A Finite-state Approach to Events in Natural Language Semantics

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Abstract

Events in natural language semantics are characterized in terms of regular languages, each string in which can be regarded as a temporal sequence of observations. The usual regular constructs (concatenation, etc.) are supplemented with superposition, inducing a useful notion of entailment, distinct from that given by models of predicate logic.

Keywords: Finite-state, events, natural language, superposition.

1 Introduction

Due in no small measure to [3], events of some form or another have become a common tool in natural language semantics for analysing expressions of change [17, 11, 1]. For example, sentence (1) is taken to describe an event of Pat walking a mile, culminating in the past.

(1) Pat walked a mile.

Such events are formulated below as runs of machines that collectively constitute a causal order around which to explain temporality in natural language [20, 16]. Similar ideas have been developed in [14, 13, 21, 15, 19, 9],[1] the distinctive feature of the present proposal being to collect model-theoretically interpretable sequences of observations in regular languages, subject to an associative binary operation $\&$ of 'superposition' that reveals temporal structure. The strings in the languages can be viewed as motion pictures (or comic strips), a logic for which can be constructed around $\&$.

Consider, for instance, (1), a crude predicate logic translation of which is $\exists x(\mathit{walk}(p,x) \wedge \mathit{mile}(x))$, abstracting away speech time (tense). Speech time or no speech time, the event described has a certain temporal structure. The un-inflected (tenseless) phrase 'Pat walk a mile' describes certain changes drawn out by the regular language (5), which is the $\&$-superposition of (2), (3) and (4). [Explanations to follow.]

(2) $\boxed{\sim(\exists x \preceq m)\mathit{walk}(p,x)}\,\boxed{(\exists x \preceq m)\mathit{walk}(p,x)}^{+}$

(3) $\boxed{\sim \mathit{walk}(p,m)}^{+}\ \boxed{\mathit{walk}(p,m)}$

(4) $\boxed{\mathit{mile}(m)}^{+}$

(5) $\boxed{\begin{array}{c}\sim(\exists x \preceq m)\mathit{walk}(p,x)\\ \sim\mathit{walk}(p,m)\quad \mathit{mile}(m)\end{array}}\,\boxed{\begin{array}{c}(\exists x \preceq m)\mathit{walk}(p,x)\\ \sim\mathit{walk}(p,m)\quad \mathit{mile}(m)\end{array}}^{*}\ \boxed{\begin{array}{c}\mathit{walk}(p,m)\\ \mathit{mile}(m)\end{array}}$

[1] Let us mention also the semantic automata of [2]. These accept languages very different from (and far simpler than) those considered here. Permutation invariance is tellingly inappropriate in the present applications — which, as discussed in the concluding paragraph below, concern essentially nonquantificational matters.

A symbol α in the regular languages above is a finite set $\boxed{\varphi \ldots}$ of formulas $\varphi \ldots$, circumscribed by a box rather than the usual curly braces $\{\cdot\}$, to mark the use of a set-as-symbol, as opposed to set-as-language.[2] The formulas inside a box are understood to hold simultaneously, with concatenation $\alpha\beta$ of boxes α and β signifying that all formulas in β hold at a temporal point that immediately succeeds one where all formulas in α hold. Relative to this notion of juxtaposition, we have the usual Kleene iteration $\alpha^+ = \alpha\alpha^*$. Thus, if we write $\Box$ for $\emptyset$-as-symbol (and henceforth we will), then the regular expression $\alpha\Box^*\beta$ signifies that a temporal point described by β comes after one described by α. Leaving the precise definition of superposition $\&$ for section 2 (below), let us ask two questions. Is (5) anything more than an analysis by comics of (1), minus tense? Does the &-factorization (2)&(3)&(4) = (5) reveal any interesting temporal structure? Very briefly, it is claimed that the answer to both questions is 'yes' and that there are many other examples (with and without tense) that can be analysed along similar lines [7, 8].

Staying with (1), the pictures (2)–(5) bear directly on a classification of events initiated by [22] (and refined in various directions since) to account for entailment patterns such as those illustated by (6)-(7).

(6) Pat walked *for* an hour.
Pat was walking $\vdash$ Pat walked

(7) Pat walked a mile *in* an hour.
Pat was walking a mile $\not\vdash$ Pat walked a mile

The tenseless phrase 'Pat walk' describes a Vendlerian *activity* depicted in (8) which, like (2), has no definite terminating condition.

$$(8) \quad \boxed{\sim(\exists x \neq \varnothing)\mathit{walk}(p,x)}\;\boxed{(\exists x \neq \varnothing)\mathit{walk}(p,x)}^+ + \boxed{(\exists x \neq \varnothing)\mathit{walk}(p,x)}^+$$

More precisely, given a formula φ, let us call a language L *φ-telic* if for every nonempty string $s\alpha s' \in L$ formed from strings s and s' and a symbol α,

$$\varphi \in \alpha \quad \text{iff} \quad s' \text{ is the empty string } \epsilon.$$

Clearly, no formula φ exists for which (2) or (8) is φ-telic. By contrast, (3) and (5) are $\mathit{walk}(p,m)$-telic. (7) exemplifies the widely accepted linguistic generalization that such telicity is associated with temporal 'in'-modification, and the so-called imperfective paradox [4]: E-ing need not imply E. On the other hand, if E is an activity such as 'Pat walk' then E-ing *does* imply E, and temporal modification is expressed by 'for' rather than 'in'. These points can, as shown in Sections 2 and 3, be formulated in terms of superposition &, with entailments dropping out from the use of $\&$ in linguistic composition (in a Davidsonian manner).

The logic involved is essentially propositional logic. The details are not complicated, but are worth spelling out to properly appreciate differences with more traditional approaches based on predicate logic or (Priorian) tense logic. To remove a formula φ from a box $\boxed{\varphi \ldots}$ we translate φ, as in tense logic, relative to an evaluation time to produce a formula interpretable in ordinary predicate logic (with time reified). But our regular languages may refer freely to time and temporal structure beyond an ordering, allowing us to capture phrases such as 'an hour' as (9) — or, for that matter, relations R between any finite number n of times, which (10) puts in increasing order for the sake of simplicity.

