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RESEARCH ARTICLE

Situations as indices and as denotations

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Abstract A distinction is drawn between situations as indices required for semantically evaluating sentences and situations as denotations resulting from such evaluation. For atomic sentences, possible worlds may serve as indices, and events as denotations. The distinction is extended beyond atomic sentences according to *formulae-as-types* and applied to implicit quantifier domain restrictions, intensionality and conditionals.

Keywords Situations · Possible worlds · Events · Types · Intensionality

1 Introduction

There are two ways situations provide an alternative to the classical view of a proposition as a set of possible worlds. These ways are most readily understood if, given some collection W of possible worlds, we reformulate a proposition $p \subseteq W$ as its *characteristic function* $\chi_p : W \rightarrow \{0, 1\}$ mapping a possible world $w \in W$ to a truth value $\chi_p(w) \in \{0, 1\}$ indicating whether or not w belongs to p

$$\chi_p(w) = \begin{cases} 1 & \text{if } w \in p \\ 0 & \text{otherwise.} \end{cases}$$

The function χ_p is an instance of an intension (in the sense of Carnap and Montague) that maps an index to an extension. The classical view of a proposition can then be

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revised by introducing situations into the function's domain, as indices, or into the function's range, as extensions or denotations.

As crisp as the distinction between the domain and range of a function is, the line between indices and denotations gets blurred when functions are loosened to relations, as in the *relation theory of meaning* adopted in Barwise and Perry (1983). The precise details of the relation theory of meaning have changed over the years (Perry 1997; Ginzburg 2008), but a key ingredient that has remained is the *described situation*. Described situations are presented in Barwise and Perry (1983) as an alternative to truth values, identified (since Frege) as the denotations of sentences (which for the moment we conflate with statements). Described situations can also serve as indices insofar as linguistic expressions may be evaluated relative to them. It is this use of described situations that informs Kratzer's "possibilistic versions of situation semantics" by which she means

> conservative extensions of possible worlds semantics that construe propositions as sets of world parts, rather than complete possible worlds (Kratzer 2008).

The aforementioned world parts are situations, relative to which the truth of a sentence is evaluated according to a binary relation support between situations s and sentences A

$$\mathsf{support}(s, A) \quad \text{iff} \quad A \text{ is true at } s.$$

The difference with the Carnap-Montague intension $i_A : W \rightarrow \{0, 1\}$ of a sentence A is simply that the possible worlds mapped by i_A to truth values are replaced by situations. While the indices change, the denotations remain the same—namely, truth values. Kratzer uses a partial order $\leq_p$ on situations to sharpen the relation support to a relation exemplify that can be equated with Davidsonian event predication in "those special cases where the sentences that are related to exemplifying situations are atomic."[1] Interestingly, linguistic expressions analyzed in terms of event predication (such as verbs) are often described in the literature as event-denoting. Event-denoting linguistic expressions take us back to the idea of situations-as-denotations, provided we count events as special kinds of situations.

The present work employs situations as indices and denotations alike, mapping a sentence A to a set $[\![A]\!]_i$ of denotations, given a suitable list $i = i_1, \ldots, i_n$ of indices i_k. Exactly what "suitable" is depends on the sentence A, as will become clear below. The details to follow flesh out the intuition that $[\![A]\!]_i$ relativizes exemplification of A to i, with

$$s \ i\text{-exemplifies} \ A \quad \text{iff} \quad s \in [\![A]\!]_i$$

and truth at i underpinned by i-exemplification in that A is defined to be true at i precisely if some situation i-exemplifies A

[1] In Schubert (2000), the converses of the relations support and exemplify are called *describe* and *characterize*, respectively. More in Sect. 4 below.

$$A \text{ is true at } i \quad \text{iff} \quad [\![A]\!]_i \neq \emptyset.$$

As denotations, situations exemplify; as indices, situations support, by virtue of their role as subscripts in $[\![A]\!]_i$.

1.1 Outline

Denotations are drawn in Sect. 2 from events to more complicated objects constructed according to the *formulae-as-types* interpretation from proof theory (Sundholm 1986; Troelstra and Schwichtenberg 2000). Rather than basing exemplification, as in possibilistic situation semantics, on dubious assumptions about the partial order $\leq_p$ on situations, we construct pairs and functions as denotations. The result is arguably what Dekker (2004) (among others) advocates, "an extended dynamic approach" that integrates the cases from Lewis (1975) with eventualities (referred to below as events, for simplicity).[2] In Sect. 3, we shift our focus from situations-as-denotations to situations-as-indices, incorporating intensionality and possibilities into formulae-as-types, while connecting truth to the partial order $\leq_p$. In Sect. 4, an approach to conditionals is outlined that brings together situations-as-indices with situations-as-denotations, evaluating antecedent and consequent relative to a common index that links their denotations. We conclude in Sect. 5 with some words on situation types and on denotations beyond truth.

1.2 An example from possibilistic situation semantics

Before proceeding to the details behind $[\![A]\!]_i$, let us pause to consider example (1a) from Kratzer (2008), a conditional with antecedent (1b) and consequent (1c).

(1) a. Whenever a man rides a donkey, the man gives a treat to the donkey.
    b. a man rides a donkey
    c. the man gives a treat to the donkey

Among the challenges posed by (1) are

(q1) how to interpret (in a principled way) the definite descriptions *the man* and *the donkey*

and

(q2) how to connect the antecedent (1b) to the consequent (1c) for a proper reading of *whenever* in (1a).

We answer (q1) in Sect. 2 below, and (q2) in Sect. 4 (taking up issues concerning truth and intensionality in Sect. 3). In the remainder of this section, we review the

[2] We will concentrate on the universal case, $Q = \forall$, of adverbial quantification, *Q if A then B*. In doing so, we put aside well-known complications having to do with weak and strong donkey readings and the proportion problem. An account of these within the proof-theoretic setting described here can be found in Fernando (2009).

analysis of (1) in Kratzer (2008). (The reader not especially interested in possibilistic situation semantics may skip ahead at this point to Sect. 2.)

The set of situations supporting the antecedent (1b) of (1a) is given by (2).

(2) $\lambda s\ \exists x\ \exists y\ [\text{man}(x)(s) \wedge \text{donkey}(y)(s) \wedge (\exists e \leq_p s)\ \text{ride}(y)(x)(e)]$

In (2), $\text{man}(x)$ and $\text{donkey}(y)$ are evaluated relative to a situation s, while $\text{ride}(y)(x)$ is evaluated relative to an event e that is $\leq_p$-part of s. Whereas e is understood to exemplify $\text{ride}(y)(x)$, s exemplifies neither $\text{man}(x)$ nor $\text{donkey}(y)$. But in order to pick out a unique man and a unique donkey for the definite descriptions in (1a)/(1c), we take a $\leq_p$-minimal situation from the set defined in (2). This is the approach to (q1) taken in possibilistic situation semantics. As for (q2), (1a) is an instance of (3a), of which "the standard analysis" is (3b), where the relation exemplify is derived from the relation support through $\leq_p$-minimization, (3c).

