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Compositionality

INDUCTIVELY, CO-INDUCTIVELY AND CONTEXTUALLY

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The meaning $[\![a]\!]$ of a term/rep/exp a

$$[\![a]\!] \quad = \quad \ldots [\![b]\!] \ldots$$

for b a part of a

$\Rightarrow$ look inside term (break a*part*) [§2]

$\Leftarrow$ look *out*side term (merge) [§3]

What about the context c underlying $[\![\cdot]\!]$?

Inject c into meaning $[\![a]\!]$, and $[\![a]\!]$ into c. [§4]

PLAN OF TALK

§1 Background: congruences and extensions

§2 Subterm extensibility *inductively*

§3 Fregean covers *co-inductively*

§4 Changing the *context*

Slide 1

f-compositional/congruence

Fix an n-ary function $f : T^n \to T$ on a set T.

A function $[\![\cdot]\!] : T \to M$ is *f-compositional* if there is a function $[\![f]\!] : M^n \to M$ such that for all $a_1, \ldots, a_n \in T$,

$$[\![f(a_1, \ldots, a_n)]\!] \quad = \quad [\![f]\!]([\![a_1]\!], \ldots, [\![a_n]\!]) \ .$$

The *kernel* of $[\![\cdot]\!]$ is the binary relation

$$\kappa[\![\cdot]\!] \quad = \quad \{(a,b) \in T \times T \ : \ [\![a]\!] = [\![b]\!]\}$$

on T.

An *f-congruence* is an equivalence relation $\equiv$ on T such that

$$\frac{a_1 \equiv b_1 \quad \cdots \quad a_n \equiv b_n}{f(a_1, \ldots, a_n) \equiv f(b_1, \ldots, b_n)}$$

for all $a_1, \ldots, a_n, b_1, \ldots, b_n \in T$.

Slide 2

From $[\![\cdot]\!]$ to $\equiv$

Given a set $\mathcal{F}$ of multi-ary functions on T, *$\mathcal{F}$-compositional/congruence* = f-compositional/congruence for every $f \in \mathcal{F}$

Fact 1 (a) $\kappa[\![\cdot]\!]$ is an equivalence reln on T.

(b) $[\![\cdot]\!]$ is $\mathcal{F}$-compositional iff $\kappa[\![\cdot]\!]$ is an $\mathcal{F}$-congruence.

Given $\equiv \subseteq T \times T$, define $\cdot^{\equiv} : T \to \mathsf{Pow}(T)$ by

$$a^{\equiv} \quad = \quad \{b \in T \ : \ a \equiv b\}$$

for every $a \in T$.

Fact 2. $\kappa[\![\cdot]\!] = \kappa(\cdot^{\kappa[\![\cdot]\!]})$

§§2, 3 make do with $\equiv$ (and $M = \mathsf{Pow}(T)$)

§4 examines meanings in $[\![\cdot]\!]$

Slide 3

Partialization of Slide 2

Fix a partial n-ary map $\alpha : T^n \rightharpoonup T$.

A partial map $[\cdot] : T \rightharpoonup M$ is *α-compositional* if there is a function $[\alpha] : M^n \to M$ such that for all $(a_1, \ldots, a_n) \in \mathsf{dom}([\cdot])^n \cap \mathsf{dom}(\alpha)$ for which $\alpha(a_1, \ldots, a_n) \in \mathsf{dom}[\cdot]$,

$$[\alpha(a_1, \ldots, a_n)] \quad = \quad [\alpha]([a_1], \ldots, [a_n]) \ .$$

Given a subset $X \subseteq T$ and $\vec{a} \in T^n$, let

$$\mathsf{d}^X_\alpha(\vec{a}) \quad \text{iff} \quad \vec{a} \in X^n \cap \mathsf{dom}(\alpha) \text{ and } \alpha(\vec{a}) \in X \ .$$

An *(α, X)-congruence* is an equivalence relation $\equiv$ on X such that

$$\frac{a_1 \equiv b_1 \quad \cdots \quad a_n \equiv b_n}{\alpha(a_1, \ldots, a_n) \equiv \alpha(b_1, \ldots, b_n)} \ \mathsf{d}^X_\alpha(\vec{a}),\ \mathsf{d}^X_\alpha(\vec{b})$$

for all $a_1, \ldots, a_n, b_1, \ldots, b_n \in T$, where $\vec{a} = (a_1, \ldots, a_n)$ and $\vec{b} = (b_1, \ldots, b_n)$.

