Types from frames as finite automata

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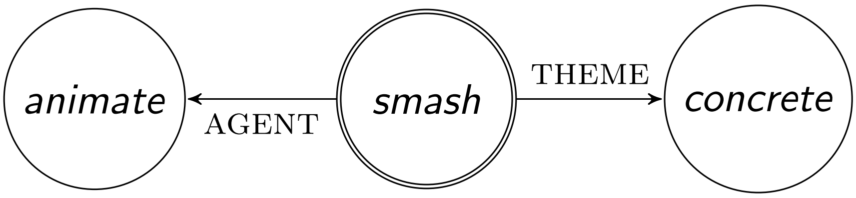


$$\begin{bmatrix} \textit{smash} & \\ \text{AGENT} & \textit{animate} \\ \text{THEME} & \textit{concrete} \end{bmatrix} \qquad \{\textit{smash},\ \text{AGENT}\ \textit{animate},\ \text{THEME}\ \textit{concrete}\}$$

Types?

Petersen (sortal)

$$\lambda e\ (\textit{smash}(e) \wedge \textit{animate}(\text{AGENT}(e)) \wedge \textit{concrete}(\text{THEME}(e))) \qquad \{\textit{smash},\ \text{AGENT}\ \textit{animate},\ \text{THEME}\ \textit{concrete}\}$$

Or Muskens

$$\lambda yx\ \lambda f\ \exists e\ [\textit{smash}\ e \circ \text{AGENT}\ ex \circ \text{THEME}\ ey]f$$

$$[\![L]\!] := \bigcap_{s \in L} \text{domain}([\![s]\!]) \qquad \text{domain}([\![\textit{smash}]\!]) \cap \text{domain}([\![\text{AGENT}]\!];[\![\textit{animate}]\!]) \cap \text{domain}([\![\text{THEME}]\!];[\![\textit{concrete}]\!])$$

$$[\![\epsilon]\!] := \lambda x.x$$

$$[\![sa]\!] := [\![s]\!];[\![a]\!] = \lambda x.[\![a]\!]([\![s]\!](x))$$

Record types?

$$\begin{bmatrix} \text{AGENT} & = & b \\ \text{THEME} & = & c \end{bmatrix} : \begin{bmatrix} \text{AGENT} & : & \mathit{animate} \\ \text{THEME} & : & \mathit{concrete} \end{bmatrix}$$

iff $b : \mathit{animate}$ and $c : \mathit{concrete}$

P $\lambda e\ (\mathit{smash}(e)\ \wedge\ \mathit{animate}(\text{AGENT}(e))\ \wedge\ \mathit{concrete}(\text{THEME}(e)))$

Cooper's meaning function $(\lambda r : bg)\ \varphi$ with

- background $bg = \begin{bmatrix} \text{AGENT} & : & \mathit{Ind} \\ \text{THEME} & : & \mathit{Ind} \end{bmatrix}$ (presuppositions)
- type $\varphi = \begin{bmatrix} p_1 & : & \mathit{smash}(r) \\ p_2 & : & \mathit{animate}(r.\text{AGENT}) \\ p_3 & : & \mathit{concrete}(r.\text{THEME}) \end{bmatrix}$ dependent on $r : bg$

$bg\ \approx$ signature/state $q = \{\text{AGENT},\ \text{THEME},\ \epsilon\}$

$\varphi\ \approx$ language $\{\mathit{smash},\ \text{AGENT}\ \mathit{animate},\ \text{THEME}\ \mathit{concrete}\} \cup q$

Finite-state calculus

- Minimal DFA (Myhill-Nerode) via Brzozowski derivatives L_a

$$L_a\ :=\ \{s \mid as \in L\}$$

- Identity as indiscernibility (Leibniz) wrt Hennessy-Milner 1985 (Blackburn 1993)

$$L\ =\ \sum_{a\in\Sigma} aL_a + o(L) \quad \text{(Taylor series – Conway)}$$

- Open-endedness of signatures (institution, Goguen & Burstall)

$$\mathbf{Sign}\ =\ \int \mathcal{Q} \quad \text{(Grothendieck construction)}$$

- Link frames with timelines as strings (runs of automata)

Open-endedness

(1) Jones did it slowly, deliberately, in the bathroom, with a knife, at midnight. (Davidson 1967)

