Prior and Temporal Sequences for Natural Language

Tim Fernando

Prior 1967 on logics of temporal sequences:

> *The usefulness of systems of this sort does not depend on any serious metaphysical assumption that time is discrete; they are applicable in limited fields of discourse in which we are concerned with what happens in a sequence of discrete states, e.g. in the workings of a digital computer*

Turing awards: Pnueli (1996), Clarke, Emerson, Sifakis (2007)

Blackburn 2006

> *tense logic has fallen into disuse in natural lang semantics*

Kamp on natural language discourse

> *when we interpret a piece of discourse — or a single sentence in the context in which it is being used — we build something like a* model *of the episode or situation described; and an important part of that model are its event structure, and the time structure that can be derived from that event structure*

> *discourse time ... made up by those comparatively few events that figure in the discourse*

applied to

punctual events described by *Passé Simple*,
contra *durative* events/states by *Imparfait*

Prior's "states" and "fields of discourse" over intervals

$$\underbrace{\text{temporal proposition over an interval } I}$$

A state is described by a fluent φ that is *pointwise*

$$I \models \varphi \text{ iff } (\forall t \in I)\ \{t\} \models \varphi$$

(e.g., Taylor 1977, Dowty 1979).

Associate a field of discourse with a set X of pointwise fluents.

An interval I is *X-homogeneous* if for all $\varphi \in X$,

$$(\exists t \in I)\ \{t\} \models \varphi \quad \text{iff} \quad (\forall t \in I)\ \{t\} \models \varphi.$$

Idea. Segment an interval into X-homogeneous subintervals
for "sequence of discrete states" $\approx$ string of subsets of X.

Segmentations and strings

A *segmentation of I* is a sequence $I_1 \cdots I_n$ of subintervals of I s.t.

$$I = \bigcup_{i=1}^{n} I_i \quad \text{and} \quad I_i \prec I_{i+1} \text{ for } 1 \leq i < n$$

where $\prec$ is *whole* precedence

$$J \prec J' \quad \text{iff} \quad (\forall t \in J)(\forall t' \in J')\ t \prec t'.$$

Fact. Given a segmentation $I_1 \cdots I_n$ of I, tfae

(i) for every $\varphi \in X$ and subinterval J of I,

$$J \models \varphi \quad \text{iff} \quad J \subseteq \bigcup \{I_i \mid I_i \models \varphi\}$$

(ii) each I_i is X-homogeneous

(iii) there is an X-morphism from I onto the string

$$\{\varphi \in X \mid I_1 \models \varphi\} \cdots \{\varphi \in X \mid I_n \models \varphi\}$$

X-segmentable intervals

An interval I is *X-segmentable* if there is a segmentation
$I_1 \cdots I_n$ of I s.t. each I_i is X-homogeneous.

A (φ, n)-*alternation in* I is a string $t_1 \cdots t_n \in I^n$ s.t. for $1 \leq i < n$,

$$t_i \prec t_{i+1} \quad \text{and} \quad \{t_i\} \models \varphi \text{ iff } \{t_{i+1}\} \not\models \varphi.$$

φ is *alternation bounded in* I if there is an integer $n > 0$ s.t.
no (φ, n)-alternation in I exists.

Fact. *For any interval I and finite set X of pointwise fluents,*

I is X-segmentable iff each $\varphi \in X$ is alternation bounded in I.

Back to Prior

limited field of discourse $\approx$ X, I where
X is a finite set of pointwise fluents
I is an interval where each φ in X is alternation bounded

F 2015, The semantics of tense and aspect: a finite-state perspective.
Handbook of Contemporary Semantic Theory, 2nd edn,
S. Lappin & C. Fox, eds., Wiley-Blackwell.

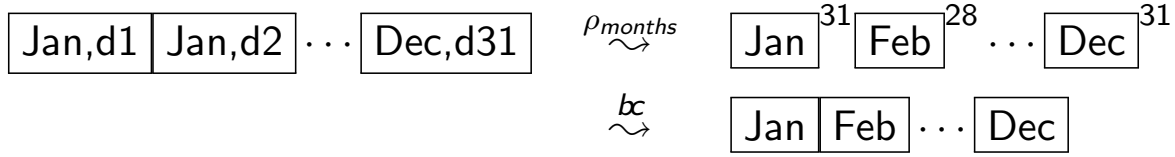
+ decidable entailments (inclusions betwen regular languages)

Egocenteric logic (1968)

> *I find myself quite unable to take 'instants' seriously as individual entities; I cannot* understand *'instants', and the earlier-later relation that is supposed to hold between them, except as logical constructions out of tensed facts. Tense logic is for me, if I may use the phrase,*
> metaphysically fundamental