(3) a. Whenever A, B
b. $\lambda s\ (\forall s' \leq_p s)\ \mathsf{exemplify}(s', A) \supset (\exists s'' \geq_p s')\mathsf{support}(s'', B(s'))$
c. $\mathsf{exemplify}(x, A)$ iff $\mathsf{support}(x, A) \wedge (\forall y <_p x)\neg\mathsf{support}(y, A)$

(3b) is the set of situations that support (3a), just as (2) is the set of situations that support (1b). In (3b), s represents the described situation for (3a), called the (Austinian) topic situation in Kratzer (2008), that restricts the A-exemplifying situations s' under consideration to be $\leq_p$-part of it. Similarly, in (2), s is the described situation for (1b) that $\leq_p$-bounds e. For the event e to be "a maximal spatiotemporally connected event of riding y by x,"[3] Kratzer puts (3c) aside, restricting the event quantifier in (2) through a "suitable counting criterion" that bans "non-identical overlapping individuals." Other examples from Kratzer (2008) that pose a challenge for defining exemplification as in (3c) are listed in (4).

(4) a. Whenever a cat eats more than one can of Super Supper in a day, it gets sick.
b. Every time I sell between two and five teapots on a single day, I am entitled to a \$5 bonus.
c. Whenever nobody showed up, we canceled the class.

In each sentence in (4), we must be careful about using the described situation of the antecedent to restrict quantification, as minimizing over that situation can be problematic. For an effectively null restriction, Kratzer takes "the actual world as a whole" as the "resource situation" for *Super Supper* in (4a) and for *teapots* in (4b) (appealing, for (4c), to "contextual restrictions" such as "those contributed by the topic-focus articulation and presuppositions"). To evaluate *it* in (4a), however, the antecedent's described situation is retained as a resource situation for *cat* before minimizing according to (3bc).

[3] The intuition (presumably) is that a donkey gets a treat for a whole ride, and not for every subpart of one, even if it is in some sense a ride. More in Sect. 4 below.

Now, the basic thrust of Sect. 2 below is to sidestep complications with $\leq_p$-minimization by proceeding not from support and (3c), but from exemplify, applying (5) to derive support from exemplify (rather than the other way around).

(5) $\mathsf{support}(x, A)$ iff $(\exists z \leq_p x)\mathsf{exemplify}(z, A)$

In fact, the full account below is slightly more complicated than (5), as exemplification is relativized there also to an index i (not to mention persistence complications, discussed in Sect. 3 .2 below). But insofar as we can put the index i in $[\![A]\!]_i$ aside, we can assert (5) alongside the equivalence

$$\mathsf{exemplify}(s, A) \quad \text{iff} \quad s \in [\![A]\!]$$

(allowing the relative priority between support and exemplify to be reversed).

2 Types and denotations

We focus in this section on situations as denotations, initially suppressing the index subscript i in $[\![A]\!]_i$ and writing simply $[\![A]\!]$. We distinguish between the object language of formulas to be interpreted and the meta-language where the interpretation takes place. Our meta-theory is ordinary set theory in classical logic, no different from the Montagovian tradition in formal semantics. We take it for granted that given sets I and J, we can form the set

$$I \times J = \{\langle i,j\rangle \mid i \in I \text{ and } j \in J\}$$

of pairs from I and J, as well as the set

$$I \to J = \{f \mid f \text{ is a function with domain } I \text{ and range } J\}$$

of functions from I to J. We follow the custom of writing $f : I \to J$ instead of $f \in I \to J$, but otherwise reserve : for the object language and $\in$ for the meta-language. See Table 1, which is explained incrementally in this section.

Table 1 Notational conventions

Object language	Meta-language
:, type	$\in$, set
$A, B, \ldots$	$I, J, \ldots$
$A \wedge B$	$I \times J$
$A \supset B$	$I \to J$
$(\exists x : A)B$	$(\sum i \in I)\hat{J}(i)$
$(\forall x : A)B$	$(\prod i \in I)\hat{J}(i)$
$\Gamma \rhd A$	$[\![A]\!]_i$ for $i \in [\![\Gamma]\!]$

2.1 Denotations minus indices

Under the *formulae-as-types* paradigm from proof theory, a formula A is a *type* whose members are its *proofs* so that we may

(i) formalize 'a is a proof of A' in the object language as '$a : A$', and
(ii) interpret 'a is a proof of A' in the meta-language as $a \in [\![A]\!]$

translating the object language expressions "type" and : to "set" and $\in$ respectively, while assuming that $[\![a]\!] = a$ (for simplicity, lest we distinguish $a = [\![\dot{a}]\!]$ from its name $\dot{a}$). The obvious question is what is a proof, in partial reply to which we can say

> a proof of an implication $A \supset B$ is a function mapping proofs of A to proofs of B

and

> a proof of a conjunction $A \wedge B$ is a pair $\langle a, b\rangle$ of a proof a of A and a proof b of B.

That is, in the meta-language, we have (6).

(6) a. $[\![A \supset B]\!] = [\![A]\!] \rightarrow [\![B]\!]$
b. $[\![A \wedge B]\!] = [\![A]\!] \times [\![B]\!]$

The connective $\supset$ provides a first approximation to the English words *if*. A useful refinement is to allow the consequent B to vary with "choices" from the antecedent A. This step leads to an analysis of Geach's donkey sentence (7a) largely along the lines (7b) of *Discourse Representation Theory* (DRT, Kamp and Reyle 1993).

(7) a. If a^x farmer owns a^y donkey, he$_x$ beats it$_y$.

b. $$\boxed{\begin{array}{c} x, y \\ \hline \text{farmer}(x) \\ \text{donkey}(y) \\ \text{own}(x, y) \end{array}} \Rightarrow \boxed{\text{beat}(x, y)}$$

The problem posed by (7a) is that the indefinites *a farmer* and *a donkey* in the antecedent must be available for pronominal reference in the consequent. DRT's solution is to devise a box (8b) for the antecedent (8a) declaring *discourse referents* x and y, over and above the truth conditions farmer(x), donkey (y) and own (x, y).

(8) a. a^x farmer owns a^y donkey

b. $$\boxed{\begin{array}{c} x, y \\ \hline \text{farmer}(x) \\ \text{donkey}(y) \\ \text{own}(x, y) \end{array}}$$

Rather than reviewing the formal details behind (7b) and (8b), we shall construct formulas that produce the same effect (and more). The idea is to extend the interpretations of $A \supset B$ and $A \wedge B$ in (6) to universal and existential quantification $(\forall x : A)B(x)$ and $(\exists x : A)B(x)$, agreeing that

a proof of $(\forall x : A)B(x)$ is a function f mapping a proof a of A to a proof of $B(a)$

and

a proof of $(\exists x : A)B(x)$ is a pair $\langle a, b\rangle$ of a proof a of A and a proof b of $B(a)$.