Slide 4

Extensions (and more)

Given a set Σ of partial multi-ary maps on T, Σ-*compositional*/(Σ, X)-*congruence* =
α-comp/(α, X)-congruence for every $\alpha \in \Sigma$

Fact 3. $[\cdot]$ is Σ-compositional iff $\kappa[\cdot]$ is a $(\Sigma, \mathsf{dom}[\cdot])$-congruence.

Given $X \subseteq Y \subseteq T$, a (Σ, X)-congruence $\equiv$ is Y-*extensible* if there is a (Σ, Y)-congruence $\equiv'$ which restricted to X is $\equiv$ (i.e. $\equiv' \cap (X \times X)$ is $\equiv$). In this case, we say $\equiv$ *extends to* $\equiv'$.

We write Σ-congruence for (Σ, T)-congruence,

d_α for d_α^T,

'$\equiv_1$ *refines* $\equiv_2$' for '$\equiv_1 \subseteq \equiv_2$,'

'finest' for '$\subseteq$-least,' and

'coarsest' for '$\subseteq$-greatest.'

Inductive = provable

Let T be a set of Σ-terms closed under subterms.

Given $\equiv \subseteq T \times T$, let $\equiv_\Sigma$ be the set of all pairs (a, b) such that $a \doteq b$ is derivable from

$$\frac{}{a \doteq b}\ a \equiv b$$

$$(\dagger)\ \frac{a \doteq b \quad b \doteq c}{a \doteq c}$$

$$\frac{a_1 \doteq b_1 \quad \cdots \quad a_n \doteq b_n}{\alpha(a_1, \ldots, a_n) \doteq \alpha(b_1, \ldots, b_n)}\ \mathsf{d}_\alpha(\vec{a}),\ \mathsf{d}_\alpha(\vec{b})$$

where $a, b, \ldots \in T$, and $\alpha \in \Sigma$ (n-ary, $n \geq 0$).

Lemma 4. Given an equivalence relation $\equiv$ on T, $\equiv_\Sigma$ is the finest Σ-congruence refined by $\equiv$.

Theorem 5 (Westerståhl). If X is closed under subterms, then every (Σ, X)-congruence is T-extensible.

Subterm extensibility as (†)-elimination

Lemma 4 provides an obvious candidate for a Σ-congruence extending a (Σ, X)-congruence $\approx$ — namely, $\equiv_\Sigma$ where $\equiv$ is the union

$$\approx\ \cup \{(a, a) : a \in T\}\ .$$

Theorem 5 then says that for all $a, b \in X$,

$$a \approx b \quad \text{iff} \quad a \equiv_\Sigma b$$

provided X is closed under subterms.

In (†), b need not be a subterm of a or c.

To prove that $\equiv_\Sigma$ restricted to X is $\approx$, let

$a \doteq_k b$ iff '$a \doteq b$' can be derived in $\leq k$ steps

and produce a contradiction from a minimal k such that for some $a, b \in X$, $a \not\approx b$ although $a \doteq_k b$ (the last step in which must be (†), applications of which unwind towards absurdum ...).

Co-inductive = satisfying

Given $g : T \to T$ and $R \subseteq T \times T$, let

$$R^g = \{(a, b) \in R\ :\ g(a)\ R\ g(b)\}\ .$$

Fact 6. Suppose $\equiv$ is an equivalence reln on T.

(a) $\equiv$ is a g-congruence iff $\underbrace{\equiv \subseteq \equiv^g}$

'$\equiv$ satisfies g-constraint'

(b) $\equiv \cap \equiv^g \cap (\equiv^g)^g \cap \cdots$ is the coarsest g-congruence refining $\equiv$.

For $f : T^{n+1} \to T$, $1 \leq i \leq n+1$ and $\vec{a} \in T^n$, let

$$f_{i,\vec{a}} : T \to T, \quad f_{i,\vec{a}}(a) = f(\underbrace{(a, \vec{a})_i})$$

$\vec{a}$ with a inserted at the ith position.

Collect f's unary projections in

$$\mathcal{U}(f) \quad = \quad \{f_{i,\vec{a}}\ :\ 1 \leq i \leq n+1 \text{ and } \vec{a} \in T^n\}.$$

Dual of Slide 6

Given $\equiv \subseteq T \times T$ and $\mathcal{F}$, let $\equiv^{\mathcal{F}}$ be $\bigcap_{k\geq 0} \equiv^{\mathcal{F}}_k$ where

$$\equiv^{\mathcal{F}}_0 \quad = \quad \equiv$$
$$\equiv^{\mathcal{F}}_{k+1} \quad = \quad \bigcap_{f\in\mathcal{F}} \bigcap_{g\in\mathcal{U}(f)} (\equiv^{\mathcal{F}}_k)^g .$$

Lemma 7. Given an equivalence relation $\equiv$ on T, $\equiv^{\mathcal{F}}$ is the coarsest $\mathcal{F}$-congruence refining $\equiv$.