[2]This notational practice is helpful given that regular expressions (adopted throughout this paper) confuse a string s with the language $\{s\}$. Within a box, let us agree to separate formulas by commas when there is not much space between them.

(9) $\boxed{\mathsf{time}(x)}\,\square^{+}\,\boxed{\mathit{hour}(x,y)\ \ \mathsf{time}(y)}$

(10) $\boxed{\mathsf{time}(x_1)}\,\square^{*}\cdots\square^{*}\,\boxed{\mathsf{time}(x_n)}\,\boxed{R(x_1,\ldots,x_n)}$

Moreover, we can capture some Priorian tense operators such as P, but *not* all — for instance, H, the universal dual of P, even though our regular languages are closed under a notion of negation. Kleene iteration stretches the temporal range of regular languages beyond any fixed number of temporal points, making them somewhat unlike the translations of tense formulas in predicate logic with distinguished evaluation times. All this is explained in Section 4, where relations with first-order logic are taken up. Insofar as transitive closure is not first-order, there is both more and less to our regular languages than first-order logic. It is comforting that entailment between our regular languages should, as detailed in Section 3 below, be so simple — perhaps deceptively so, given how easy it is to overlook the aforementioned differences. Connections with the situation calculus ([12]) are mentioned towards the end of Section 4.

2 Superposition and subsumption

Given a finite set Φ of formulas and languages $L, L' \subseteq \mathit{Pow}(\Phi)^*$, let us define the *superposition* $L\&L'$ *of* L *and* L' to be the language

$$L \,\&\, L' \quad = \quad \bigcup_{k\geq 1} \{(\alpha_1 \cup \alpha_1')\cdots(\alpha_k \cup \alpha_k') \mid \alpha_1\cdots\alpha_k \in L \text{ and } \alpha_1'\cdots\alpha_k' \in L'\}$$

formed from componentwise unions $(\alpha_i \cup \alpha_i')$ of nonempty strings from L and L' of equal length (k).[3]

PROPOSITION 2.1
Let $L, L', L'' \subseteq \mathit{Pow}(\Phi)^*$.

(i) $\&$ is associative and commutative, with $L \,\&\, \square^{+} = L$ if $\epsilon \notin L$.

(ii) If L and L' are regular languages, then so is $L\&L'$.

(iii) If $L \subseteq L'$ and $L \subseteq L''$ then $L \subseteq L'\&L''$.

PROOF. Immediate, except perhaps for (ii). Given finite automata A and A' accepting L and L' respectively, we form a finite automaton accepting $L\&L'$ as follows. Its set of states is the (Cartesian) product $Q \times Q'$ of the state sets of A and A' (respectively). Its initial state is the pair (q_0, q_0') of initial states of A and A'. Its set of final states is the product $F \times F'$ of those of A and A'. And its transition set consists of the transitions

$$(q, q') \overset{\sigma\cup\sigma'}{\longrightarrow} (r, r')$$

for all A-transitions $q \overset{\sigma}{\to} r$ and A'-transitions $q' \overset{\sigma'}{\to'} r'$. (Had we required that $\sigma = \sigma'$, we would have the usual construction for intersection $L \cap L'$.) ∎

[3] The alert reader will notice that line (5) from the introduction is, under this definition of &, not quite equal to (2)&(3)&(4). The formula $(\exists x \preceq m)\mathit{walk}(p,x)$ is missing from the last box in (5). Since it is entailed by the formula $\mathit{walk}(p,m)$ *in* (5)'s last box, it got dropped so as to fit (5) in one line. We can establish the equality by adding the omitted formula or else weaken equality to equivalence up to Φ-entailments, which we consider in the next section.

How can we compare the information content of languages using $\&$? Were we to equate $L \trianglerighteq L'$ with the equality $L = L\&L'$, then $\trianglerighteq$ would not be reflexive. (Consider $L = \boxed{\varphi} + \boxed{\psi}$.) Taking a hint from Proposition 2.1(iii), let us instead define *subsumption* $\trianglerighteq$ by

$$L \trianglerighteq L' \quad \text{iff} \quad L \subseteq L\&L'.$$

Applying $\trianglerighteq$ also to strings s, s', let us agree to write $s \trianglerighteq s'$ for $\{s\} \trianglerighteq \{s'\}$ (as languages). It is easy to prove

PROPOSITION 2.2
Let $L, L', L'' \subseteq Pow(\Phi)^+$ and $\alpha_1, \ldots, \alpha_k, \alpha'_1, \ldots, \alpha'_n \in Pow(\Phi)$.

(i) $\alpha_1 \cdots \alpha_k \trianglerighteq \alpha'_1 \cdots \alpha'_n$ iff $k = n$ and for $1 \leq i \leq k$, $\alpha_i \supseteq \alpha'_i$.
(ii) $L \trianglerighteq L'$ iff $(\forall s \in L)(\exists s' \in L')\ s \trianglerighteq s'$. (That is, $\trianglerighteq$ is a Hoare pre-order.)
(iii) $\trianglerighteq$ is reflexive and transitive, with $\emptyset \trianglerighteq L \trianglerighteq \Box^+$ and $L\&L' \trianglerighteq L$.
(iv) $L \trianglerighteq L'\&L''$ iff $L \trianglerighteq L'$ and $L \trianglerighteq L''$.

Returning to the inclusion $L \subseteq L\&L'$ defining $L \trianglerighteq L'$, note that we have not lost much by weakening $=$ to $\subseteq$. Parts (iii) and (iv) of Proposition 2.2 imply that $L \subseteq L\&L'$ iff $L \equiv L\&L'$, where we write $\equiv$ for the equivalence induced by $\trianglerighteq$

$$L_1 \equiv L_2 \quad \text{iff} \quad L_1 \trianglerighteq L_2 \text{ and } L_2 \trianglerighteq L_1 \ .$$

Parts (i) and (ii) of Proposition 2.2 suggest an alternative approach to (the same relation) $\trianglerighteq$, which is resisted so as to link $\trianglerighteq$ more directly to superposition &.