To capture the dependency of $B(a)$ on a proof a of A, we work with a set-valued function $\hat{J}$ with domain I, and define dependent forms $(\prod i \in I)\hat{J}(i)$ and $(\sum i \in I)\hat{J}(i)$ of $I \to J$ and $I \times J$ (respectively), putting functions that map $i \in I$ to objects in $\hat{J}(i)$ into

$$\left(\prod i \in I\right)\hat{J}(i) = \left\{f : I \to \bigcup_{i \in I} \hat{J}(i) \mid f(i) \in \hat{J}(i) \text{ for each } i \in I\right\}$$

and input/output pairs for such functions into

$$\left(\sum i \in I\right)\hat{J}(i) = \{\langle i, j\rangle \mid i \in I \text{ and } j \in \hat{J}(i)\}.$$

Notice that if $\hat{J}$ is the constant function mapping each $i \in I$ to $\hat{J}(i) = J$, then

$$\left(\prod i \in I\right)\hat{J}(i) = I \to J \quad \text{and} \quad \left(\sum i \in I\right)\hat{J}(i) = I \times J.$$

Accordingly, we say (6) generalizes to (9).

(9) a. $[\![(\forall x : A)B(x)]\!] = (\prod a \in [\![A]\!])[\![B(a)]\!]$
 b. $[\![(\exists x : A)B(x)]\!] = (\sum a \in [\![A]\!])[\![B(a)]\!]$

In (9), $B(a)$ is assumed to be a type for every $a \in [\![A]\!]$. For instance, given a unary predicate R over a type O of objects (or entities e in Montague grammar), we can form $(\exists x : O)R(x)$ to say there is a proof of $R(a)$ for some $a \in O$. It will be convenient to form the subtype

$$R_O = \{x : O | R(x)\}$$

of O, interpreted according to (10), assuming $B(a)$ is a type for every $a \in [\![A]\!]$.

(10) $[\![\{x : A | B(x)\}]\!] = \{a \in [\![A]\!] \mid [\![B(a)]\!] \neq \emptyset\}$

We can now rewrite (8b), the DRT box for (8a), as (11).

(11) $(\exists x : \text{farmer}_O)(\exists y : \text{donkey}_O)\ \text{own}(x, y)$

Table 2 Situations from denotations

Formula	Situation
Atomic formula φ	Event[uality]
$(\exists x : A)B$	Pair
$(\forall x : A)B$	Function

To interpret (11) via (9b), we work from the inside out, forming the type

$$(\exists y : \text{donkey}_O)\ \text{own}(a, y) \quad \text{for each } a \in [\![\text{farmer}_O]\!]$$

so that we can regard $[\![(\exists y : \text{donkey}_O)\ \text{own}(x, y)]\!]$ as a function mapping $a \in [\![\text{farmer}_O]\!]$ to $[\![(\exists y : \text{donkey}_O)\ \text{own}(a, y)]\!]$. The upshot is that (11) is interpreted as the set

$$[\![(11)]\!] = \{\langle a, \langle d, o\rangle\rangle \mid a \in [\![\text{farmer}_O]\!],\ d \in [\![\text{donkey}_O]\!] \text{ and } o \in [\![\text{own}(a, d)]\!]\}$$

of triples, given sets $[\![\text{farmer}_O]\!]$ of farmers, $[\![\text{donkey}_O]\!]$ of donkeys, and $[\![\text{own}(a, d)]\!]$ of $\text{own}(a, d)$-proofs for each $a \in [\![\text{farmer}_O]\!]$ and $d \in [\![\text{donkey}_O]\!]$. Proofs of farmer$(a)$, donkey$(d)$ and own(a, d) can be regarded as events (or states) so that, as in DRT (Kamp and Reyle 1993), we can write '$x : \text{own}(a, d)$' for x is an $\text{own}(a, d)$-event(uality). See Table 2.

Next, the DRT analysis (7b) of the donkey sentence (7a) becomes (12), where l and r are functions mapping pairs $\langle i, j\rangle$ to their left and right components, i and j, respectively

$$l(\langle i, j\rangle) = i \quad \text{and} \quad r(\langle i, j\rangle) = j.$$

(12) $(\forall z : (\exists x : \text{farmer}_O)(\exists y : \text{donkey}_O)\ \text{own}(x, y))\ \text{beat}(l(z), l(r(z)))$

Notice that the formula in (12) has the form $(\forall z : A)B$, with A instantiated to the formula in (11), and B containing terms $l(z)$ and $l(r(z))$ that range over farmers and donkeys, as outlined in (13a–c), where three typing conclusions (to the right of the turnstile $\vdash$) are drawn from the typing assumption

$$z : (\exists x : \text{farmer}_O)(\exists y : \text{donkey}_O)\text{own}(x, y)$$

(to the left of $\vdash$).

(13) a. $z : (\exists x : \text{farmer}_O)(\exists y : \text{donkey}_O)\ \text{own}(x, y) \ \vdash\ l(z) : \text{farmer}_O$
b. $z : (\exists x : \text{farmer}_O)(\exists y : \text{donkey}_O)\ \text{own}(x, y) \ \vdash\ r(z) : (\exists y : \text{donkey}_O)\ \text{own}(x, y)$
c. $z : (\exists x : \text{farmer}_O)(\exists y : \text{donkey}_O)\ \text{own}(x, y) \ \vdash\ l(r(z)) : \text{donkey}_O$

(13a) and (13c) say that the terms for *he* and *it* are indeed of type farmer_O and donkey_O, respectively.

We can sidestep the pesky projections l and r, and bring the linear expression (11) closer to the DRT box (8b) if, as advocated in Cooper (2005), we instead use *record types*, transforming (11) into (14a), and (12) into (14b).

(14) a. $$\begin{bmatrix} x & : & \text{farmer}_O \\ y & : & \text{donkey}_O \\ e & : & \text{own}(x, y) \end{bmatrix}$$

b. $$\left(\forall z : \begin{bmatrix} x & : & \text{farmer}_O \\ y & : & \text{donkey}_O \\ e & : & \text{own}(x, y) \end{bmatrix}\right) [\, e' \quad : \quad \text{beat}(z.x, z.y)\,]$$

The variables x, y, e and e' in (14) are called *labels*, and $[\![(14a)]\!]$ is the set

$$\left\{ \begin{bmatrix} x & = & a \\ y & = & d \\ e & = & o \end{bmatrix} \mid a \in [\![\text{farmer}_O]\!],\ d \in [\![\text{donkey}_O]\!] \text{ and } o \in [\![\text{own}(a, d)]\!] \right\}.$$

A record type is essentially a context such as that occurring to the left of $\vdash$ in (13a–c) assigning variables types. More generally, contexts are generated simultaneously with pairs (Γ, A) of contexts Γ and formulas A, written

$$\Gamma \rhd A$$

and pronounced 'A is a type relative to Γ,' such that

(c1) the empty sequence is a context (assigning *no* variable a type), and
(c2) whenever $\Gamma \rhd A$ and x is a variable *not* already assigned a type in Γ, the concatenation of Γ with the typing $x : A$, written

$$\Gamma,\ x : A$$

is a context (assigning x the type A).

In other words, a context is a finite sequence of variable typings

$$x_1 : A_1,\ x_2 : A_2, \ldots, x_n : A_n$$

where for each $i \leq n$, x_i is a variable different from x_j for $1 \leq j < i$, and

$$x_1 : A_1,\ x_2 : A_2, \ldots, x_{i-1} : A_{i-1} \rhd A_i.$$

Among the rules for $\rhd$ are (15abc).