(2) $\begin{bmatrix} \text{AGENT} & \textit{jones} \\ \text{HOW} & \begin{bmatrix} \textit{slow} \\ \textit{deliberate} \end{bmatrix} \\ \text{WHERE} & \begin{bmatrix} \textit{bathroom} \end{bmatrix} \\ \text{WHEN} & \begin{bmatrix} \textit{midnight} \end{bmatrix} \\ \text{WITH-WHAT} & \begin{bmatrix} \textit{knife} \end{bmatrix} \end{bmatrix}$

(3) $\begin{bmatrix} a_1 & q_1 \\ & \vdots \\ a_k & q_k \end{bmatrix}$ $\qquad q_0 \overset{a_i}{\rightsquigarrow} q_i$

- set Σ of labels a
- relations $\overset{a}{\rightsquigarrow} \subseteq Q \times Q$ for $a \in \Sigma$

Hennessy-Milner & traces

Σ*-deterministic system* $\delta : Q \times \Sigma \rightharpoonup Q$ $\qquad q \xrightarrow{a} \delta(q,a)$

$trace_\delta(q) \; := \; \mathrm{domain}(\delta_q) \; \subseteq \; \Sigma^*$ where $\qquad \downarrow a'$

$$\delta_q : \Sigma^* \rightharpoonup Q, \; \epsilon \mapsto q, \quad sa \mapsto \delta(\delta_q(s), a) \qquad \delta_q(aa')$$

$$(\Phi_\Sigma) \qquad \varphi \; ::= \; \top \mid \langle a \rangle \varphi \mid \varphi \wedge \varphi' \mid \neg\varphi \qquad (a \in \Sigma)$$

$$\begin{aligned} q \models \langle a \rangle \varphi \quad & \text{iff} \quad (q,a) \in \mathrm{domain}(\delta) \text{ and } \delta(q,a) \models \varphi \\ & \text{iff} \quad a \in trace_\delta(q) \text{ and } \delta_q(a) \models \varphi \end{aligned}$$

$$\begin{aligned} \langle \epsilon \rangle \varphi \; &:= \; \varphi \\ \langle as \rangle \varphi \; &:= \; \langle a \rangle \langle s \rangle \varphi \end{aligned}$$

$$q \models \langle s \rangle \varphi \quad \text{iff} \quad s \in trace_\delta(q) \text{ and } \delta_q(s) \models \varphi$$

Identity of indiscernibles (Leibniz)

$$trace_\delta(q) \; = \; \{s \in \Sigma^* \mid q \models \langle s \rangle \top\}$$

Does $\models$ depend on more than $trace_\delta(q)$?

$$\begin{aligned} \Phi_\Sigma(q) \; &:= \; \{\varphi \in \Phi_\Sigma \mid q \models \varphi\} \\ trace_\delta(q) \; &= \; \{s \in \Sigma^* \mid \langle s \rangle \top \in \Phi_\Sigma(q)\} \end{aligned}$$

Fact. $\qquad \Phi_\Sigma(q) = \Phi_\Sigma(q') \quad \text{iff} \quad trace_\delta(q) = trace_\delta(q')$

Holds also with Φ_Σ closed under $\Diamond$ where

$$\begin{aligned} q \models \Diamond\varphi \quad & \text{iff} \quad (\exists s \in trace_\delta(q)) \; \delta_q(s) \models \varphi \\ \Diamond\varphi \quad & \approx \bigvee_{s \in trace_\delta(q)} \langle s \rangle \varphi \end{aligned}$$

Components as derivatives (Brzozowski)

$$L_s \;:=\; \{s' \mid ss' \in L\}$$

$$\begin{aligned} L_\epsilon &= L \\ L_{sa} &= (L_s)_a \end{aligned}$$

$$L \;=\; \{s \mid \epsilon \in L_s\}$$

$$\begin{aligned} aa'a'' \in L \;\; &\text{iff} \;\; a'a'' \in L_a \\ &\text{iff} \;\; a'' \in L_{aa'} \\ &\text{iff} \;\; \epsilon \in L_{aa'a''} \end{aligned}$$

$$L_\epsilon \xrightarrow{a} L_a \xrightarrow{a'} L_{aa'} \xrightarrow{a''} L_{aa'a''}$$

Minimal DFA & finality

For all $s, s' \in \Sigma^*$ and $L \subseteq \Sigma^*$,

$$L_s = L_{s'} \;\;\text{iff}\;\; (\forall w \in \Sigma^*)\; (sw \in L \text{ iff } s'w \in L)$$

so that the **Myhill-Nerode Theorem** says:

L is regular iff $\{L_s \mid s \in \Sigma^*\}$ is finite.

