text{iff} & \vec{e} = \vec{e'} \text{ and } \alpha = \alpha'\ . \end{array}$$

[4] Imposing (a$_\circ$) for $\Phi = E$ would, as previously noted, trivialize translation. A more interesting adaptation of (a$_\circ$) to $\mathcal{L}$ would for each $\alpha \in I$, use some subset of E for Φ.

3.1 Factoring via merged translations

To factor an $\mathcal{L}$-transition of the form (4) through the relations $\xrightarrow{e}$, $\xrightarrow{-}$ and $\xrightarrow{\Diamond}$, let us consider the assumption

(a_+) for every pair $\alpha, \beta \in I$, there is an index $\gamma \in I$ such that

$$\text{prefix-closure}(\mathit{Stage}_\alpha \cup \mathit{Stage}_\beta) \quad \subseteq \quad \mathit{Stage}_\gamma$$

where prefix-closure(S) is (by definition)

$$\{(\vec{e}, \vec{\varphi}) \quad : \quad (\vec{e}\vec{e'}, \vec{\varphi}\vec{\varphi'}) \in S \text{ for some } \vec{e'}, \vec{\varphi'} \text{ of the same length}\} \ .$$

Note that (a_+) holds vacuously if there is just one index in I, the stage set of which is prefix-closed (a natural assumption for $\mathcal{L}_\circ$). We get more than one index in I once we differentiate in $\mathcal{L}$ between different contextual assumptions α and β. From here, it is a small step to forming disjunctions of these assumptions — suggesting that I be closed under a binary function $+$ such that for all $\alpha, \beta \in I$, prefix-closure($\mathit{Stage}_\alpha \cup \mathit{Stage}_\beta$) $= \mathit{Stage}_{\alpha+\beta}$. In fact, ($a_+$) is weaker than this requirement, as (a_+) can also be met in one fell swoop by a prefix-closed "monster" index $\star$ that is maximally underspecified/uncommitted/ignorant/indeterminate in that $\mathit{Stage}_\star = \bigcup_{\alpha \in I} \mathit{Stage}_\alpha$. At any rate, ($a_+$) allows us to factor $(\vec{e}, \vec{\varphi})_\alpha \overset{e}{\rightsquigarrow} (\vec{e}e, \vec{\psi}\psi)_\beta$, as

$$(\vec{e}, \vec{\varphi})_\alpha \ \xrightarrow{-} \ (\vec{e}, \vec{\varphi})_\gamma \ \xrightarrow{\Diamond} \ (\vec{e}, \vec{\psi})_\gamma \ \xrightarrow{e} \ (\vec{e}e, \vec{\psi}\psi)_\gamma \ \xrightarrow{-} \ (\vec{e}e, \vec{\psi}\psi)_\beta \ ,$$

with an index γ mediating the transformation of α into β.

More generally, given $e_1 \cdots e_n \in E^*$, let $\langle\!\langle e_1 \cdots e_n \rangle\!\rangle$ be the regular language consisting of the string $\langle e_1 \rangle \cdots \langle e_n \rangle$ and all other strings obtained from it by inserting any number of occurrences of $\langle - \rangle$ or $\Diamond$. That is,

$$\langle\!\langle e_1 \cdots e_n \rangle\!\rangle \quad = \quad L \cdot \langle e_1 \rangle \cdot L \cdots L \cdot \langle e_n \rangle \cdot L$$

where L is $\{\langle - \rangle, \Diamond\}^*$. The closure condition (a_+) on I validates the $\mathcal{L}$-scheme

(+1) $\quad \sigma A \supset \langle - \rangle \Diamond \langle e_1 \rangle \langle e_2 \rangle \cdots \langle e_n \rangle \langle - \rangle A \quad$ for $\sigma \in \langle\!\langle e_1 \cdots e_n \rangle\!\rangle$, $A \in \mathcal{L}$

and, if, moreover, Stage_α is prefix-closed for every $\alpha \in I$,

(+2) $\quad \sigma A \supset \langle - \rangle \Diamond \langle - \rangle \langle e_1 \rangle \langle e_2 \rangle \cdots \langle e_n \rangle A \quad$ for $\sigma \in \langle\!\langle e_1 \cdots e_n \rangle\!\rangle$, $A \in \mathcal{L}$.