(15) a. $\Gamma \rhd (\forall x : A)B(x)$ if $\Gamma, x : A \rhd B(x)$
b. $\Gamma \rhd (\exists x : A)B(x)$ if $\Gamma, x : A \rhd B(x)$
c. $\Gamma \rhd \{x : A | B(x)\}$ if $\Gamma, x : A \rhd B(x)$

The condition

$$\Gamma, x : A \rhd B(x)$$

in (15abc) expresses the assumption in (9) and (10) that $B(a)$ is a type for every $a \in [\![A]\!]$. If we read

$$\Gamma \rhd A \quad \text{as} \quad \text{‘}\Gamma \text{ satisfies the presuppositions of } A\text{’}$$

then (15ab) can explain why neither (16a) nor (16b) presupposes *John is married* even though *John's wife is lucky* does (Beaver 1997; Ranta 1994; Fernando 2001).

(16) a. If John is married, John's wife is lucky.
b. John is married and John's wife is lucky.

2.2 Denotations as indices and contexts as types

From the incremental build-up (c2) of contexts and lines such as (13) of the form $\Gamma \vdash t : A$, it is clear that although : is binary, there can be any number of background assumptions behind a typing statement $t : A$. These assumptions constitute an index i for the set $[\![A]\!] = [\![A]\!]_i$ interpreting A. The index is implicit in the interpretation (9) of $(\forall x : A)B$ and $(\exists x : A)B$, which we make explicit in (17) through subscripts.

(9) a. $[\![(\forall x : A)B(x)]\!] = (\prod a \in [\![A]\!])[\![B(a)]\!]$
b. $[\![(\exists x : A)B(x)]\!] = (\sum a \in [\![A]\!])[\![B(a)]\!]$

(17) a. $[\![(\forall x : A)B]\!]_i = (\prod a \in [\![A]\!]_i)[\![B]\!]_{i,x/a}$
b. $[\![(\exists x : A)B]\!]_i = (\sum a \in [\![A]\!]_i)[\![B]\!]_{i,x/a}$

In (17), the denotation a in $[\![A]\!]_i$ appears as an index in $[\![B]\!]_{i,x/a}$ although not in $[\![(\forall x : A)B]\!]_i$ or $[\![(\exists x : A)B]\!]_i$. The substitution $B(a)$ in (9) is expressed through the subscript x/a on $[\![B]\!]_{i,x/a}$ in (17), paralleling the expansion in (15) of the context Γ (for $(\forall x : A)B(x)$ and $(\exists x : A)B(x)$) to $\Gamma, x : A$ (for $B(x)$). Indeed, a context Γ can be viewed as a type of indices, interpreted as a set $[\![\Gamma]\!]$ such that

(i) for Γ equal to the empty sequence $\emptyset$, $[\![\Gamma]\!] = \{\emptyset\}$

and whenever A is a type relative to Γ, $\Gamma \rhd A$,

(ii) we can interpret A, relative to every $i \in [\![\Gamma]\!]$, as a set $[\![A]\!]_i$, and

Table 3 Interpreting contexts and types

Index i	A type	Denotation a	Γ-novel x	Expanded context
$i \in [\![\Gamma]\!]$	$\Gamma \rhd A$	$a \in [\![A]\!]_i$	$x \notin \mathrm{Var}(\Gamma)$	'$i, x/a$' $\in [\![\Gamma, x : A]\!]$

(iii) $[\![\Gamma, x : A]\!]$ is the set consisting of elements i of $[\![\Gamma]\!]$ extended to map x to an element of $[\![A]\!]_i$

$$[\![\Gamma, x : A]\!] = \{\text{`}i, x/a\text{'} \mid i \in [\![\Gamma]\!] \text{ and } a \in [\![A]\!]_i\}$$

(Table 3). A context Γ does not require an index for interpretation, but a formula A that is a type relative only to a non-empty context does. A variable x in a formula A translating an English sentence may represent a dependence not only on anaphora but also on speaker, addressee, speech time, speech location or some other deictic element.

As the formulation (9) of (17) suggests, information in indices can be moved into formulas. Consider (18).

(18) a. I_x am_y speaking.
b. Pat is_y speaking.
c. $[\![(18\mathrm{a})]\!]_{x/\mathrm{Pat},y/\mathrm{now}} = [\![(18\mathrm{b})]\!]_{y/\mathrm{now}}$

In (18c), we identify (18a) and (18b) with their respective translations as formulas (for simplicity). The equation in (18c) fails to explain an important difference between (18a) and (18b)—(18a) is indisputable in a way that (18b) is not (e.g. [Kaplan 1989]). But then (18c) leaves out any account of the constraints on utterances of (18a)—namely, that *I* is the speaker. To correct this defect, we might structure the indices into situations such as utterances (in accordance with Barwise and Perry's relation theory of meaning), imposing suitable requirements on these situations for a proper context. We shall not do this here, but shall consider in the next section what it means for a formula to be true.

3 Indices and intensionality

In possible worlds semantics, the truth of a proposition p is evaluated relative to a possible world w.

(pw) p is true at w iff $w \in p$

Under formulae-as-types, truth is nothing more than the existence of a proof.

(at) A is true iff there is a proof of A—i.e. $[\![A]\!] \neq \emptyset$
Compared to the formulation (pw) of truth in possible worlds, it would appear (at) describes a notion of truth that is *absolute*, leaving out any qualification or notion of

Table 4 Interpretations of formulas

Formula A	Interpretation $[\![A]\!]_i$
Atomic formula φ	??
$(\exists x : A)B$	$(\sum a \in [\![A]\!]_i)[\![B]\!]_{i,x/a}$
$(\forall x : A)B$	$(\prod a \in [\![A]\!]_i)[\![B]\!]_{i,x/a}$
$\{x : A \mid B\}$	$\{a \in [\![A]\!]_i \mid [\![B]\!]_{i,x/a} \neq \emptyset\}$

possibility. In fact, however, as we observed in Sect. 2.2, a formula A is, in general, interpreted relative to a list $i = i_1, \ldots, i_n$ of indices, turning (at) to (rt).

(rt) $\quad A$ is true at i iff $[\![A]\!]_i \neq \emptyset$

The interpretation $[\![A]\!]_i$ of a formula A is fixed according to Table 4 once the interpretations of its atomic subformulas are fixed. But how are the interpretations $[\![\varphi]\!]_i$ of atomic formulas φ fixed? Formulae-as-types is silent on this question, and it is far from clear how to choose one interpretation of say, farmer(a) over another. If $[\![\cdot]\!]_i$ cannot be determined from i alone, we ought perhaps rewrite (rt) to (Rt).

