MSO with tests and reducts

Tim Fernando, David Woods & Carl Vogel (Dublin)

Dresden, 25 Sep 2019 (FSMNLP)

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- $aab \models \exists x(P_a(x) \wedge \exists y(xSy \wedge P_b(y)))$

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  $q \in (b_1 + \overline{b_1}) \cdots (b_n + \overline{b_n})$ Kleene algebra with tests (Kozen)

  e.g., $[\![b_1]\!] = b_1(b_2 + \overline{b_2}) \cdots (b_n + \overline{b_n})$

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- for all models M and sentences φ,

$$M \models \varphi \iff M \upharpoonright \text{vocabulary}(\varphi) \models \varphi$$

Natural language

Reichenbach

Amundsen flew to the North Pole

fly(A,NP)		*speech*

Natural language events (Davidson . . .)

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fly_0(A,NP)	fly_1(A,NP), reach(A,NP)

Events as partial descriptions of intervals — James Allen

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Events as partial descriptions of intervals — James Allen

Statives as the basis of Dowty's aspect calculus

events in NL	KAT (Kozen)
statives	tests/Booleans
non-statives	actions

Main points

- MSO_A-model = string $s \in (2^A)^+$ (rather than alphabet A)

$$\rho_B(\alpha_1 \cdots \alpha_n) \ := \ (\alpha_1 \cap B) \cdots (\alpha_n \cap B)$$

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- compression bc from test-as-program to test-as-formula

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Time$_S$ *as change*$_B$

$$xSy \;\supset\; \neg \bigwedge_{a \in B} (P_a(x) \equiv P_a(y)) \qquad (\dagger)$$

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$$s \models \forall x \forall y (\dagger) \quad \iff \quad bc(\rho_B(s)) = \rho_B(s)$$

Outline

§1 Transitions and reducts

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§2 Tests and observable change

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§3 Projections and superpositions

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- guarded strings in MSO

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KA with tests: add states $q_i \subseteq B$

	alphabet
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Concatenation modified

$$s\hat{q} \diamond_k \hat{q}' s' \simeq s\hat{q}s' \text{ if } q = q'$$

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$$sq \bullet_\Sigma \alpha s' \simeq s\alpha s' \text{ if } q = \alpha \setminus \Sigma$$

Reducts and guarded strings

$$\rho_C(\alpha_1 \cdots \alpha_n) \;:=\; (\alpha_1 \cap C) \cdots (\alpha_n \cap C)$$

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$$\Sigma_{\Box} \ := \ \{\boxed{p} \mid p \in \Sigma\}$$

$$\mathcal{G}^B_{\Sigma} \ := \{s \in (2^{B \cup \Sigma})^+ \mid \rho_{\Sigma}(s) \in {\Sigma_{\Box}}^* \Box\}$$

Reducts and guarded strings

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$$\begin{aligned} \Sigma_{\square} \;&:=\; \{\boxed{p} \mid p \in \Sigma\} \\ \mathcal{G}^B_{\Sigma} \;&:=\; \{s \in (2^{B \cup \Sigma})^+ \mid \rho_{\Sigma}(s) \in {\Sigma_{\square}}^* \square\} \\ &=\; \{s \in (2^{B \cup \Sigma})^+ \mid s \models \forall x\; \chi_{\Sigma}(x)\} \end{aligned}$$

$\chi_{\Sigma}(x)$ says

(i) x is non-final iff some $a \in \Sigma$ occurs in x

$$\exists y (xSy) \equiv \bigvee_{a \in \Sigma} P_a(x)$$

(ii) *no* two symbols from Σ occur in x

$$\neg \bigvee_{a \in \Sigma} (P_a(x) \wedge \bigvee_{a' \in \Sigma \setminus \{a\}} P_{a'}(x))$$

Outline

§1 Transitions and reducts
- guarded strings in MSO

§2 Tests and observable change
- compressing reducts

§3 Projections and superpositions

Tests as programs

$$[\![b]\!] \;:=\; \{q \mid q \subseteq B \text{ and } b \in q\}$$

$$[\![b?]\!] \;:=\; \{(q \cup \{b?\})q \mid q \in [\![b]\!]\}$$

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$$P_{b?}(x) \wedge xSy \;\supset\; P_b(x) \;\wedge\; x \equiv_B y$$

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Tests as programs executing in isolation

