Two perspectives on change & institutions

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- Ontological analysis as a search for truth makers
- Episodes as truth makers for material relations

(1) John works for Mary

Perspective One:
find truth makers in a timeline (where episodes occur)

Perspective Two:
find tm's behind episodes/timelines in "rules and regulations" (*G. Carlson* 1995) / causal structures (*M. Steedman* 2005)

Complications

(2) Tess eats dal $\neq$ Tess is eating dal

Tess is not eating dal today; she will tomorrow

(3) Bishops move diagonally
- Genericity (*Carlson*)

(4) John was drawing a circle
$\not\Rightarrow$ John drew a circle
- Imperfective Paradox (*Dowty*)

Episode occurs in a "maximally connected time interval" (*Guarino*)

(5) Pat spoke until noon $^?$but not a picosecond later
- Sorites (heap) paradox

Bound granularity
$\leadsto$ relativize $\models$ to *signature* in an **institution** (*Goguen & Burstall*)

Antony Galton . . .

G 2008:

a fundamental ontological distinction between

EXP, the dynamic experiential world of objects and processes as they exist at one time, and

HIST, the static historical overview populated by events that are generated by the ongoing processes in EXP

modifying *Grenon & Smith* 2004

SNAP	SPAN
objects	events
	processes

EXP	HIST
objects	events
processes	

G 20012:

processes as abstract patterns of behaviour which may be realised in concrete form as actually occurring states or events

Proposal

$$\frac{\text{EXP-process}}{\text{HIST-event}} \approx \frac{\text{internal mechanism}}{\text{external timeline}}$$

$$\approx_{\Sigma} \frac{\text{automaton}}{\text{string}}$$

$$\approx_{\Sigma} \frac{\text{Hennessy-Milner}(\Diamond)}{\text{Monadic Second-Order Logic}}$$

$$\approx_{\Sigma} \frac{\text{type}}{\text{particular}}$$

Granularity bounded by a signature Σ within an institution

1. Perspective One: strings
2. Perspective Two: languages
3. Relating the perspectives

Strings for natural language semantics

W. Klein

> *The expression of time in natural languages relates a clause-internal temporal structure to a clause-external temporal structure.*
> *The latter may shrink to a single interval, for example, the time at which the sentence is uttered; but this is just a special case.*
> *The clause-internal temporal structure may also be very simple – it may be reduced to a single interval without any further differentiation, the 'time of the situation'; but if this ever happens, it is only a borderline case.*
> *As a rule, the clause-internal structure is much more complex.*

`Ed exhaled`		E		S	

H. Reichenbach

		tense	aspect
`it rained`	E,R | | S	R | | S	E,R
`it has rained`	E | | R,S	R,S	E | | R

Inside E: *Aristotle* . . .

Al was running towards the post-office
$\therefore$ Al ran towards the post-office

Al was running to the post-office
$\not\therefore$ Al ran to the post-office

at(al,post-office) holds at the end of an interval

Partition an interval into
a sequence $I_1 \cdots I_n$ of intervals with $I_1 < I_2 < \cdots < I_n$
to interpret a string $\alpha_1 \cdots \alpha_n$ of boxes α_i

$$I_1 \cdots I_n \models \alpha_1 \cdots \alpha_n \quad \text{iff} \quad (\forall i \in \{1, \ldots, n\})(\forall \varphi \in \alpha_i)\ I_i \models \varphi$$

$\alpha_1 \cdots \alpha_n$ is *telic* if $n \geq 2$ and there is some φ in α_n such that
the negation $\sim\varphi$ of φ appears in α_i for $1 \leq i < n$

~at(al,post-office)	~at(al,post-office)	at(al,post-office)

Intervals strung out

days in a year $\rightsquigarrow$ months in a year

Jan,d1	Jan,d2	$\cdots$	Dec,d31

$\overset{\rho_{months}}{\rightsquigarrow}$ Jan[31] Feb[28] $\cdots$ Dec[31]