(Rt) $\quad A$ is true at $[\![\cdot]\!], i$ iff $[\![A]\!]_i \neq \emptyset$

The choice of $[\![\cdot]\!]$ in (Rt) essentially corresponds to a possible world w in (pw), fixing, for instance, the interpretations of farmer(a), donkey(d) and of own(a, d). To formalize variations in the interpretation of A, it suffices to expand the n-ary predicates in A to $(n+1)$-ary predicates with a fresh possible world argument x_0, turning, for instance, own(a, d) into own(a, d, x_0), and interpreting own(a, d, x_0) relative to $x_0/[\![\cdot]\!]$ as $[\![$own$(a, d)]\!]$. The general idea (familiar from Montague) is to "intensionalize" A into a formula $A'(x_0)$ where a novel variable x_0 occurs freely. In the present framework, we require that whenever $[\![A]\!]_i$ is a set (i.e., whenever $i \in [\![\Gamma]\!]$ for some context Γ such that $\Gamma \rhd A$), $A'(x_0)$ is interpreted relative to $x_0/[\![\cdot]\!], i$ as $[\![A]\!]_i$, yielding (RT).

(RT) $\quad A'(x_0)$ is true at $x_0/[\![\cdot]\!], i$ iff $[\![A]\!]_i \neq \emptyset$

The pair $[\![\cdot]\!], i$ in the left hand side of (Rt) becomes the list $x_0/[\![\cdot]\!], i$ of indices for $A'(x_0)$ in (RT). Furthermore, the interpretation of $A'(x_0)$ is fixed by the list $x_0/[\![\cdot]\!], i$ of indices. The problem above of applying (rt) to A = farmer(a) (leading to (Rt)) goes away when farmer(a) is expanded to farmer(a, x_0) and $[\![\cdot]\!]$ is smuggled in through $x_0/[\![\cdot]\!]$.

In the present section, we will consider different choices of $[\![\cdot]\!]$ that can be made through a set bSitn of *basic situations* to be described shortly. For each basic situation $s \in$ bSitn, the instantiation of x_0 given by s will be written $[\![\cdot]\!]_s$, clashing with the notation above for $[\![\cdot]\!]_i$. But as the only index that will concern us in the remainder of this paper is the instantiation of x_0, we will tolerate the clash, and will (what's more) suppress the variable x_0 and write, for instance $[\![\text{farmer}(a)]\!]_s$, leaving *no* trace of x_0 within or underneath $[\![\cdot]\!]$.

3.1 Atomic formulas and partially ordered situations

Let Φ be a set of atomic formulas φ with *no* variables (all arguments being instantiated, as in the case of farmer(a) and own(a, d), so that apart from the instantiation of x_0 via s, φ can be interpreted without i, reducing the subscript on $[\![\cdot]\!]$ to s). Let us work with the intuition that a denotation in $[\![\varphi]\!]_s$ is a *way* φ *is true that is part of* s, so that the equivalence (19a) comes down to (19b).

(19) a. φ is true at s iff $[\![\varphi]\!]_s \neq \emptyset$
b. φ is true at s iff there is a way φ is true that is part of s

Suppose that we could abstract s away from $[\![\varphi]\!]_s$ to pick out a subset $\|\varphi\|$ of bSitn consisting of *ways* φ *is true*, and that *part-of* is given by a partial order $\leq_p$ on bSitn.

(20) a. The set $\|\varphi\|$ of ways φ is true is a subset of bSitn.
b. $\leq_p$ partially orders bSitn and $a \leq_p s$ says: a is part of s.

Then it is plausible to equate $[\![\varphi]\!]_s$ with the set of situations in $\|\varphi\|$ that are $\leq_p$-contained in s, whence it follows from (19a) that φ is true at s exactly if there is way φ is true $\leq_p$-contained in s.

(21) a. $[\![\varphi]\!]_s = \{a \in \|\varphi\| \mid a \leq_p s\}$
b. φ is true at s iff $(\exists a \in \|\varphi\|)\ a \leq_p s$

But how are we to meet the assumptions (20ab) on which (21) rests? A simple answer that will suffice for our present purposes[4] is to let

(i) $\mathsf{bSitn} \subseteq \text{Power}(\Phi)$ consist of subsets of Φ
(ii) $\{\varphi\}$ be the only way φ is true

$$\|\varphi\| = \{\{\varphi\}\}$$

(iii) $\leq_p$ be the subset relation $\subseteq$

so that under (21b),

$$s = \{\varphi \in \Phi \mid \varphi \text{ is true at } s\}$$

for every $s \in \mathsf{bSitn}$. To formulate the notion of a Φ-world, it is useful also to build negation into Φ, and require basic situations to respect it, leading respectively to

[4] A more elaborate account, in the spirit of what Parsons (1990) calls *subatomic semantics*, is explored in Fernando (2009).

(22) and (23). (The formulas in Φ may, if the reader prefers, be called basic, rather than atomic.)

(22) Every $\varphi \in \Phi$ has a negative form $\overline{\varphi} \in \Phi$ different from φ ($\overline{\varphi} \neq \varphi$), whose negative form $\overline{\overline{\varphi}}$ is φ ($\overline{\overline{\varphi}} = \varphi$).

(23) For every $s \in \mathsf{bSitn}$, whenever $\varphi \in s$, $\overline{\varphi} \notin s$.

We can arrange (22) by doubling predicate symbols, if necessary, expressing negative extensions via the additional predicates. Complementing the Φ-non-contradictoriness (23) imposes on every basic situation, we define $s \in \mathsf{bSitn}$ to be a Φ-*world* if s is Φ-complete inasmuch as for all $\varphi \in \Phi$,

$$\varphi \in s \quad \text{or} \quad \overline{\varphi} \in s.$$

An immediate consequence of (23) is that Φ-worlds are $\subseteq$-maximal elements of bSitn and that for every Φ-world s,

$$\overline{\varphi} \in s \quad \text{iff} \quad \varphi \notin s$$

for all $\varphi \in \Phi$. Without a constraint such as (23), we run the risk that bSitn has exactly one $\subseteq$-maximal element, Φ.

We can extend negation $\overline{\cdot}$ beyond Φ, defining

$$\overline{\left(\sum x : A\right)B} = \left(\prod x : A\right)\overline{B}$$

$$\overline{\left(\prod x : A\right)B} = \left(\sum x : A\right)\overline{B}$$

$$\overline{\{x : A|B\}} = \{x : A|\overline{B}\}$$

so that $\overline{A} \neq A = \overline{\overline{A}}$. The question arises: can we apply our definition (21a) of $[\![\varphi]\!]_s$ to possibly non-atomic formulas A, putting

$$\|A\| = \bigcup_{s \in \mathsf{bSitn}} [\![A]\!]_s$$

and extending the domain bSitn of $\leq_p$ to include pairs and functions, so as to assert (24)?

(24) $[\![A]\!]_s \; = \; \{a \in \|A\| \mid a \leq_p s\}$

We take up a basic obstacle to the generalization (24) next.

3.2 Non-persistence and index-denotation pairs

An immediate consequence of (24) and the transitivity of $\leq_p$ is that

$$s \leq_p s' \quad \text{implies} \quad [\![A]\!]_s \subseteq [\![A]\!]_{s'}$$

from which it follows that the truth of A is *persistent*:

$$s \leq_p s' \text{ and } [\![A]\!]_s \neq \emptyset \quad \text{implies} \quad [\![A]\!]_{s'} \neq \emptyset.$$

As is well-known, however, persistence is problematic for universal formulas A. Consider (25).