Finality: given a relation $\rightsquigarrow \subseteq Q \times \Sigma \times Q$ and $q \in Q$, let

$$L \;:=\; \{a_1 \cdots a_n \in \Sigma^* \mid q \in \text{domain}(\overset{a_1}{\rightsquigarrow}; \overset{a_2}{\rightsquigarrow}; \cdots ; \overset{a_n}{\rightsquigarrow})\}$$

for a unique morphism to $\{L_s \mid s \in L\}$

$$\bigcup_{a_1 \cdots a_n \in \Sigma^*} \{(q', L_{a_1 \cdots a_n}) \mid q \;\overset{a_1}{\rightsquigarrow}; \overset{a_2}{\rightsquigarrow}; \cdots ; \overset{a_n}{\rightsquigarrow}\; q'\}$$

from the subset of Q accessible from q via $\rightsquigarrow$.

Prefix-closed languages & coderivatives

Fact. *For* $L \subseteq \Sigma^*$,

$$L \;=\; \sum_{a\in\Sigma} aL_a + o(L) \qquad \text{where} \;\; o(L) \;:=\; \begin{cases} \epsilon & \text{if } \epsilon \in L \\ \emptyset & \text{otherwise} \end{cases}$$

and the following are equivalent

(i) $L = trace_\delta(q)$ *for some* $\delta : Q \times \Sigma \rightharpoonup Q$ *and* $q \in Q$

(ii) L *is prefix-closed* ($s \in L$ *whenever* $sa \in L$) *and non-empty*

(iii) $\epsilon \in L = pref(L)$ *where*

$$pref(L) \;:=\; \{s \mid L_s \neq \emptyset\}.$$

a-coderivative of L $\qquad {}_aL \;:=\; \{s \mid sa \in L\}$

Fact. *For any* $L \subseteq \Sigma^*$ *and* $a \notin \Sigma$,

$$L \;=\; {}_a\mathrm{pref}(La).$$

Types as formulas

Encode a type t as $\mathit{wff}(t)$ — e.g., $\langle a_t \rangle \top$
a particular a as singleton type $\{a\}$

$$\begin{aligned}\text{subtype}_\Sigma(t, t') &:= \{\neg\langle s\rangle(\mathit{wff}(t) \wedge \neg \mathit{wff}(t')) \mid s \in \Sigma^*\} \\ &\equiv \neg\Diamond(\mathit{wff}(t) \wedge \neg \mathit{wff}(t')) \\ &\equiv \Box(\mathit{wff}(t) \supset \mathit{wff}(t')) \quad \text{subtype as entailment}\end{aligned}$$

$$\text{in}_\Sigma(a, t) := \text{subtype}_\Sigma(\{a\}, t) \cup \underbrace{\text{nominal}_\Sigma(\mathit{wff}(\{a\}))}_{\text{sortal presupposition in Hybrid Logic}}$$

$$\begin{aligned}\text{nominal}_\Sigma(\varphi) &:= \{\neg(\langle s'\rangle(\varphi \wedge \langle s\rangle\top) \wedge \langle s''\rangle(\varphi \wedge \neg\langle s\rangle\top)) \mid s, s', s'' \in \Sigma^*\} \\ &\equiv \{\Diamond(\varphi \wedge \psi) \supset \Box(\varphi \supset \psi) \mid \psi \in \Phi_\Sigma\}\end{aligned}$$

Singletons, terminals & record labels

For $L \subseteq \Sigma^*$ with $a_L \notin \Sigma$

$$\begin{aligned}\text{singleton}_\Sigma(L) := \; &\{\Box(\langle a_L\rangle\top \supset \langle s\rangle\top) \mid s \in L\} \cup \\ &\{\Box(\langle a_L\rangle\top \supset \neg\langle s\rangle\top) \mid s \in \Sigma^* - L\}\end{aligned}$$

$$L \quad \mapsto \quad L + a_L$$

$$\begin{aligned}\text{terminal}_\Sigma(a) &:= \{\neg\langle sab\rangle\top \mid s \in \Sigma^* \text{ and } b \in \Sigma\} \\ &\equiv \bigwedge_{b \in \Sigma} \neg\Diamond\langle ab\rangle\top\end{aligned}$$

$$\begin{aligned}&\langle \mathit{smash}\rangle\top \wedge \\ &\langle \text{AGENT}\rangle\langle \mathit{animate}\rangle\top \wedge \\ &\langle \text{THEME}\rangle\langle \mathit{concrete}\rangle\top\end{aligned} \qquad \begin{bmatrix} \mathit{smash} & \\ \text{AGENT} & \mathit{animate} \\ \text{THEME} & \mathit{concrete}\end{bmatrix}$$