Factoring $(\vec{e}, \vec{\varphi})_\alpha \overset{e}{\rightsquigarrow} (\vec{e}e, \vec{\psi}\psi)_\beta$ as

$$(\vec{e}, \vec{\varphi})_\alpha \ \xrightarrow{e} \ (\vec{e}e, \vec{\varphi}\varphi)_\alpha \ \xrightarrow{-} \ (\vec{e}e, \vec{\varphi}\varphi)_\gamma \ \cdots \ (\vec{e}e, \vec{\psi}\psi)_\beta$$

might be blocked by presupposition failure of e at $(\vec{e}, \vec{\varphi})_\alpha$

3.2 Further axioms

But if the presupposition $\langle e \rangle \top$ of e holds, $\stackrel{e}{\to}$ commutes with $\stackrel{\Diamond}{\to}$ and with $\stackrel{-}{\to}$

$$
\begin{array}{lll}
(e\Diamond) & \langle e \rangle \top \ \supset \ (\langle e \rangle \Diamond A \ \equiv \ \Diamond \langle e \rangle A) \\
(e-) & \langle e \rangle \top \ \supset \ (\langle e \rangle \langle - \rangle A \ \equiv \ \langle - \rangle \langle e \rangle A)
\end{array}
$$

provided, for every α, Stage_α is prefix-closed. The closure condition (a_+) yields

$$
(\mathcal{E}-) \qquad \langle - \rangle A \wedge \langle - \rangle B \ \supset \ \langle - \rangle (A \wedge \Diamond B) \qquad \text{for } A \text{ and } B \in \mathcal{E}
$$

where the set $\mathcal{E}$ of "existential" $\mathcal{L}$-formulas A is generated by

$$
A \quad ::= \quad \varphi \ | \ \neg\varphi \ | \ \langle e \rangle A \ | \ \langle - \rangle A \ | \ \Diamond A \ | \ A \wedge A \ | \ A \vee A
$$

(recalling that $A \vee B$ abbreviates $\neg(\neg A \wedge \neg B)$). Beyond (+1), (+2), $(e\Diamond)$, $(e-)$ and $(\mathcal{E}-)$, we have the S5 axioms for $\langle - \rangle$ and for $\Diamond$. But we lose symmetry for combinations $\langle - \rangle \Diamond$ etc because $\stackrel{-}{\to}$ may fail to commute with $\stackrel{\Diamond}{\to}$.

4 Processing layers of representations

To factor the transition $(\vec{e}, \vec{\varphi})_\alpha \rightsquigarrow (\vec{\varphi}, \vec{\psi})_\beta$, let us cheat a bit, and rewrite it as

$$
(\vec{e}, \vec{\varphi})_\alpha \quad \stackrel{;}{\to} \quad (\vec{e}, \vec{\psi})_{\alpha;\beta}
$$

where $\mathit{Stage}_{\alpha;\beta} \ = \ \{(\vec{e}, \vec{\psi}) \ : \ (\exists \vec{\varphi}) \ (\vec{e}, \vec{\varphi}) \in \mathit{Stage}_\alpha \ \& \ (\vec{\varphi}, \vec{\psi}) \in \mathit{Stage}_\beta\}$. By passing to $\stackrel{;}{\to}$, we preserve not only the inputs $\vec{e}$, but also the intuition that $\vec{\varphi}$ is the underspecified expression obtained from translating $\vec{e}$ using α, while $\vec{\psi}$ is its refinement from β. Now, we can factor $(\vec{e}, \vec{\varphi})_\alpha \stackrel{;}{\to} (\vec{e}, \vec{\psi})_{\alpha;\beta}$ as

$$
(\vec{e}, \vec{\varphi})_\alpha \ \stackrel{-}{\to} \ (\vec{e}, \vec{\varphi})_\gamma \ \stackrel{\Diamond}{\to} \ (\vec{e}, \vec{\psi})_\gamma \ \stackrel{-}{\to} \ (\vec{e}, \vec{\psi})_{\alpha;\beta} \ ,
$$

appealing to the closure condition (a_+) on I for a γ such that

$$
\text{prefix-closure}(\mathit{Stage}_\alpha \cup \mathit{Stage}_{\alpha;\beta}) \quad \subseteq \quad \mathit{Stage}_\gamma \ .
$$