(25) Every mistake was corrected.
$(\forall x : \text{mistake})\ \text{corrected}(x)$

While (25) might be true at a situation s, it would fail if we extend s to include a mistake that was *not* corrected.

Although existential quantification does not (in itself) pose problems for persistence, we should think twice about asserting (24) for existential formulas, given that Φ is assumed closed under $\dot{\neg}$-negation and that $\overline{(\exists x : A)B}$ is universal.

In view of the untenability of (24) for arbitrary formulas A, a more useful set to associate with A than $\|A\| = \bigcup_{s \in \mathsf{bSitn}} [\![A]\!]_s$ is the set

$$|A| = \left(\sum s \in \mathsf{bSitn}\right) [\![A]\!]_s$$

of pairs $\langle s, a \rangle$ of $s \in \mathsf{bSitn}$ and $a \in [\![A]\!]_s$. (That is, a denotation a is coupled with the index s inducing the set to which a belongs.) Given a set $I \subseteq \mathsf{bSitn}$ of basic situations, we might then define

$$\begin{array}{lll} A \text{ is } I\text{-possible} & \text{iff} & (\exists i \in I)\ A \text{ is true at } i \\ & \text{iff} & (\exists p \in |A|)\ l(p) \in I \end{array}$$

where l, recall, extracts the left component s of a pair $\langle s, a \rangle$.

4 Conditionals via denotations and indices

The points about conditionals of interest in the present section are most conveniently made by moving from the donkey sentence (7a) to example (26) from Kratzer (2008).

(7) a. If a^x farmer owns a^y donkey, he$_x$ beats it$_y$.

(26) Whenever a^x man rides a^y donkey, the$_x$ man gives a treat to the$_y$ donkey.

(26) illustrates the usefulness in associating events with predicates. These events are written e and e' in the treatments (27abc) of (26) paralleling the analyses (7b), (12) and (14b) of (7a) from Sect. 2.

(27)

a. $$\begin{array}{|c|}\hline x, y, e \\ \hline \text{man}(x) \\ \text{donkey}(y) \\ e : \text{ride}(x, y) \\ \hline \end{array} \Rightarrow \begin{array}{|c|}\hline e' \\ \hline e' : \text{giveTreat}(x, y) \\ \hline \end{array}$$

b. $$(\forall z : (\exists x : \text{man}_o)(\exists \text{y} : \text{donkey}_o)\text{ride}(x, y))$$
$$\text{giveTreat}(l(z), l(r(z)))$$

c. $$\left(\forall z : \begin{bmatrix} x & : & \text{man}_o \\ y & : & \text{donkey}_o \\ e & : & \text{ride}(x, y) \end{bmatrix}\right)[e' : \text{giveTreat } (z, x, z, y)]$$

In Kratzer (2008), (26) is analyzed by identifying the set of situations supporting (28a) with (28b).

(28) a. Whenever A, B
b. $\lambda s\ (\forall z \leq_p s)\ \mathsf{exemplify}(z, A) \supset (\exists v \geq_p z)\mathsf{support}(v, B(z))$

(28b) restricts the A-exemplifying situation z to be $\leq_p$-contained in s, and requires the $B(z)$-supporting situation v to $\leq_p$-contain z. Assuming that

$$\begin{array}{lll} \mathsf{support}(x, X) & \text{iff} & X \text{ is true at } x \\ \mathsf{exemplify}(x, X) & \text{iff} & x \in \|X\| \quad \text{for atomic } X \end{array}$$

so that

$$\mathsf{support}(x, X) \quad \text{iff} \quad (\exists u \leq_p x)\ u \in \|X\| \quad \text{for atomic } X$$

(not unlike (5) from Sect. 1.2), we can reduce (28b) to (29) provided A and $B(z)$ are atomic.

(29) $\lambda s\ (\forall z \leq_p s)\ z \in \|A\| \ \supset\ (\exists v \geq_p s)(\exists u \leq_p v)\ u \in \|B(z)\|$

For atomic A and $B(z)$, the sole difference between (29) and the set (30) of situations s at which $(\forall z : A)B(z)$ is true is that (30) simplifies v in (29) to s.

(30) $\lambda s\ (\forall z \leq_p s)\ z \in \|A\| \ \supset\ (\exists u \leq_p s)\ u \in \|B(z)\|$

To defend this simplification, we consider modifications to the antecedent A and to the consequent $B(z)$ in turn.

4.1 Spelling out implicit restrictions

To ensure that both the A-exemplifying situation z and the $B(z)$-exemplifying situation u in (30) are $\leq_p$-bound by the index s in $[\![(\forall z : A)B(z)]\!]_s$, we can express whatever restriction s makes on z over and beyond that on u explicitly in A. In the case of (26), this may involve adding to A a condition $\chi(x, y, e)$ specifying, for instance, the spatio-temporal location of x, y and e.

(26) Whenever a^x man rides a^y donkey, the_x man gives a treat to the_y donkey.

(31)

a. $$\boxed{\begin{array}{c} x, y, e \\ \hline \text{man}(x) \\ \text{donkey}(y) \\ e : \text{ride}(x, y) \\ \chi(x, y, e) \end{array}} \Rightarrow \boxed{\begin{array}{c} e' \\ \hline e' : \text{giveTreat}(x, y) \end{array}}$$

b. $$(\forall z : (\exists x : \text{man}_o)(\exists y : \text{donkey}_o)(\exists e : \text{ride}(x, y)) \\ \chi(x, y, e))\text{giveTreat}(l(z), l(r(z)))$$

c. $$\left(\forall z : \begin{bmatrix} x & : & \text{man}_o \\ y & : & \text{donkey}_o \\ e & : & \text{ride}(x, y) \\ l & : & \chi(x, y, e) \end{bmatrix}\right) [e' : \text{giveTreat}\ (z, x, z, y)]$$

The spatio-temporal location of the antecedent event may differ from that of the consequent—as illustrated by the elaborations (32) of (26), for which I thank an anonymous referee.

(32)
a. Whenever a^x man rides a^y donkey in the desert, the_x man gives a treat to the_y donkey.
b. Whenever a^x man rides a^y donkey in the desert, the_x man gives a treat to the_y donkey before entering the desert.
c. Whenever a^x man rides a^y donkey in the desert, the_x man gives a treat to the_y donkey after returning from the desert.

The reader resistant to partializing worlds into situations might note that (31a) is *not* interpreted in Kampe and Reyle (1993) relative to a situation. That is, one may assume the index s in $[\![(\forall z : A)B(z)]\!]_s$ is a full world, given that restrictions such as $\chi(x, y, e)$ above might be added to the antecedent A. Such an assumption is, however, not necessary under the present framework. Nor is it clear to me what closure conditions on indices s can safely be taken for granted.[5] What is clear is that restrictions on conditionals are routinely left implicit, but that uncovering these restrictions is often (if not always) the key to understanding the claims at stake.

4.2 Linking the antecedent to the consequent

If the antecedent A of a conditional $(\forall z : A)B(z)$ might be extended to step from (27) to (31), the consequent $B(z)$ too might be extended to explicitly connect the situations exemplifying the antecedent and the consequent. Hence, the step from (31) to (33), for some connection $R(e, e')$ implicit in (26).