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$$(q \cup \{b?\})q \;\overset{\rho_B}{\rightsquigarrow}\; qq \;\overset{bc}{\rightsquigarrow}\; q$$

Compressing reducts

s is *stutterless* if $s = \mathit{bc}(s)$ where

$$
\begin{aligned}
\mathit{bc}(s) &:= s \quad \text{if length}(s) < 2 \\
\mathit{bc}(\alpha\alpha' s) &:= \begin{cases} \mathit{bc}(\alpha' s) & \text{if } \alpha = \alpha' \\ \alpha\, \mathit{bc}(\alpha' s) & \text{otherwise.} \end{cases}
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$$s \models \forall x \forall y\ (xSy \supset x \not\equiv_B y) \iff \rho_B(s) \text{ is stutterless}$$

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$$s \models \langle C \rangle \varphi \iff bc(\rho_C(s)) \models \varphi$$

satisfaction condition for *institution* (Goguen & Burstall)

Outline

§1 Transitions and reducts
- guarded strings in MSO

§2 Tests and observable change
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§3 Projections and superpositions
- partial overlapping descriptions

Superposing projections

For $C \subseteq A$,

$$\mathcal{L}_C := \{bc(s) \mid s \in (2^C)^+\}.$$

For $s \in \mathcal{L}_C$ and $s' \in \mathcal{L}_{C'}$,

$$\begin{aligned} s \,\&_{C,C'}\, s' \;:=\; \{s'' \in \mathcal{L}_{C \cup C'} \mid\; & bc(\rho_C(s'')) = s \text{ and} \\ & bc(\rho_{C'}(s'')) = s'\} \end{aligned}$$

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$$\boxed{}\boxed{b}\boxed{} \;\&_{\{b\},\{b'\}}\; \boxed{}\boxed{b'}\boxed{} \;=\; \boxed{}\boxed{b, b'}\boxed{} + \boxed{}\boxed{b}\boxed{b'}\boxed{} + \cdots$$

13 Allen interval relations

$$\boxed{}\boxed{b}\boxed{b'}\boxed{} \;\&_{\{b,b'\},\{b',b''\}}\; \boxed{}\boxed{b', b''}\boxed{b'}\boxed{} \;=\; \boxed{}\boxed{b}\boxed{b', b''}\boxed{b'}\boxed{}$$

transitivity table (Allen)

$\&_{C,C'}$ inductively

$$\frac{}{\&(\epsilon, \epsilon, \epsilon)} \text{ (s0)}$$

$$\frac{\&(s, s', s'')}{\&(\alpha s, \alpha' s, (\alpha \cup \alpha')s'')} \text{ (s1)}$$

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$$\frac{\&(s, \alpha' s', s'')}{\&(\alpha s, \alpha' s', (\alpha \cup \alpha') s'')} \text{ (b2)}$$

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$$\frac{\&(s, \alpha' s', s'')}{\&(\alpha s, \alpha' s', (\alpha \cup \alpha')s'')} \text{ (b2)}$$

For $C \cap C' \neq \emptyset$, impose side conditions

$$\alpha \cap C' \subseteq \alpha' \quad \text{and} \quad \alpha' \cap C \subseteq \alpha$$

on (s1), (b1), (b2)

So what?

$s \,\&_{C,C'}\, s'$ combines partial overlapping descriptions $(C, s), (C', s')$

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MSO$_A$-model $\hat{s} \in (2^A)^+$ is C-approximated by s and
C'-approximated by s' (for $C, C' \subseteq B$)

$$\hat{s} \text{ is } C\text{-approximated by } s \iff bc(\rho_C(\hat{s})) = s$$

So what?

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MSO$_A$-model $\hat{s} \in (2^A)^+$ is C-approximated by s and
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$$\hat{s} \text{ is } C\text{-approximated by } s \iff \mathit{bc}(\rho_C(\hat{s})) = s$$

$$\rho_B(\hat{s}) = \mathit{bc}(\rho_B(\hat{s})) \iff \hat{s} \models \forall x \forall y (xSy \supset x \not\equiv_B y)$$

$$x \not\equiv_B y \;\equiv\; \bigvee_{a \in B} \underbrace{(P_a(x) \wedge \neg P_a(y))}_{P_{r(a)}(x)} \vee \underbrace{(P_a(y) \wedge \neg P_a(x))}_{P_{l(a)}(x)}$$

actions localised to a

Thank You

☺