Next, let us give some form $\Sigma \subseteq Pow(\Phi)$ to the information content measured by $\trianglerighteq$, relativizing $\&$ to Σ according to the definition

$$L \;\&_\Sigma\; L' \quad = \quad (L\&L') \cap \Sigma^+ \ .$$

The intuition is that Σ consists of subsets of Φ that can be observed/realized. Different subfamilies Σ induce different notions of what is observable.

PROPOSITION 2.3
Let $L, L', L'' \subseteq Pow(\Phi)^+$ and $\Sigma \subseteq Pow(\Phi)$.

(i) For all $\Sigma' \subseteq \Sigma$, $(L \;\&_\Sigma\; L') \;\&_{\Sigma'}\; L'' = (L\&L'\&L'') \cap \Sigma'^+$.
(ii) $L \;\&_\Sigma\; L'$ is regular if L and L' are.

PROOF. Easy, with part (ii) a consequence of the closure of regular languages under intersection. ■

Now, let us apply $\trianglerighteq$ and $\&_\Sigma$ to lines (1)–(8) in the introduction, under the assumption that the set Φ of formulas φ is closed under negations $\sim\varphi$ which Σ respects according to (11) and (12).

(11) $(\forall \varphi \in \Phi)\ \boxed{\varphi, \sim\varphi} \notin \Sigma$
(12) $(\forall \sigma \in \Sigma)(\forall \sigma' \subseteq \sigma)\ \sigma' \in \Sigma$

To characterize telicity, let us extract from a language $L \subseteq Pow(\Phi)^*$ its set $L_\omega \subseteq Pow(\Phi)$ of last symbols

$$L_\omega \quad = \quad \{\alpha \subseteq \Phi \mid L \cap Pow(\Phi)^*\alpha \neq \emptyset\}$$

where the regular expression $Pow(\Phi)^*\alpha$ denotes the language of $Pow(\Phi)$-strings ending in α. For example, for L given by line (2), $L_\omega = \boxed{(\exists x \preceq m)walk(p,x)}$, and for L given by (3), $L_\omega = \boxed{walk(p,m)}$. Next, we define the *$\sim$-complement* $\tilde{\Gamma}$ *of* a subfamily $\Gamma \subseteq Pow(\Phi)$ so that $\tilde{\Gamma} = \boxed{\sim\varphi}$ for the special case where $\Gamma = \boxed{\varphi}$. To define $\tilde{\Gamma}$ for an arbitrary $\Gamma \subseteq Pow(\Phi)$, let $\tilde{\emptyset} = \{\Box\}$ and for Γ with $n \geq 1$ distinct elements $\alpha_1 \ldots \alpha_n$,

$$\tilde{\Gamma} \quad = \quad \{\boxed{\sim\varphi_1, \ldots, \sim\varphi_n} \mid \varphi_1 \in \alpha_1, \ldots, \varphi_n \in \alpha_n\}$$

(so that $\tilde{\Gamma} = \emptyset$ if $\Box \in \Gamma$).[4] Now, let us call L *telic* if $L \trianglerighteq \mathsf{telic}(L)$, where by definition,

$$\mathsf{telic}(L) \quad = \quad \tilde{L_\omega}^{+}\,\Box\ .$$

Neither (2) nor (8) is telic, whereas both (3) and (5) are. Let us call L *iterative* if $L \trianglerighteq \mathsf{iter}(L)$, where by definition,

$$\mathsf{iter}(L) \quad = \quad \Box\,({L_\omega}^{+})\ .$$

This time, (2) and (8) are iterative, whereas neither (3) nor (5) is. Next, writing $\mathcal{L}(P)$ for the regular language we associate with the phrase P, let us suppose that 'in'- and 'for'-modification of a phrase E by an interval I are analysed according to

$$\begin{aligned}
\mathcal{L}(E \text{ in } I) \quad &= \quad \mathcal{L}(E)\ \&_\Sigma\ \mathsf{telic}(\mathcal{L}(E))\ \&_\Sigma\ \mathcal{L}(I) \\
\mathcal{L}(E \text{ for } I) \quad &= \quad \mathcal{L}(E)\ \&_\Sigma\ \mathsf{iter}(\mathcal{L}(E))\ \&_\Sigma\ \mathcal{L}(I)
\end{aligned}$$

and that, as in (9), $\mathcal{L}(I) \trianglerighteq \Box\Box^{+}\Box$ (insuring that the interval I has a middle as well as a beginning and end). As for the progressive, let us assume

$$\mathcal{L}(E\text{-ing}) \quad \subseteq \quad \{s \mid \text{length}(s) \geq 2 \text{ and } (\exists s' \neq \epsilon)\ ss' \in \mathcal{L}(E)\}\ .$$

It now follows that if $\mathcal{L}(E)$ is telic then

$$\begin{aligned}
\mathcal{L}(E \text{ in } I) \quad &\equiv \quad \mathcal{L}(E)\ \&_\Sigma\ \mathcal{L}(I) \qquad [\text{recall } \equiv \text{ is } \trianglerighteq \cap \trianglelefteq] \\
\mathcal{L}(E \text{ for } I) \quad &= \quad \emptyset \quad (\text{marking the oddness of `}E \text{ for } I\text{'}) \\
\mathcal{L}(E\text{-ing}) \quad &\ntrianglerighteq \quad \Box^*\mathcal{L}(E)_\omega \quad (\text{signalling the imperfective paradox})
\end{aligned}$$

whereas if $\mathcal{L}(E)$ were iterative then

$$\begin{aligned}
\mathcal{L}(E \text{ in } I) \quad &= \quad \emptyset \quad (\text{marking the oddness of `}E \text{ in } I\text{'}) \\
\mathcal{L}(E \text{ for } I) \quad &\equiv \quad \mathcal{L}(E)\ \&_\Sigma\ \mathcal{L}(I) \\
\mathcal{L}(E\text{-ing}) \quad &\trianglerighteq \quad \Box^*\mathcal{L}(E)_\omega \quad (\text{no imperfective paradox})\ .
\end{aligned}$$

Stepping back from the example above, let us say two languages L and L' are *Σ-incompatible* and write $L \perp_\Sigma L'$ if $L\ \&_\Sigma\ L' = \emptyset$. Note that $\{\alpha\}\perp_\Sigma\{\alpha'\}$ iff $\alpha \cup \alpha' \notin \Sigma$. Turning $\perp_\Sigma$ sideways, we consider $\vdash_\Sigma$ in the next section.