Proposition 8. Given an equivalence relation $\approx$ on a subset $X \subseteq T$, every Fregean cover of $\approx$ extends to the $\mathcal{F}$-congruence $\equiv^{\mathcal{F}}$, where $\equiv$ is

$$\approx \cup ((T-X)\times(T-X)) .$$

Definition (Hodges). Given equivalence relns $\approx$ and $\approx'$ on subsets of T, $\approx'$ is a *Fregean cover of* $\approx$ if for $X = \mathsf{dom}(\approx)$,

F(a): if $a \approx' b$ and $t(a|x) \in X$ then $t(b|x) \in X$

F(b): if $a \approx' b$ and $t(a|x), t(b|x) \in X$ then $t(a|x) \approx t(b|x)$

F(c): if $a \not\approx' b$ then $t(a|x) \not\approx t(b|x)$ for some t.

N.B. Proposition 8 says Fregean covers of $\approx$ are restrictions of $(\approx \cup ((T-X)\times(T-X)))^{\mathcal{F}}$.

[$\subseteq$ by F(a), F(b); $\supseteq$ on $\mathsf{dom}(\approx')$ by F(c)]

$\approx$ need *not* extend to a Fregean cover — e.g. $a\approx b$, $f(a) \in X$, $f(b) \notin X$ then $a \not\equiv^{\{f\}} b$ [but $a \equiv_{\{f\}} b$; slide 7] non-*Husserlian* (Hodges)

The co-induction defining $\cdot^{\mathcal{F}}$ sidesteps the partiality of $\Sigma, \mathsf{d}^X_\alpha$, focusing on *refinements*, instead of extensions.

Lemma 7 and $(\approx \cup ((T-X)\times(T-X)))^{\mathcal{F}}$ were applied in

> Ambiguity under changing contexts, *Linguistics & Philo* 20(6):575-606, 1997

with the intuition that $\bigcup_{f\in\mathcal{F}} \mathcal{U}(f)$ provides *contexts* a compositional semantics must respect.

Slide 9

Slide 10

Reflecting on meaning

The meaning $[\![a]\!]$?

A context c is presupposed defining $[\![a]\!] = [\![a]\!]_{\mathsf{c}}$.

E.g. c is a first-order model M

$a_1 =$ *Pat's spouse is lucky*

$a_0 =$ *Pat is married*

$[\![a_1]\!]_M$ is undefined for M where a_0 is false.

Step 1. Build context into meaning $\{\!|\cdot|\!\}$

$$[\![a]\!]_{\mathsf{c}} \simeq m \quad \rightsquigarrow \quad (\mathsf{c}, m) \in \{\!|a|\!\}$$

with (following Karttunen et al)

$$\mathsf{dom}\{\!|a|\!\} \quad = \quad \{\text{contexts satisfying } a\text{'s presuppositions}\} .$$

$a_2 =$ *if* a_0 *then* a_1

a_2 does *not* presuppose a_0 though a_1 does — within a_2, a_1's context differs from a_2's.

Slide 11

Meaning as context update

Step 2 (K, Heim). Meaning as context change potential $(\!|\cdot|\!)$

$$\mathsf{c}_{in}\ (\!|a|\!)\ \mathsf{c}_{out} \quad \text{iff} \quad (\exists \mathsf{c}, m)\ [\![a]\!]_{\mathsf{c}} \simeq m,\ \mathsf{c} \text{ 'in' } \mathsf{c}_{in} \text{ and } \mathsf{c}_{in}, \underbrace{m \text{ 'in' } \mathsf{c}_{out}}$$

builds meaning into context
– Kamp (DRT), Stalnaker, . . .

More examples

(1) *Pat is married, and Pat's spouse is lucky.*

(2) *Every ant bites.*
Every ant is an ant that bites.

(3) *Some ant bites.*
Some ant is an ant that bites.

(4) *Every farmer who owns a donkey beats it.*
Every f-w-o-a-d is a f-w-o-a-d that beats it.