$\overset{bc}{\rightsquigarrow}$ | Jan | Feb | $\cdots$ | Dec |

ρ_Σ "see only Σ"

$$\rho_\Sigma(\alpha_1 \cdots \alpha_n) \quad := \quad (\alpha_1 \cap \Sigma) \cdots (\alpha_n \cap \Sigma)$$

bc "no time without change" (McTaggart's dictum)
compress α^+ to α

$\alpha_1 \cdots \alpha_n$ is *stutterless* if $\alpha_i \neq \alpha_{i+1}$ for $1 \leq i < n$
— i.e. if $bc(\alpha_1 \cdots \alpha_n) = \alpha_1 \cdots \alpha_n$

bc_Σ is $\rho_\Sigma; bc$ [vocabulary ; ontology]

Institutions (*Goguen & Burstall*)

$$M \models_\Sigma \varphi \qquad \Sigma \xrightarrow{\sigma} \Sigma' \qquad \frac{\varphi \in sen(\Sigma)}{\sigma(\varphi) \in sen(\Sigma')} \qquad \frac{M' \in Mod(\Sigma')}{M'|_\sigma \in Mod(\Sigma)}$$

$$M'|_\sigma \models_\Sigma \varphi \quad \text{iff} \quad M' \models_{\Sigma'} \sigma(\varphi)$$

$sen(\Sigma)$ = Monadic Second-Order logic (MSO) over Σ
= regular languages over Σ (*Büchi, Elgot, Trakhtenbrot*)

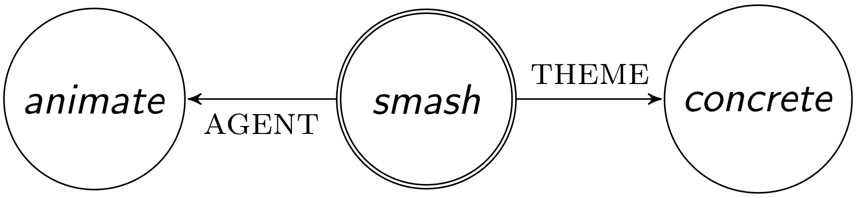
$Mod(\Sigma)$ = strings over alphabet 2^Σ (not Σ)

$$\rho_\Sigma(s') \models_\Sigma \varphi \quad \text{iff} \quad s' \models_{\Sigma'} \varphi$$

For stutterless strings, apply *bc* after ρ_Σ for bc_Σ

- make $\boxed{a}\boxed{a}$ stutterless via *bc* or extra symbol TIC for $\boxed{a,\text{TIC}}\boxed{a}$

Frames (*Fillmore, Barsalou,...*)



$$\begin{bmatrix} \textit{smash} & & \\ \text{AGENT} & : & \textit{animate} \\ \text{THEME} & : & \textit{concrete} \end{bmatrix} \qquad \begin{array}{l} \{\textit{smash}, \\ \ \ \text{AGENT}\ \textit{animate}, \\ \ \ \text{THEME}\ \textit{concrete}\} \end{array}$$

$$\begin{array}{rcl} [\![L]\!] & := & \bigcap_{s \in L}\ \textit{domain}([\![s]\!]) \\ [\![\epsilon]\!] & := & \lambda x.x \\ [\![sa]\!] & := & [\![s]\!];[\![a]\!] \end{array} \qquad \begin{array}{l} \textit{domain}([\![\textit{smash}]\!]) \cap \\ \textit{domain}([\![\text{AGENT}]\!];[\![\textit{animate}]\!]) \cap \\ \textit{domain}([\![\text{THEME}]\!];[\![\textit{concrete}]\!]) \end{array}$$

Hennessy-Milner & traces

Σ-*deterministic system* $\delta : Q \times \Sigma \rightharpoonup Q \qquad q \xrightarrow{a} \delta(q, a)$