[4] The sense in which $\tilde{\Gamma}$ is a negation is the content of Proposition 3.2 in the next section.

3 Σ-completion, entailment and complements

The purpose of this section is to boost subsumption $\unrhd$ to a notion $\vdash_\Sigma$ of entailment between languages (over the alphabet *Pow*(Φ)) implicit in a notion $\Sigma \subseteq$ *Pow*(Φ) of consistency on Φ. The basic tool is that of a Σ-*completion* L_Σ of a language $L \subseteq$ *Pow*$(\Phi)^*$, formed from the set $\Sigma_\sharp$ of $\subseteq$-maximal elements of Σ

$$\Sigma_\sharp \quad = \quad \{\sigma \in \Sigma \mid (\forall \sigma' \in \Sigma)(\sigma \subseteq \sigma' \text{ implies } \sigma = \sigma')\}$$

by collecting all of L's $\unrhd$-extensions in ${\Sigma_\sharp}^+$; that is,

$$L_\Sigma \quad = \quad \{s \in {\Sigma_\sharp}^+ \mid (\exists s' \in L)\ s \unrhd s'\}\ .$$

Σ-entailment between languages can then be reduced to $\unrhd$

$$L \vdash_\Sigma L' \quad \text{iff} \quad L_\Sigma \unrhd L'\ .$$

To relate $\vdash_\Sigma$ to $\perp_\Sigma$, it is useful (as the next proposition suggests) to form the Σ-*complement of* a language L, defined to be

$${}_\Sigma L \quad = \quad \Sigma_\sharp^+ - L_\Sigma\ .$$

(In writing ${}_\Sigma L$ rather than $\neg_\Sigma L$, I have opted for brevity over the clarity that perhaps favors the latter.)

PROPOSITION 3.1
Let $L, L' \subseteq$ *Pow*$(\Phi)^+$.

(i) The following are equivalent to $L \vdash_\Sigma L'$.
  (a) $L_\Sigma \ \subseteq\ L'_\Sigma$
  (b) $L_\Sigma\ \cap\ {}_\Sigma L' = \emptyset$
  (c) $L \perp_\Sigma {}_\Sigma L'$

(ii) If L is regular then so are L_Σ and ${}_\Sigma L$.

(iii) $L \unrhd L'$ implies $L \vdash_\Sigma L'$.

PROOF. As with Proposition 2.1, the interesting bit is the construction of a finite automaton, this time one accepting L_Σ from one, A, accepting $L - \{\epsilon\}$. The automaton for L_Σ is the same as A, except that A's transitions $\rightarrow$ are modified to

$$q \overset{\alpha}{\Rightarrow} q' \quad \text{iff} \quad \alpha \in \Sigma_\sharp \text{ and for some } \beta \subseteq \alpha,\ q \overset{\beta}{\rightarrow} q'\ .$$

∎

We can expand Proposition 3.1 by any number of standard facts, including ${}_\Sigma{}_\Sigma L = L_\Sigma$. The *double negation* map sending L to L_Σ transports us to Boolean logic, with $\&$ as intersection

$$(L\&L')_\Sigma \quad = \quad L_\Sigma \cap L'_\Sigma\ ,$$

Σ-complements as set complementation, and $L \Rightarrow_\Sigma L'$ as ${}_\Sigma(L\&_\Sigma({}_\Sigma L'))$, yielding the expected deduction theorem over the pre-order $\vdash_\Sigma$. The point of $\&$, $\sim$ and $\unrhd$, however, is to *avoid the costs in forming* Σ*-completions*; hence, the definition above of $\&_\Sigma$ (which does not involve Σ-completions) before $\vdash_\Sigma$ (which does). It is difficult to see how to side-step L_Σ when reducing $L \vdash_\Sigma L'$ to $\trianglerighteq$, in view of examples such as

$$\boxed{\varphi} \quad \vdash_\Sigma \quad \boxed{\varphi, \psi} + \boxed{\varphi, \sim\psi}$$

unless we transform L'. But then what are we to do with

$$\boxed{\varphi} \quad \vdash_\Sigma \quad \boxed{\psi}$$

where $\psi \neq \varphi$?

As for the $\sim$-complement $\tilde{\Gamma}$ of a subfamily $\Gamma \subseteq Pow(\Phi)$, it is useful to supplement lines (11)–(12) from the previous section with (13).

(13) $(\forall \sigma \in \Sigma)(\forall \varphi \in \Phi)\ \sigma \cup \boxed{\varphi} \in \Sigma$ or $\sigma \cup \boxed{\sim\varphi} \in \Sigma$

PROPOSITION 3.2
If Σ satisfies (11)-(13), then for every $\Gamma \subseteq Pow(\Phi)$, $\tilde{\Gamma}_\Sigma \ = \ _\Sigma\Gamma$.

PROOF. A tedious but easy verification of $\subseteq$ and $\supseteq$. ■

4 Model-theoretic interpretation

The present section interprets a language L over the alphabet $Pow(\Phi)$ in models of predicate logic. Let us fix a set Var of variables, a subset $\mathsf{Var}_\tau \subseteq \mathsf{Var}$ of which is designated temporal. Let us associate with Φ a map fvar such that for every $\varphi \in \Phi$, $\mathsf{fvar}(\varphi)$ is a nonrepeating list of variables occurring freely in φ, and let us assume Φ comes with a subset $\Phi_\tau \subseteq \Phi$ of formulas such that for some temporal variables $x, y \in \mathsf{Var}_\tau$,

$$\mathsf{time}(x) \in \Phi_\tau \quad \text{and} \quad succ(x,y),\ \mathcal{O}(x) \in \Phi - \Phi_\tau\ .$$