Slide 12

Presupposition linked to conservativity

$$[\![Q(A,B)]\!]_{\mathsf{c}} \quad = \quad [\![Q]\!]_{\mathsf{c}}([\![A]\!]_{\mathsf{c}}, [\![B]\!]_{\mathsf{c}})$$

Classical generalized quantifier theory uses the same c, formulating the conservativity of q as

$$q(\mathsf{A},\mathsf{B}) \text{ iff } q(\mathsf{A},\mathsf{A}\cap\mathsf{B})$$

(2) $\mathsf{ant} \subseteq \mathsf{bite}$ iff $\mathsf{ant} \subseteq \mathsf{ant} \cap \mathsf{bite}$

(3) $\mathsf{ant} \cap \mathsf{bite} \neq \emptyset$ iff $\mathsf{ant} \cap (\mathsf{ant} \cap \mathsf{bite}) \neq \emptyset$

(4) $[\![\textit{beats it}]\!]_{\mathsf{c}}$?

Interpret *it* relative to *f-w-o-a-d*.

In general, interpret B relative to A

as the set $\{x : \mathsf{A} \mid \mathsf{B}\}$ of A's that B

$$[\![Q(A,B)]\!]_{\mathsf{c}} = [\![Q]\!]_{\mathsf{c}}([\![A]\!]_{\mathsf{c}}, [\![B]\!]_{\mathsf{c},x:[\![A]\!]_{\mathsf{c}}})$$

updating the context evaluating B by '$x : [\![A]\!]_{\mathsf{c}}$'

$$= [\![Q]\!]_{\mathsf{c}}([\![A]\!]_{\mathsf{c}}, [\![B]\!]_{\mathsf{c}})$$

by conservativity, provided $[\![B]\!]_{\mathsf{c}}$ is defined.

Details in e.g. Fernando, SALT 2001

Conclusion

Compositionality can be approached from below (inductively) or above (co-inductively).

The projections $f_{i,\vec{a}}$ from the latter comprise one of the many dimensions of context.

Is the context c in $[\![a]\!]_{\mathsf{c}}$ beyond compositionality?

Not if we allow changes in meaning & context.

Beyond piecing together meanings (for extensions and/or refinements of congruences; §§2, 3), we have (from §4)

$$[\![\cdot]\!] \quad \rightsquigarrow \quad \{\!|\cdot|\!\},\ (\!|\cdot|\!)$$

that reflect changes in meaning and context.

Should we keep context and meaning apart?

Yes, insofar as semantic interpretation involves two distinct tasks

- identifying what is taken for granted
- capturing what is asserted

plus integrating them — compositionally!?

Appendix for Slide 7

Fix (towards a contradiction) a k-length derivation $\mathcal{D}$ of $a \doteq b$ with $a, b \in X$, $a \not\approx b$, and for all $a', b' \in X$ and $k' < k$ s.t. $a' \doteq_{k'} b'$, $a' \approx b'$. $\mathcal{D}$'s last step must be (†) — say, $a \doteq_{k-1} x$ and $x \doteq_{k-1} b$. Working within $\mathcal{D}$ and expanding out uses of (†), we can appeal to the minimality of k to convert the sequence a, x, b to a sequence $t_1 \ldots t_l$ of terms occurring in $\mathcal{D}$ such that $t_1 = a$, $t_l = b$ and for $1 \leq j < l$, $\mathcal{D}$ contains a derivation of $t_j \doteq t_{j+1}$ ending with an α-rule. As $a = t_1$ and $b = t_l$, we get $a = \alpha(a_1 \ldots a_n)$ and $b = \alpha(b_1 \ldots b_n)$ for some $a_1 \ldots a_n$ and $b_1 \ldots b_n$. The contradiction $\alpha(a_1 \ldots a_n) \approx \alpha(b_1 \ldots b_n)$ then follows from k's minimality (again) and the assumption that $\approx$ is a congruence on the subterm-closed set X.

Appendix for Proposition 8

Lemma A. If $a \equiv^{\mathcal{F}} b$ then for all t,

$$t(a|x) \equiv^{\mathcal{F}} t(b|x).$$

Proof. Induct on t.

Lemma B. If $a \not\equiv^{\mathcal{F}}_k b$ then for some t,

$$t(a|x) \not\equiv t(b|x).$$

Proof. Induct on k.

Let $\approx'$ be a Fregean cover of $\approx$, and $Y = \mathsf{dom}(\approx')$.

Claim. For all $a, b \in Y$, if $a \equiv^{\mathcal{F}} b$ then $a \approx' b$.

Proof. Lemma A + F(c).

Claim. $a \approx' b$ implies $a \equiv^{\mathcal{F}} b$.

Proof. F(a), F(b) and Lemma B.