Strings under $bc_X(s) := bc(\rho_X(s))$

days in a year $\rightsquigarrow$ months in a year

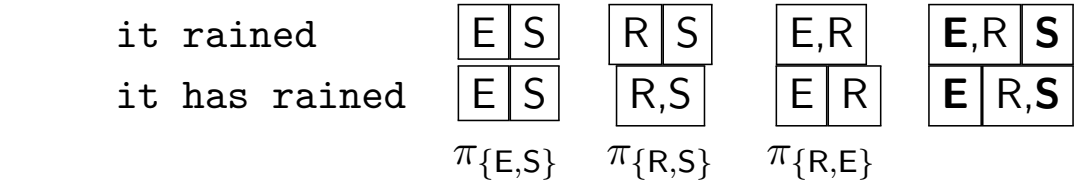


ρ_X sees only X

bc sees time only via change : compress α^+ to α

$\pi_X := bc_X$; *unpad* where *unpad* removes any initial or final $\square$

a is an *interval in* s iff $\pi_{\{a\}}(s) = \boxed{a}$ (e.g. Feb but not d2)



Interval relations + Russell-Wiener-Kamp event structures

RWK	**Allen**	$(2^{\{a,a'\}})^+$
$a \prec a'$	a m a'	$a \mid a'$
	$a < a'$	$a \mid \mid a'$
$a' \prec a$	a mi a'	$a' \mid a$
	$a > a'$	$a' \mid \mid a$

Allen	$(2^{\{a,a'\}})^+$
$a = a'$	a, a'
a s a'	$a, a' \mid a'$
a si a'	$a, a' \mid a$
a f a'	$a' \mid a, a'$
a fi a'	$a \mid a, a'$
a d a'	$a' \mid a, a' \mid a'$
a di a'	$a \mid a, a' \mid a$
a o a'	$a \mid a, a' \mid a'$
a oi a'	$a' \mid a, a' \mid a$

$a \mid \mid a' \quad \rightsquigarrow$

$a, past_{a'}$	$fut_a, past_{a'}$	fut_a, a'

$past_{a'} \rightsquigarrow \neg a' \wedge Fa'$

$fut_a \rightsquigarrow \neg a \wedge Pa$

Fact. For finite A, every RWK-event structure $\langle A, \bigcirc, \prec \rangle$ is representable as a string over the alphabet 2^A.

Instants as "tensed facts"

Egocentric logic (1968)

> *Philosophically the most interesting proposition which is true at a given instant only is the conjunction of all the propositions which are then true, but for formal purposes any proposition true at that instant only will do as its tense-logical "representative."*

$$\boxed{a}\,\Box\,\boxed{a'} \rightsquigarrow \quad a' \wedge \neg a \;\wedge\; \mathbb{Y}(\neg a \wedge \neg a' \;\wedge\; \mathbb{Y}(a \wedge \neg a' \;\wedge\; \neg \mathbb{Y}\top))$$

Instant carries its past
branching any number of ways into the future

$$s \leq_{prefix} s' \quad \text{iff} \quad (\exists \hat{s})\; s\hat{s} = s'$$

Relativize $\mathbb{Y}$ to a set X of fluents

Beyond finite X

For infinite Φ, let $Fin(\Phi)$ be the set of finite subsets of Φ.

The *inverse limit of* $\{bc_X\}_{X \in Fin(\Phi)}$ is the set of functions $f : Fin(\Phi) \to (2^{\Phi})^+$ s.t.

$$(\forall A \in Fin(\Phi))(\forall X \subseteq A) \quad f(X) \;=\; bc_X(f(A)).$$

Tree-like ordering

$$f \prec_{\Phi} f' \quad \text{iff} \quad f \neq f' \text{ and } (\forall X \in Fin(\Phi))\; f(X) \leq_{prefix} f'(X)$$

bc reduces intervals to nominals —

$$a \text{ is an interval in } s \;\text{ iff }\; bc_{\{a\}}(s) \in \Box^*\boxed{a}\Box^*$$

Finite automata and non-determinism

For fixed X, nominals are 1st-order variables in

Monadic 2nd-order Logic (MSO) = finite-state (Büchi 1960)

MSO and model checking, Turing award 2007

Rabin and Scott, Turing award 1976

> *For their joint paper "Finite Automata and Their Decision Problems" (1959) which introduced the idea of nondeterministic machines, which has proved to be an enormously valuable concept.*

Prior's lifelong interest in indeterminism – Kenny 1970, Copeland

Branching time in PPF (1967)

instants individuated by their present and past

$$\boxed{a}\,\boxed{}\,\boxed{a'} \quad \rightsquigarrow \quad a' \wedge \neg a \ \wedge \ \mathbb{Y}(\neg a \wedge \neg a' \ \wedge \ \mathbb{Y}(a \wedge \neg a' \ \wedge \ \neg \mathbb{Y}\top))$$

The index X

Øhrstrøm and Hasle 1993

> *according to Prior events do not "exist" at all; strictly speaking, only things exist. "Events are just what things do and what happens to them", he said. ... Points of time, instants and events seemed as mythical to him as matter did to Berkeley ...*

Blackburn 2006

> *Prior's dominant theme (the importance of the internal perspective) was coupled with what is (by contemporary standards) a narrow view of the role of tense logic in natural language. ... the real division was not tensed versus untensed, but indexical versus non-indexical.*

Temporal sequences/granularity from X (homogeneity)

- X varies with discourse (open-ended)
  down to the "sub-atomic" (T. Parsons)