(The intuition is that the formulas in $\Phi - \Phi_\tau$ can be interpreted without an evaluation time.) We then form a vocabulary/signature $v(\Phi)$ consisting of relation symbols $R[\varphi]$ for $\varphi \in \Phi$, with arity equal to the number $|\mathsf{fvar}(\varphi)|$ of free variables in φ plus, in case $\varphi \in \Phi_\tau$, 1

$$\text{arity}(R[\varphi]) \quad = \quad \begin{cases} |\mathsf{fvar}(\varphi)| + 1 & \text{if } \varphi \in \Phi_\tau \\ |\mathsf{fvar}(\varphi)| & \text{otherwise.} \end{cases}$$

When do two formulas $\varphi, \varphi' \in \Phi$ induce the same relation symbol $R[\varphi] = R[\varphi']$? To answer this question, let us define

$$\begin{aligned} \varphi \approx \varphi' \quad \text{iff} \quad & \varphi' = \varphi[\rho] \text{ for some bjiection } \rho \text{ on } \mathsf{Var} \text{ such that} \\ & \text{for all } x \in \mathsf{Var},\ \rho(x) \in \mathsf{Var}_\tau \text{ iff } x \in \mathsf{Var}_\tau \end{aligned}$$

where $\varphi[\rho]$ is φ with all free occurrences of a variable x in φ replaced by $\rho(x)$. Now, let us agree that $R[\varphi] = R[\varphi']$ iff $\varphi \approx \varphi'$.[5] We write time, *succ* and $\mathcal{O}$ for $R[\mathsf{time}(x)]$, $R[succ(x,y)]$ and $R[\mathcal{O}(x)]$ respectively, without worrying about the exact choice of distinct $x, y \in \mathsf{Var}_\tau$.

[5]For $\varphi \in \Phi - \Phi_\tau$, we may set $R[\varphi] = (\lambda \mathsf{fvar}(\varphi))\varphi$ for an extensional system of λ-abstraction that distinguishes Var_τ from Var.

Relative to a temporal variable $x \in \mathsf{Var}_\tau$, a formula φ in Φ translates to a $v(\Phi)$-formula φ_x given by

$$\varphi_x \quad = \quad \begin{cases} R[\varphi](\vec{x}, x) & \text{if } \varphi \in \Phi_\tau \\ R[\varphi](\vec{x}) & \text{otherwise} \end{cases}$$

for $\mathsf{fvar}(\varphi) = \vec{x}$. For a proper interpretation relative to $v(\Phi)$-structures, a few more definitions are useful. Given a $v(\Phi)$-model M, let $|M|$ be the universe/domain of M, and $R[\varphi]_M$ be M's interpretation of $R[\varphi]$. A $v(\Phi)$-model M is defined to be *suited* if

(i) $\mathcal{O}_M$ is *succ*$_M$*-connected* in that for all $m, m' \in \mathcal{O}_M$,

$$m = m' \text{ or } \mathit{succ}_M{}^+(m, m') \text{ or } \mathit{succ}_M{}^+(m', m)$$

where $\mathit{succ}_M{}^+$ is the transitive closure of succ_M

(ii) for $\varphi \in \Phi_\tau$, $R[\varphi]_M \subseteq |M|^n \times \mathcal{O}_M$ where $n = |\mathsf{fvar}(\varphi)|$

(iii) if $\mathsf{fvar}(\varphi) = x_1, \ldots, x_n$ and $x_i \in \mathsf{Var}_\tau$, then the ith projection[6] of $R[\varphi]_M$ is a subset of $\mathcal{O}_M$, and

(iv) time_M is equality on $\mathcal{O}_M$.

A *sorted* M*-assignment* is a function $f : \mathsf{Var} \to |M|$ such that $f(x) \in \mathcal{O}_M$ for all $x \in \mathsf{Var}_\tau$. A set $\alpha \subseteq \Phi$ of formulas in Φ is interpreted conjunctively relative to a suited $v(\Phi)$-model M, a sorted M-assignment f and a temporal variable $x \in \mathsf{Var}_\tau$

$$M, f \models_x \alpha \quad \text{iff} \quad (\forall \varphi \in \alpha)\; M, f \models \varphi_x \;.$$

To apply a suited $v(\Phi)$-model M to nonempty strings of subsets of Φ, let us define the set $ch(M)$ of M*-chains* to consist of nonempty finite sequences $m_1 \cdots m_k \in \mathcal{O}_M{}^+$ such that

$$\mathit{succ}_M(m_i, m_{i+1}) \quad \text{for } 1 \le i < k \;.$$

Given $\alpha_1 \cdots \alpha_k \in \mathit{Pow}(\Phi)^+$, a sorted M-assignment f, and a temporal variable $x \in \mathsf{Var}_\tau$ not occurring in $\alpha_1 \cdots \alpha_k$, let $M_{f,x}[\alpha_1 \cdots \alpha_k]$ be the set of M-chains that componentwise witness $\alpha_1 \cdots \alpha_k$

$$M_{f,x}[\alpha_1 \cdots \alpha_k] \quad = \quad \{m_1 \cdots m_k \in ch(M) \mid M, f[x/m_i] \models_x \alpha_i \text{ for } 1 \le i \le k\}$$

where $f[x/m]$ maps x to m and is otherwise identical to f. Clearly, for all $x, y \in \mathsf{Var}_\tau$ that do not occur in a string $s \in \mathit{Pow}(\Phi)^+$, $M_{f,x}[s] = M_{f,y}[s]$. Accordingly, let us simply write $M_f[s]$ for $M_{f,x}[s]$ where x is some temporal variable that does not occur in s. Given a language L, let

$$M_f[L] \;=\; \bigcup_{s \in L} M_f[s]$$

and let us say L M*-portrays* a $v(\Phi)$-formula χ if

(i) $L \trianglerighteq \Box^* \boxed{\mathsf{time}(x)} \Box^*$ for every $x \in \mathsf{Var}_\tau$ that occurs freely in χ, and

(ii) for every sorted M-assignment f, we have $M, f \models \chi$ iff $M_f[L] \neq \emptyset$.

[6]The ith projection of a relation $r \subseteq A^k$ is $\{a \in A \mid (\exists a_1 \cdots a_k \in r)\; a_i = a\}$.