$(\Phi_\Sigma) \qquad \varphi \ ::= \ \top \mid \langle a \rangle \varphi \mid \varphi \wedge \varphi' \mid \neg\varphi \qquad (a \in \Sigma)$

$$q \models \langle a \rangle \varphi \ \text{ iff } \ (q, a) \in \textit{domain}(\delta) \text{ and } \delta(q, a) \models \varphi$$

$$\begin{array}{rcl} \langle \epsilon \rangle \varphi & := & \varphi \\ \langle as \rangle \varphi & := & \langle a \rangle \langle s \rangle \varphi \end{array}$$

$$\textit{trace}_\delta(q) \ = \ \{s \in \Sigma^* \mid q \models \langle s \rangle \top\}$$

For $\models$, we can reduce q, δ to $\textit{trace}_\delta(q) \subseteq \Sigma^*$.

Identity of indiscernibles (*Leibniz*) & derivatives

$$\Phi_\Sigma(q) \;:=\; \{\varphi \in \Phi_\Sigma \mid q \models \varphi\}$$
$$trace_\delta(q) \;=\; \{s \in \Sigma^* \mid \langle s\rangle\top \in \Phi_\Sigma(q)\}$$

Fact. $\quad \Phi_\Sigma(q) = \Phi_\Sigma(q') \quad \text{iff} \quad trace_\delta(q) = trace_\delta(q')$

Transitions as derivatives (Brzozowski)

$$L_s \;:=\; \{s' \mid ss' \in L\}$$

For all $s, s' \in \Sigma^*$ and $L \subseteq \Sigma^*$,

$$L_s = L_{s'} \;\text{ iff }\; (\forall w \in \Sigma^*)\ (sw \in L \text{ iff } s'w \in L)$$

so that the **Myhill-Nerode Theorem** says:

$$L \textit{ is regular} \quad \textit{iff} \quad \{L_s \mid s \in \Sigma^*\} \textit{ is finite.}$$

A monster $\mathcal{A}$-deterministic system $\hat{\delta}$

$$\mathrm{Fin}(\mathcal{A}) \;:=\; \{\Sigma \subseteq \mathcal{A} \mid \Sigma \text{ is finite}\}$$

For $X \in \mathrm{Fin}(\mathcal{A}) \cup \{\mathcal{A}\}$,

an X-*state* is a non-empty prefix-closed subset q of X^*

$\hat{\delta} = \{(q, a, q_a) \mid q \text{ is an } \mathcal{A}\text{-state and } a \in q \cap \mathcal{A}\}$

Fact. *For every $\Sigma \in Fin(\mathcal{A})$, $\varphi \in \Phi_\Sigma$ and $\mathcal{A}$-state q,*

$$q \models \varphi \quad \textit{iff} \quad q \cap \Sigma^* \models \varphi$$

and if, moreover, $s \in q \cap \Sigma^*$, then

$$q \models \langle s\rangle\varphi \;\; \textit{iff} \;\; (q \cap \Sigma^*)_s \models \varphi.$$

The functor $Q : \mathsf{Fin}(\mathcal{A})^{op} \to \mathbf{Cat}$

For $\Sigma \in \mathsf{Fin}(\mathcal{A})$,

$Q(\Sigma)$ is the category with
object non-empty prefix-closed $q \subseteq \Sigma^*$
morphisms (q,s) from q to q_s, for $q \in Q(\Sigma)$ and $s \in q$
$(q,s);(q_s,s') = (q,ss')$ with identities (q,ϵ)

$Q(\Sigma',\Sigma) : Q(\Sigma') \to Q(\Sigma)$ for $\Sigma \subseteq \Sigma' \in \mathsf{Fin}(\mathcal{A})$