$$L \quad \mapsto \quad \textstyle\bigwedge_{s \in L}\langle s\rangle\top$$

Record types from relations

$$\begin{bmatrix} \text{x} & : & \mathit{Real} \\ \text{loc} & : & \mathit{Loc} \\ \text{e} & : & \text{temp(loc,x)} \end{bmatrix} \qquad \begin{array}{l} [\![\text{temp(loc,x)}]\!]_r = \\ \{(c, \surd) \mid [\![\text{temp}]\!]([\![\text{loc}]\!]_r, [\![\text{x}]\!]_r, c)\} \end{array}$$

$$\mathcal{L}(\begin{bmatrix} \text{x} & : & \mathit{Real} \\ \text{loc} & : & \mathit{Loc} \\ \text{e} & : & \text{temp(loc,x)} \end{bmatrix}) \quad = \quad \begin{array}{l} \text{x}\,\mathcal{L}(\mathit{Real}) + \\ \text{loc}\,\mathcal{L}(\mathit{Loc}) + \\ \text{e}\,\mathcal{L}(\text{temp(loc,x)}) \ + \ \epsilon \end{array}$$

with

$$T \in \mathcal{L}(T) \quad \text{for } T \in \{\mathit{Real},\ \mathit{Loc},\ \text{temp(loc,x)}\}$$

and

$$\mathcal{L}(\begin{bmatrix} \text{x} & : & \mathit{Real} \\ \text{loc} & : & \mathit{Loc} \\ \text{e} & : & \text{temp(loc,x)} \\ \text{l} & : & R \end{bmatrix}) \quad = \quad \begin{array}{l} \text{x}\,\mathcal{L}(\mathit{Real}) + \\ \text{loc}\,\mathcal{L}(\mathit{Loc}) + \\ \text{e}\,\mathcal{L}(\text{temp(loc,x)}) + \\ \text{l}\,\mathcal{L}(R) \ + \ \epsilon \end{array}$$

A monster $\mathcal{A}$-deterministic system $\hat{\delta}$

$$\mathrm{Fin}(\mathcal{A}) \;:=\; \{\Sigma \subseteq \mathcal{A} \mid \Sigma \text{ is finite}\}$$

For $X \in \mathrm{Fin}(\mathcal{A}) \cup \{\mathcal{A}\}$,

an X-*state* is a non-empty prefix-closed subset q of X^*

$\hat{\delta} = \{(q, a, q_a) \mid q \text{ is an } \mathcal{A}\text{-state and } a \in q \cap \mathcal{A}\}$
making $\hat{\delta}_q = \{(s, q_s) \mid s \in q\}$

$$(sen(\Sigma)) \quad \varphi \;::=\; \top \mid \langle a\rangle\varphi \mid \varphi \wedge \varphi' \mid \neg\varphi \mid \Diamond_Y\varphi \qquad (a \in \Sigma,\;\; Y \subseteq \Sigma)$$

$$q \models \Diamond_Y\varphi \quad \text{iff} \quad (\exists s \in q \cap Y^*)\; q_s \models \varphi$$

Shorten $\Diamond_\Sigma$ to $\Diamond$

Σ-reducts for satisfaction

For $\Sigma \subseteq \Sigma' \in \mathrm{Fin}(\mathcal{A})$ and Σ'-state q,

$$sen(\Sigma) \subseteq sen(\Sigma')$$
$$q \cap \Sigma^* \text{ is a } \Sigma\text{-state}$$
$$q_s \text{ is a } \Sigma'\text{-state}, \quad \text{for } s \in q$$

Fact. *For every $\Sigma \in Fin(\mathcal{A})$, $\varphi \in sen(\Sigma)$ and $\mathcal{A}$-state q,*

$$q \models \varphi \quad \textit{iff} \quad q \cap \Sigma^* \models \varphi$$

and if, moreover, $s \in q \cap \Sigma^*$, then

$$q \models \langle s\rangle\varphi \quad \textit{iff} \quad (q \cap \Sigma^*)_s \models \varphi.$$

The functor $\mathcal{Q} : \mathsf{Fin}(\mathcal{A})^{op} \to \mathbf{Cat}$

For $\Sigma \in \mathsf{Fin}(\mathcal{A})$,

$\mathcal{Q}(\Sigma)$ is the category with
object non-empty prefix-closed $q \subseteq \Sigma^*$
morphisms (q, s) from q to q_s, for $q \in |\mathcal{Q}(\Sigma)|$ and $s \in q$
$(q, s); (q_s, s') = (q, ss')$ with identities (q, ϵ)