Note that condition (i) has the consequence that the temporal variable x used in $M_{f,x}[s]$ for $s \in L$ does not occur freely anywhere in L (or χ).

To portray $v(\Phi)$-formulas, let us define the *padded superposition* $L \&^\circ L'$ of languages $L, L' \subseteq Pow(\Phi)^*$ by

$$L \&^\circ L' \quad = \quad (\Box^* L \Box^*) \;\&\; (\Box^* L' \Box^*) \;.$$

For every finite subset X of Var_τ, let $\mathsf{TIME}(X)$ be the language given by

$$\begin{aligned} \mathsf{TIME}(\emptyset) &= \Box \\ \mathsf{TIME}(X \cup \{x\}) &= \mathsf{TIME}(X) \;\&^\circ\; \boxed{\mathsf{time}(x)} \quad \text{for } x \notin X \end{aligned}$$

and for every $\varphi \in \Phi$, let $\mathsf{Var}_\tau(\varphi)$ be the set of temporal variables that occur freely in φ. Now, for every $\varphi \in \Phi$ and $x \in \mathsf{Var}_\tau$, let us form the language

$$\mathcal{L}[\varphi_x] \quad = \quad \begin{cases} \mathsf{TIME}(\mathsf{Var}_\tau(\varphi)) \;\&^\circ\; \boxed{\varphi,\ \mathsf{time}(x)} & \text{if } \varphi \in \Phi_\tau \\ \mathsf{TIME}(\mathsf{Var}_\tau(\varphi)) \;\&^\circ\; \boxed{\varphi} & \text{otherwise.} \end{cases}$$

Assuming (for convenience) that Φ is closed under renaming of variables in Var,[7] every atomic $v(\Phi)$-formula has the form φ_x for some $\varphi \in \Phi$ and $x \in \mathsf{Var}_\tau$. Next, we interpret disjunction $\vee$ as nondeterministic choice $+$

$$\mathcal{L}[\chi \vee \chi'] \quad = \quad \mathcal{L}[\chi] + \mathcal{L}[\chi']$$

and conjunction $\wedge$ as padded superposition $\&^\circ$

$$\mathcal{L}[\chi \wedge \chi'] \quad = \quad \mathcal{L}[\chi] \;\&^\circ\; \mathcal{L}[\chi'] \;.$$

As for negation $\neg$, let us reduce $\mathcal{L}[\neg\chi]$ to $\mathcal{L}[\chi^\sim]$ where

$$\begin{aligned} (\varphi_x)^\sim &= (\sim\varphi)_x \quad \text{for } \varphi \in \Phi \text{ and } x \in \mathsf{Var}_\tau \\ (\chi \vee \chi')^\sim &= \chi^\sim \wedge \chi'^\sim \\ (\chi \wedge \chi')^\sim &= \chi^\sim \vee \chi'^\sim \;. \end{aligned}$$

The following lemma justifies the application of padded superposition in $\mathcal{L}[\chi]$ above.

LEMMA 4.1
For every $v(\Phi)$-model M, $\mathcal{O}_M$ is $succ_M$-connected iff for every finite nonempty $A \subseteq \mathcal{O}_M$, there is an M-chain $m_1 \cdots m_k$ such that $A \subseteq \{m_1, \ldots, m_k\}$.

Lemma 4.1 is proved by induction on the size of A. Another routine inductive argument, this time on $v(\Phi)$-formulas χ, yields

THEOREM 4.2
Every quantifier-free $v(\Phi)$-formula χ is M-portrayed by $\mathcal{L}[\chi]$, for every suited $v(\Phi)$-model M.

[7]That is, if necessary, let us replace Φ by $\{\varphi \mid (\exists\varphi' \in \Phi)\ \varphi \approx \varphi'\}$.

To bring out what quantification there is in portraying $v(\Phi)$-formulas, let us relativize $\exists$ and $\forall$ to $\mathcal{O}$. Given a list $x_1 \cdots x_n$ of variables, let $\mathcal{O}(\vec{x})$ abbreviate the conjunction $\mathcal{O}(y_1) \wedge \cdots \wedge \mathcal{O}(y_k)$ where $y_1 \cdots y_k$ is the sublist of $x_1 \cdots x_n$ consisting of temporal variables. (If $k = 0$, $\mathcal{O}(\vec{x})$ is set to some tautology.) Now, for $\vec{x} = x_1 \cdots x_n$, let us write $(\exists_{\mathcal{O}} \vec{x})\chi$ and $(\forall_{\mathcal{O}} \vec{x})\chi$ for

$$(\exists x_1) \cdots (\exists x_n)(\mathcal{O}(\vec{x}) \wedge \chi) \;\text{ and }\; (\forall x_1) \cdots (\forall x_n)(\mathcal{O}(\vec{x}) \supset \chi)$$

respectively. Can we portray universal $v(\Phi)$-formulas $(\forall_{\mathcal{O}} \vec{x})\chi$, for quantifier-free χ? Basic model-theoretic notions applied to the present context show that we can*not*. Let us call a $v(\Phi)$-model M' an *extension of* a $v(\Phi)$-model M and write $M \sqsubseteq M'$ if $|M| \subseteq |M'|$ and for every atomic $v(\Phi)$-formula χ and every sorted M-assignment f,

$$M, f \models \chi \quad \text{iff} \quad M', f \models \chi \; .$$

A $v(\Phi)$-formula χ is M*-persistent* if for every extension M' of M and for every sorted M-assignment f such that $M, f \models \chi$, $M', f \models \chi$. Obviously, if $M \sqsubseteq M'$ then for every $s \in \mathit{Pow}(\Phi)^+$,

$$M_f[s] \subseteq {M'}_f[s] \;\text{ for every sorted } M\text{-assignment } f$$

whence

PROPOSITION 4.3
Every $v(\Phi)$-formula M-portrayed by a language is M-persistent.