$q \mapsto q \cap \Sigma^*$

$(q,s) \mapsto (q \cap \Sigma^*, \pi_\Sigma(s))$

where $\pi_\Sigma(s)$ is the longest prefix of s in Σ^*

$$\pi_\Sigma(\epsilon) := \epsilon$$

$$\pi_\Sigma(as) := \begin{cases} a\,\pi_\Sigma(s) & \text{if } a \in \Sigma \\ \epsilon & \text{otherwise.} \end{cases}$$

$\int Q$ (*Grothendieck*) & institutions

$\mathbf{Sign}^{op} = \int Q$

- objects (Σ, q) where $\Sigma \in \mathsf{Fin}(\mathcal{A})$ and $q \in Q(\Sigma)$
- morphisms from (Σ', q') to (Σ, q) are pairs

$$((\Sigma',\Sigma),(q'',s))$$

of $\mathsf{Fin}(\mathcal{A})^{op}$-morphisms (Σ',Σ) and
$Q(\Sigma)$-morphisms (q'',s) s.t. $q'' = q' \cap \Sigma^*$ and $q = q''_s$

$sen : \mathbf{Sign} \to \mathbf{Set}$

- $sen(\Sigma, q) := \Phi_\Sigma$
- $sen((\Sigma',\Sigma),(q'',s)) :\ \varphi \mapsto \langle s \rangle \varphi$

$Mod : \mathbf{Sign}^{op} \to \mathbf{Cat}$

- $|Mod(\Sigma,q)| := \{q' \in |Q(\Sigma)| \ :\ q \subseteq q'\}$
- $Mod((\Sigma',\Sigma),(q'',s)) : \hat{q} \mapsto (\hat{q} \cap \Sigma^*)_s$

Back to intuitions

$$\frac{\text{HIST-event}}{\text{EXP-process}} \approx \frac{\text{external timeline (temporal)}}{\text{internal mechanism (causal)}}$$

$$\approx_{\Sigma} \frac{\text{string (timeline)}}{\text{automaton (language)}}$$

Finite approximability hypothesis: timeline as string and processes as finite automata

$\Sigma \in |\mathbf{Sign}|$ in an institution $(\mathbf{Sign}, sen, Mod, \models)$

timeline $\approx$ where many different processes meet (events)

Ontology & the satisfaction condition

Recall *Guarino*'s dictum

Ontological analysis as a search for truth makers

What makes a sentence φ true?

Find $\Sigma_\circ \stackrel{\sigma}{\to} \Sigma$ and $\varphi_\circ \in sen(\Sigma_\circ)$ s.t. $\sigma(\varphi_\circ) = \varphi \in sen(\Sigma)$ and

$$M|_\sigma \models_{\Sigma_\circ} \varphi_\circ \quad \text{iff} \quad M \models_\Sigma \varphi$$

Institution 1: φ from MSO
σ as inclusion $\subseteq$
M as string s and $M|_\sigma = \rho_{\Sigma_\circ}(s)$

Institution 2: φ from Hennessy-Milner
σ from $\int Q$
M as language q and $M|_\sigma = (q \cap {\Sigma_\circ}^*)_s$

From strings to types & back

$\models$ organizes models into types

$$[\![\varphi]\!] \; := \; \{M \in Mod(\Sigma) \mid M \models_\Sigma \varphi\}$$

Inst 1: adjust Büchi-Elgot-Trakhtenbrot theorem:

$$\begin{array}{rlcl} & \text{MSO}_\Sigma & = & \text{regular languages over } \Sigma \\ \leadsto & \text{MSO}^\Sigma & = & \text{regular languages over } 2^\Sigma \end{array}$$

$$\rho_\Sigma(\alpha_1 \cdots \alpha_n) \; := \; (\alpha_1 \cap \Sigma) \cdots (\alpha_n \cap \Sigma)$$

Inst 2: interpret Hennessy-Milner over determinized transitions
- subset construction NFA $\leadsto$ DFA (Rabin-Scott)

Bottom-up & top-down

$(*)$ over any stretch of time, any number of processes may run, some interfering with others.