(33)

a. $$\begin{array}{|c|}\hline x, y, e \\ \hline \text{man}(x) \\ \text{donkey}(y) \\ e : \text{ride}(x, y) \\ \chi(x, y, e) \\ \hline \end{array} \Rightarrow \begin{array}{|c|}\hline e' \\ \hline e' : \text{giveTreat}(x, y) \\ R(e, e') \\ \hline \end{array}$$

b. $$(\forall z : (\exists x : \text{man}_o)(\exists y : \text{donkey}_o)(\exists e : \text{ride}(x, y)) \chi(x, y, e))(\exists e' : \text{giveTreat}(l(z), l(r(z))))R(e, e')$$

c. $$\left(\forall z : \begin{bmatrix} x & : & \text{man}_o \\ y & : & \text{donkey}_o \\ e & : & \text{ride}(x, y) \\ l & : & \chi(x, y, e) \end{bmatrix}\right) \begin{bmatrix} e' & : & \text{giveTreat } (z, x, z, y) \\ c & : & R(e, e') \end{bmatrix}$$

A problem with (31c) that (33c) is intended to fix is that under (31c), a single giveTreat$(z.x, z.y)$-event e' may be associated with two different ride$(z.x, z.y)$-events e_1 and e_2. This is fine if e_1 is $\leq_p$-part of e_2, but not if e_1 and e_2 are $\leq_p$-part of two distinct $\leq_p$-maximal ride$(z.x, z.y)$-events. One approach then is to ensure that the events in question are (as Kratzer 2008 puts it) maximal spatiotemporally connected. In particular, we might revise the basic clause defining $[\![A]\!]_s$, replacing $[\![\varphi]\!]_s$ for atomic φ by the set $[\![\varphi]\!]_s^\sharp$ of $\leq_p$-maximal elements of $[\![\varphi]\!]_s$

$$[\![\varphi]\!]_s^\sharp = \{a \in [\![\varphi]\!]_s \mid \neg(\exists x \in [\![\varphi]\!]_s)\ a <_p x\}$$

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[5] Understood as an index for evaluation, the described situation is notoriously difficult to pin down. Its slippery nature lies at the heart of the treatment of the Liar paradox in Barwise and Etchemendy (1987).

> The main claim of this book is that the explicit introduction of the parameter for the described situation allows us to see quite clearly why the Liar behaves in the way it does. We have shown that if it is used to make a claim about some particular portion of the world, it always gives you a fact that lies outside the portion being described. (Postscript to the second printing of Barwise and Etchemendy 1987)

It would appear the Liar may provide a counter-example to the assumption above that the index s through which x_0 is interpreted contains all the denotations it indexes.

(assuming every element of $[\![\varphi]\!]_s$ has a $\leq_p$-maximal extension). Working with non-overlapping ride$(z.x, z.y)$-events, we might then require R to be 1-1 in that

$$\text{whenever } R(a,b) \text{ and } R(a',b), \quad a = a'.$$

Or, rather than simply requiring R to be 1-1, we might flesh it out, reading $R(a,b)$ as 'b is a consequence of a.' But whatever "consequence" means exactly, note that the temporal relation between the antecedent and consequent situations can vary—a point about *when* familiar from Moens and Steedman (1988). In (32b) and (34a), the A-events happen after the B-events, whereas in (32c) and (34b), they happen before.

(34) a. Whenever John won, Mary predicted it.
b. Whenever John won, Mary reported it.

When saying that A-events happen after or before B-events, we have in mind a particular pairing of A and B-events. In particular, (34a) and (34b) can both be true, which we can report as (35a).

(35) a. Whenever John won, Mary predicted and reported it.
b. Whenever John won, Mary predicted or reported it.

(35a) shows that the consequent B need not consist of a single event (understood as having an interval for its temporal trace), while (35b) suggests that we had better not fix a particular temporal ordering between A- and B-events.

Nevertheless, some connection R between A- and B-events is required to keep (26) from being true simply because the man gave a treat to the donkey some time in the past.

(36) a. Whenever a man rides a donkey, the man gives a treat to the donkey for the ride.
b. Whenever a man rides a donkey, the man gives a treat to the donkey after the ride.
c. Whenever a man rides a donkey, the man gives a treat to the donkey before the ride.
d. Whenever a man rides a donkey, the man gives a treat to the donkey within two days of the ride.

(36a) links a ride to a treat more tightly than does (36b) or (36c); a treat can lie in the future (or past) of several rides, as the past (future) of a treat stretches indefinitely far back (forward). Specifying a temporal measure as in (36d) helps, but our tendency to equate (26) with (36a) illustrates the importance of some loosely causal configuration of events, called an *episode* in Moens and Steedman (1988).[6]

[6] It is noteworthy that causation is the motivation in Schubert (2000) for distinguishing exemplify from support (or, in Schubert's terminology, characterize from describe) and developing that distinction beyond atomic sentences.

Although (26) is compatible with both (36b) and (36c), there are examples where a temporal relationship between A and B can be inferred from an episode. A *push-causes-fall* episode is the basis in Asher and Lascarides (2003) for assigning (37b) the push-and-then-fall interpretation given to (37a).

(37) a. Mary pushed Max. He fell.
b. Max fell. Mary pushed him.

We can flesh out a relation R left implicit in *Whenever A, B* along lines similar to that in the calculation of rhetorical relations (even though (38a) and (38b) are by no means equivalent in the way (37a) and (37b) are).

(38) a. Whenever Mary pushed Max, he fell.
b. Whenever Max fell, Mary pushed him.

Such calculations can be avoided in cases where a connection between A and B is spelled out, as in (39).

(39) a. Whenever Mary pushed Max, he fell as a result.
b. Whenever Max fell, it was because Mary pushed him.

But just as a sequence of unrelated sentences lacks coherence (e.g. Hobbs 1979), a conditional with a consequent that is completely disconnected from its antecedent is arguably ill-formed. Stepping from $(\forall z : A)\ B(z)$ to $(\forall z : A)(\exists v : B(z))\ R(z, v)$, we require that a situation z in the denotation $[\![A]\!]$ map to a situation v in the denotation $[\![B(z)]\!]$ relative to an index supporting the connection $R(z, v)$.[7]

5 Conclusion

It is not uncommon for a semantic account based on possible worlds to eschew the formal use of events (e.g. Dowty 1979) or for an analysis employing events to make do without mentioning possible worlds (e.g. Kamp and Reyle 1993). This may, in part, be due to the differing aims of the accounts. Above, we have brought together possible worlds and events, as situations (broadly construed), presenting possible worlds as indices and events primarily as denotations, without restricting indices to possible worlds or denotations to events. Indeed, the type-theoretic approach to anaphora reviewed in Sect. 2 largely concerns denotations given by pairs and

[7] Is a link between antecedent and consequent situations required because the word *whenever* suggests repeated occurrences? So-called *anankastic conditionals* Sæbø (2001) link antecedent and consequent situations in a particular way without overt use of *whenever*. Fleshing out that connection is arguably crucial to their interpretation. As an anankastic conditional, (40a) is to be read as (40b).