By contrast, universal sentences are not, in general, persistent. In particular, there is no portraying the temporal logic modal operator H given by the translation $(\mathsf{H}p)_x = (\forall y < x)\; p_y$ for some ordering $<$ on $\mathcal{O}$. Existential sentences, on the other hand, are persistent, and we have, for the record,

PROPOSITION 4.4
For every language L that M-portrays a $v(\Phi)$-formula χ with free variables $\vec{x}$,

$$M \models (\exists_{\mathcal{O}} \vec{x})\chi \quad \textit{iff} \quad M_f[L] \neq \emptyset \textit{ for some sorted } M\textit{-assignment } f \; .$$

We can sharpen the languages $\mathcal{L}[\chi]$ for quantifier-free $v(\Phi)$-formulas χ by means of the following regular (finite-state) constructions. Let

(i) *unpad* be the set of nonempty $\mathit{Pow}(\Phi)$-strings that, except for $\Box$, neither begin nor end with $\Box$

$$\mathit{unpad} \quad = \quad \mathit{Pow}(\Phi) \;+\; \mathit{Pow}_+(\Phi)\, \mathit{Pow}(\Phi)^*\, \mathit{Pow}_+(\Phi)$$

where $\mathit{Pow}_+(\Phi) = \mathit{Pow}(\Phi) - \{\Box\}$

(ii) $\mathit{uniq}(\varphi)$ consist of $\mathit{Pow}(\Phi)$-strings in which φ occurs at most once

$$\mathit{uniq}(\varphi) \quad = \quad \mathit{Pow}(\Phi - \{\varphi\})^* \;(\epsilon \;+\; \mathit{Pow}(\Phi))\mathit{Pow}(\Phi - \{\varphi\})^*$$

(iii) $\mathit{del}(L, \varphi)$ be L with φ deleted from its symbols

$$\mathit{del}(L, \varphi) \quad = \quad \{(\alpha_1 - \{\varphi\}) \cdots (\alpha_k - \{\varphi\}) \mid k \geq 1 \text{ and } \alpha_1 \cdots \alpha_k \in L\}$$

which is accepted by the same automaton M for L, except that M's transitions $\rightarrow$ are revised to

$$q \overset{\alpha}{\Rightarrow} q' \quad \text{iff} \quad \varphi \notin \alpha \text{ and } (q \overset{\alpha}{\rightarrow} q' \text{ or } q \overset{\alpha \cup \{\varphi\}}{\rightarrow} q') \; .$$

A formula $\varphi \in \Phi$ is L, M*-redundant* if for every $\alpha_1 \cdots \alpha_k \in L$ and sorted M-assignment f, if $\beta_i = \alpha_i - \{\varphi\}$ for $1 \leq i \leq k$, then

$$M_f[\beta_1 \cdots \beta_k] \quad = \quad M_f[\alpha_1 \cdots \alpha_k] \ .$$

For example, if $L \trianglerighteq \Box^* \boxed{\mathsf{time}(x)}\,\boxed{\mathsf{time}(y)} \Box^*$ then *succ*(x, y) is L, M-redundant for every $v(\Phi)$-model M.

PROPOSITION 4.5
If L M-portrays χ, then so do

(a) $L \cap$ *unpad*

(b) $L \cap$ *uniq*$(\mathsf{time}(x))$ for every $x \in \mathsf{Var}_\tau$ that occur freely in χ, provided the transitive closure of *succ*$_M$ is irreflexive, and

(c) *del*(L, φ) for every L, M-redundant φ.

Proposition 4.5 suggests sharpening $\&^\circ$ to $\&^\bullet$ obtained by intersection with *unpad*

$$L \,\&^\bullet\, L' \quad = \quad (L \,\&^\circ\, L') \cap \textit{unpad} \ .$$

For a concrete example, consider the Priorian past operator P described by the translation $(\mathsf{P}\varphi)_x = (\exists_{\mathcal{O}} y)(y < x \wedge \varphi_y)$. Assuming $<_M$ is the transitive closure of *succ*$_M$, we can appeal to Proposition 4.5 to M-portray $y < x \wedge \varphi_y$ by

$$\boxed{\mathsf{time}(y)} \Box^* \boxed{\mathsf{time}(x)} \quad \&^\bullet \quad \mathcal{L}[\varphi_y]$$

assuming $\mathcal{L}(\varphi_y)$ M-portrays φ_y. Notice that, in general, $\mathsf{time}(z)$ is L, M-redundant if for every $s \in L$, there is at most one occurrence of a formula in s that mentions z. Thus, if $\mathcal{L}(\varphi_y) = \boxed{\varphi, \mathsf{time}(y)}$ with y *not* occurring in φ, then (on the basis of Proposition 4.5(c)) we may delete $\mathsf{time}(y)$ (in line with the existential quantification $\exists_{\mathcal{O}} y$) to M-portray $(\mathsf{P}\varphi)_x$ by $\boxed{\varphi} \Box^* \boxed{\mathsf{time}(x)}$.

As Proposition 4.5(b) suggests, a language L that M-portrays χ generally includes spurious possibilities; i.e. strings s for which there is *no* sorted f satisfying $M_f[s] \neq \emptyset$. Could such fat spoil the soundness of $\vdash_\Sigma$? To analyse this question formally, let us say Σ Φ*-covers* M if Σ contains every $\alpha \subseteq \Phi$ witnessed by M and some sorted M-assignment f — that is, if

$$\{\alpha \subseteq \Phi \mid M_f[\alpha] \neq \emptyset \text{ for some sorted } M\text{-assignment } f\} \quad \subseteq \quad \Sigma \ .$$

Next, given a language L and a sorted M-assignment f, let us define the notion of an M, f-*completion* $L_{M,f}$ in analogy with that of a Σ-completion L_Σ from Section 3. More precisely, let us collect all nonempty strings s witnessed by M, f in

$$\mathcal{L}(M, f) \quad = \quad \{s \in \textit{Pow}(\Phi)^+ \mid M_f[s] \neq \emptyset\}$$

and equate $L_{M,f}$ with the set of $\trianglerighteq$-maximal extensions in $\mathcal{L}(M, f)$ of L-strings; that is,

$$L_{M,f} \quad = \quad \{s \in \trianglerighteq\text{-max}(\mathcal{L}(M, f)) \mid (\exists s' \in L)\ s \trianglerighteq s'\}$$

where $\trianglerighteq$-max$(\mathcal{L}(M, f))$ is the set of $s \in \mathcal{L}(M, f)$ such that for all $s' \in \mathcal{L}(M, f)$, if $s' \trianglerighteq s$ then $s' = s$. We then define $L \models^M L'$ to mean $L_{M,f} \subseteq L'_{M,f}$ for every sorted M-assignment f.