4.1 Monotonic interpretation (adding translation to entailment)

The index $\alpha;\beta$ need *not* be introduced above through a global closure condition on I taking ; to be a total binary function on I. Instead, we might, as in category theory, conceive of ; as a partial operation that is defined only when certain conditions are met (matching domains and co-domains, etc).[5] With this in mind, define $\stackrel{;}{\to} \subseteq \mathit{STAGE} \times \mathit{STAGE}$ by

$$
\begin{array}{lll}
(\vec{e}, \vec{\varphi})_\alpha \ \stackrel{;}{\to} \ (\vec{e'}, \vec{\varphi'})_{\alpha'} & \text{iff} & \vec{e} = \vec{e'} \text{ and for some } \beta \text{ s.t. } (\alpha, \beta) \in \mathit{dom}(;), \\
& & \alpha' = \alpha;\beta \text{ and } (\vec{\varphi}, \vec{\varphi'})_\beta \in \mathit{STAGE} \ ,
\end{array}
$$

the modal operator for which let us denote $\langle ; \rangle$. Beyond the $\mathcal{L}$-scheme

[5]The same conditions presumably for $(\vec{e}, \vec{\varphi})_\alpha \quad (\vec{\varphi}, \vec{\psi})_\beta$.

$$(;-\Diamond) \qquad \langle ; \rangle A \;\supset\; \langle - \rangle \Diamond \langle - \rangle A \qquad \text{for } A \in \mathcal{L}$$

(recording our factorization of $\stackrel{;}{\rightarrow}$ above), we might also stipulate

$$(;\text{mon}) \qquad \varphi \;\supset\; [;]\varphi \qquad \text{for } \varphi \in E$$

(where $[;]$ is $\neg\langle ; \rangle\neg$) which unwinds to the constraint that for all $(\alpha, \beta) \in dom(;)$,

$$\text{whenever } (\vec{e}, \vec{\varphi})_\alpha \stackrel{;}{\rightarrow} (\vec{\varphi}, \vec{\psi})_\beta \text{ and } \vec{\varphi} \vdash_\alpha \varphi, \;\; \vec{\psi} \vdash_\beta \varphi.$$

Thus, to interpret underspecified representations of α monotonically (i.e. in accordance with (;mon)), it suffices, assuming $Stage_\beta$ is prefix-closed for every $\beta \in I$, that

$$\varphi_1 \cdots \varphi_n \vdash_\alpha \varphi \quad \text{iff} \quad (\epsilon, \epsilon)_\alpha \mid\mid\!\!- [;][\varphi_1] \cdots [\varphi_n]\varphi \tag{8}$$

for all $\varphi_1, \ldots, \varphi_n, \varphi \in E$. Given definitions of $\vdash_\beta$ for every β such that $(\alpha, \beta) \in dom(;)$, (8) can serve as a recipe for cooking up $\vdash_\alpha$.

4.2 Conclusion

In closing, let us turn to the obvious question: what about an axiomatization of $\mathcal{L}$, extending say that in §1.1? Short of establishing a completeness theorem (which I believe is within reach), I hope to have outlined what $\mathcal{L}$ ought to look like for such a theorem to be interesting. A distinctive feature of $\mathcal{L}$ is its representationalism, eschewing reductions of expressions in E to sets of possible worlds. In this connection, one further construct might be considered for $\mathcal{L}$ — viz. atomic $\mathcal{L}$-formulas of the form $\underline{\varphi}$, for $\varphi \in E$, marking explicit (as opposed to merely entailed) information inasmuch as

$$(\vec{e}, \vec{\varphi})_\alpha \mid\mid\!\!- \underline{\varphi} \quad \text{iff} \quad \varphi \text{ is listed among } \vec{\varphi}\;.$$

Alternatively, $\underline{\varphi}$ might be reconceptualized as $\underline{\Diamond}\varphi$ for some modal operator $\underline{\Diamond}$ expressing a transition from α to an index whose notion of entailment is look-up.

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