(40) a. If you want to go to Harlem, you must take the A-train.
b. If you want to go to Harlem *then if you go to Harlem*, you must, *as part of going to Harlem*, take the A-train.

functions (rather than events, understood as denotations of atomic formulas), and forms indices from denotations to update contexts. The crucial point behind the distinction between denotations and indices is that denotations *exemplify* what a sentence is *about*, whereas indices *support* the *truth* of sentences. The structural complexity of denotations reflects the complexity of the sentences they exemplify; for atomic formulas, denotations are indices and their $\leq_p$-parts (Sect. 3), while for universal and existential formulas, denotations are functions and pairs (Sect. 2). This fine structure on denotations is useful not only for anaphora (Sect. 2) but also for connecting the consequent of a conditional to its antecedent (Sect. 4). More generally, denotations may serve as *main eventualities* underlying rhetorical relations for discourse coherence (Asher and Lascarides 2003).

Can we make do with the truth sets

$$\begin{aligned} \text{tr(A)} &= \{s \in \mathsf{bSitn} \mid A \text{ is true at } s\} \\ &= \{s \in \mathsf{bSitn} \mid [\![A]\!]_s \neq \emptyset\} \end{aligned}$$

rather than the full set $(\sum s \in \mathsf{bSitn})[\![A]\!]_s$ of index-denotations pairs? For atomic sentences φ, we can recover $[\![\varphi]\!]_s$ from $\text{tr}(\varphi)$

$$[\![\varphi]\!]_s = \{a \in \text{tr}(\varphi) \mid a \leq_p s\}$$

given the partial order $\leq_p$ on bSitn. As noted in Sect. 3.2, however, complications with persistence block such a reduction for universal sentences A. Are there other grounds for doubting denotations can be reduced to truth sets? In an influential, early critique of Barwise and Perry (1997), Soames points out that

> if direct reference is possible and propositional attitude verbs have a relational semantics, then the semantic values of sentences (objects of propositional attitudes) cannot be collections of truth-supporting circumstances

(whether these circumstances be possible worlds or situations)

> but rather are single, composite entities with structures related to those of the sentences that express them. They are, I suggest, essentially Russellian propositions.

(Soames 1985, p. 63). Under the present proposal, the semantic values are neither collections of truth-supporting circumstances nor the Russellian propositions Soames suggests. They are instead collections of exemplifying situations.[8] Collections rather than "single, composite entities" are arguably more faithful to the spirit of the Austinian view that a sentence is mapped to a type (or collection) of situations by the language's descriptive conventions. It is a separate, contentious matter whether or not demonstrative conventions always single out a described situation (Ginzburg and Sag 2000, p. 93). Types containing more (or less) than one situation are compatible with the claim that a unique event need not be described by a statement,

[8] Indices are not mentioned here because "whenever I speak of *the semantic value of a sentence*, this should be understood as short for *the semantic value of a sentence relative to a context*" (Soames 1985, p. 67).

for instance, of *Amundsen flew to the North Pole in May 1926* (Davidson 1967, p. 91), not to mention generics such as *birds fly*.

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References

Asher, N., & Lascarides, A. (2003). *Logics of conversation*. Cambridge: Cambridge University Press.
Barwise, J., & Etchemendy, J. (1987). *The Liar: An essay on truth and circularity*. Oxford: Oxford University Press.
Barwise, J., & Perry, J. (1983). *Situations and attitudes*. Cambridge: MIT Press.
Beaver, D. I. (1997). Presupposition. In van J. Benthem & ter A. Meulen (Eds.), *Handbook of logic and language* (pp. 939–1008). Amsterdam: Elsevier.
Cooper, R. (2005). Austinian truth, attitudes and type theory. *Research on Language and Computation, 3*(4), 333–362.
Davidson, D. (1967). The logical form of action sentences. In N. Rescher (Ed.), *The logic of decision and action* (pp. 81–95). Pittsburgh: University of Pittsburgh Press.
Dekker, P. (2004). Cases, adverbs, situations and events. In H. Kamp & B. Partee (Eds.), *Context dependence in the analysis of linguistic meaning* (pp. 383–404). Amsterdam: Elsevier.
Dowty, D. R. (1979). *Word meaning and montague grammar*. Dordrecht: Reidel.
Fernando, T. (2001). Conservative generalized quantifiers and presupposition. In *Proceeding of the semantics and linguistic theory XI* (pp. 172–191). New York: Cornell University.
Fernando, T. (2009). Situations in LTL as strings. *Information and Computation*, doi:10.1016/j.ic. 2008.11.003 (in press, available online at ScienceDirect).
Ginzburg, J. (2008). Situation semantics: From indexicality to metacommunicative interaction. Draft chapter for *Handbook of Semantics*.
Ginzburg, J., & Sag, I. (2000). *Interrogative investigations: The form, meaning and use of english interrogatives*. CSLI, Stanford.
Hobbs, J. R. (1979). Coherence and coreference. *Cognitive Science, 3*(1), 67–90.
Kamp, H., & Reyle, U. (1993). *From discourse to logic*. Dordrecht: Kluwer Academic Publishers.
Kaplan, D. (1989). Demonstratives: An essay on the semantics, logic, metaphysics and epistemology of demonstratives and other indexicals. In J. Almog, J. Perry, & H. Wettstein (Eds.), *Themes from Kaplan* (pp. 481–614). Oxford: Oxford University Press.
Kratzer, A. (2008). Situations in natural language semantics. In *Stanford encyclopedia of philosophy*. http://plato.stanford.edu/entries/ situations-semantics.
Lewis, D. (1975). Adverbs of quantification. In E. Keenan (Ed.), *Formal semantics of natural language* (pp. 3–15). Cambridge: Cambridge University Press.
Moens, M., & Steedman, M. (1988). Temporal ontology and temporal reference. *Computational Linguistics, 14*(2), 15–28.
Parsons, T. (1990). *Events in the semantics of English*. Cambridge: MIT Press.
Perry, J. (1997). Situation semantics. In *Routledge encyclopedia of philosophy*. London: Routledge.
Ranta, A. (1994). *Type-theoretical grammar*. Oxford: Oxford University Press.
Sæbø, K. J. (2001). Necessary conditions in natural language. In C. Féry & W. Sternefeld (Eds.), *Audiatur Vox Sapientiae: A Festschrift for Arnim von Stechow* (pp. 427–449). Berlin: Akademie Verlag.
Schubert, L. (2000). The situations we talk about. In J. Minker (Ed.), *Logic-based artificial intelligence* (pp. 407–439). Dordrecht: Kluwer Academic Publishers.
Soames, S. (1985). Lost innocence. *Linguistics and Philosophy, 8*, 59–71.
Sundholm, G. (1986). Proof theory and meaning. In D. Gabbay & F. Guenthner (Eds.), *Handbook of philosophical logic* (Vol. 3, pp. 471–506). Dordrecht: Reidel.
Troelstra, A. S., & Schwichtenberg, H. (2000). *Basic proof theory* (2nd ed.). Cambridge: Cambridge University Press.