PROPOSITION 4.6 (Soundness of $\vdash_\Sigma$)
If Σ Φ-covers M and $L \vdash_\Sigma L'$ then

(a) $L \models^M L'$, and
(b) for every χ that is M-portrayed by L, and every χ' M-portrayed by L',

$$M \quad \models \quad (\forall_{\mathcal{O}} \vec{x})(\chi \Rightarrow \chi')$$

where $\vec{x}$ lists all free variables in $\chi \Rightarrow \chi'$.

Notice that the set $\trianglerighteq$-max$(\mathcal{L}(M,f))$ is given by the set $ch(M)$ of M-chains as follows

$$\trianglerighteq\text{-max}(\mathcal{L}(M,f)) \quad = \quad \{\Phi(M,f,m_1)\cdots\Phi(M,f,m_k) \mid m_1\cdots m_k \in ch(M)\}$$

where for every $m \in \mathcal{O}_M$,

$$\Phi(M,f,m) \quad = \quad \{\varphi \in \Phi \mid M, f[x/m] \models \varphi_x \text{ for } x \in \mathsf{Var}_\tau \textit{ not} \text{ occurring in } \varphi\} \ .$$

$\Phi(M,f,m)$ is what [12] refers to as a *situation* over the *propositional fluents* $\varphi \in \Phi$ inasmuch as $\Phi(M,f,m)$ is a snapshot of M,f at m. Since $\vdash_\Sigma$ fails to check for M-chains, we cannot expect $\models^M$ to co-incide exactly with $\vdash_\Sigma$, for nontrivial $succ_M$. Σ-completions are simply too crude to meet, on their own, the challenge the *frame problem* [12] poses for natural language [20]. At the very least, we might intersect L_Σ with some other language L that encodes constraints on *succ*, refining the test $L_\Sigma \subseteq L'_\Sigma$ (for L entailing L') to $L_\Sigma \cap \mathsf{L} \subseteq L'_\Sigma \cap \mathsf{L}$. An example of L above is *uniq*$(\mathsf{time}(x))$, but there are far subtler inertial laws and inter-situational constraints to consider. We may well need to resort to more sophisticated tools such as forcing (explored in section 3 of [6]). [8] That said, much can already be done with finite-state methods, and it bears noting that Proposition 2.3 from Section 2 holds with $\&_\Sigma$ generalized to $\&^{\mathsf{L}}$, under the definition

$$L \,\&^{\mathsf{L}}\, L' \quad = \quad (L \& L') \cap \mathsf{L}$$

for all $L, L' \subseteq Pow(\Phi)^*$. (Observe that $\&_\Sigma$ is just $\&^{\mathsf{L}}$ for $\mathsf{L} = \Sigma^+$.) For this approach to be finite-state, it is crucial that L be regular. It should perhaps be emphasized that although Proposition 4.3 imposes obvious limitations on the part of first-order logic that we can capture, our regular languages are *not* strictly contained in first-order logic. It is well-known that even over finite models connectivity is *not* first-order; and throughout this section, we have made heavy use of the assumption that $\mathcal{O}_M$ is $succ_M$-connected.

5 Conclusion

A slew of constructions on regular languages are introduced above, chief among which is superposition $\&$, giving rise to an approximation $\trianglerighteq$ of entailments $\vdash_\Sigma$ based on Σ-completions L_Σ. In addition, we have negations $\tilde{\Gamma}$ and ${}_\Sigma L$, padding $\Box^* L \Box^*$, *unpad*, *uniq*(φ) and *del*(L,φ), each of which is put to some model-theoretic use. What does all this machinery come to?

Consider the following passage from [20].

> The proposal is that the so-called temporal semantics of natural language is not primarily to do with time at all. Instead, the formal devices we need are those related to the representation of causality and goal-directed action.

[8] The present paper borrows freely from the conference paper [6] without containing or being contained by it.

It is not claimed that the present approach does full justice to the aforementioned proposal. Even so, it is fair to say that a form of causality is implicit in the state transitions of finite automata (accepting regular languages), and that the account of telicity in Section 2 gets at some notion of goal-directed action. As for the claim that temporal semantics is 'not primarily to do with time at all', let me return to the construal of the strings above as sequences of observations, distinct from some temporal reality that is presumably observed. While it is natural to interpret the relation symbols $\mathcal{O}$ and *succ* in Section 4 as bits of that temporal reality, these bits are of interest only to the extent that they throw light on the strings of observations. Nor is anything made of finite automata beyond the regular languages they accept.[9] A language offers only a faint trace of a machine that accepts/generates it, of which there may be any number. And while there is considerable scientific interest in identifying the computational mechanism underlying natural language, my own ignorance inclines me towards more modest goals. An abstract stance may have limited predictive power, but it also has a better chance of getting something right (the step from procedural to declarative semantics gaining us, with any luck, simplicity and clarity).

There is a further sense in which the system of observations above is of bounded scope. And that has to do with the persistence of $v(\Phi)$-formulas that can be portrayed (according to Proposition 4.3), and indeed *are* portrayed (according to Theorem 4.2 and Proposition 4.4). What about the non-persistent formulas of predicate logic? The present finite-state approach was, in fact, conceived as a complement, at what [17] calls the 'sub-atomic' level, to a more wide-ranging model-theoretic re-interpretation of *propositions-as-types* (applied to natural language discourse in [18]). That is, beyond the sub-atomic realm of regular languages wait typed λ-calculi (functional programming). Should events, as observations, be equated with objects that are observed? For a perspective that instead links events with proofs, the interested reader is referred to the *constructive eventuality assumption* in [5].

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[9] That is, the present system of observations occupies some middle ground between what Jackendoff calls (following Chomsky) the E[xternalized]-semantics of the model-theoretic tradition and the I[nternalized]-semantics of conceptual structure ([10]).

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