$\mathcal{Q}(\Sigma', \Sigma) : \mathcal{Q}(\Sigma') \to \mathcal{Q}(\Sigma)$ for $\Sigma \subseteq \Sigma' \in \mathsf{Fin}(\mathcal{A})$
$q \mapsto q \cap \Sigma^*$
$(q, s) \mapsto (q \cap \Sigma^*, \pi_\Sigma(s))$
where $\pi_\Sigma(s)$ is the longest prefix of s in Σ^*

$$\pi_\Sigma(\epsilon) := \epsilon$$

$$\pi_\Sigma(as) := \begin{cases} a\,\pi_\Sigma(s) & \text{if } a \in \Sigma \\ \epsilon & \text{otherwise.} \end{cases}$$

$\int \mathcal{Q}$ (Grothendieck) & institutions (Goguen & Burstall)

$\mathbf{Sign}^{op} = \int \mathcal{Q}$

- objects (Σ, q) where $\Sigma \in \mathsf{Fin}(\mathcal{A})$ and $q \in |\mathcal{Q}(\Sigma)|$
- morphisms from (Σ', q') to (Σ, q) are pairs

$$((\Sigma', \Sigma), (q'', s))$$

of $\mathsf{Fin}(\mathcal{A})^{op}$-morphisms (Σ', Σ) and
$\mathcal{Q}(\Sigma)$-morphisms (q'', s) s.t. $q'' = q' \cap \Sigma^*$ and $q = q''_s$

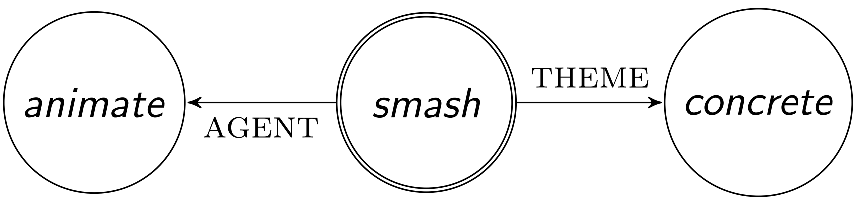
$sen : \mathbf{Sign} \to \mathbf{Set}$

- $sen(\Sigma, q) := sen(\Sigma)$
- $sen((\Sigma', \Sigma), (q'', s)) : \ \varphi \mapsto \langle s \rangle \varphi$

$Mod : \mathbf{Sign}^{op} \to \mathbf{Cat}$

- $|Mod(\Sigma, q)| := \{q' \in |\mathcal{Q}(\Sigma)| \ : \ q \subseteq q'\}$
- $Mod((\Sigma', \Sigma), (q'', s)) : \hat{q} \mapsto (\hat{q} \cap \Sigma^*)_s$

Back to *smash*



P $\quad \lambda e\ (smash(e)\ \wedge\ animate(\textsc{agent}(e))\ \wedge\ concrete(\textsc{theme}(e)))$

C $$(\lambda r : \underbrace{\begin{bmatrix} \textsc{agent} & : & Ind \\ \textsc{theme} & : & Ind \end{bmatrix}}_{bg})\ \underbrace{\begin{bmatrix} p_1 & : & smash(r) \\ p_2 & : & animate(r.\textsc{agent}) \\ p_3 & : & concrete(r.\textsc{theme}) \end{bmatrix}}_{\varphi}$$

$$bg \ \approx\ \text{signature} \begin{cases} \Sigma = \{\textsc{agent},\ \textsc{theme},\ smash,\ animate,\ concrete\} \\ q = \{\textsc{agent},\ \textsc{theme},\ \epsilon\} \end{cases}$$

$$\varphi\ \approx\ \text{language}\ \ \{smash,\ \textsc{agent}\ animate,\ \textsc{theme}\ concrete\} \cup q$$

Main ideas

- Centralized abstraction
  - from DFA's initial state
- Identity of indiscernibles (Leibniz)
  - relativize to finite set Σ of attributes
- Open-endedness
  - let Σ vary over $\mathrm{Fin}(\mathcal{A})$ within an institution
- Run many finite automata $\rightsquigarrow$ timeline

$$\frac{\text{causal}}{\text{temporal}} \approx \frac{\text{mechanism}}{\text{timeline}} \approx_\Sigma \frac{\text{language}}{\text{string}}$$

$$\approx_\Sigma \frac{\text{type}}{\text{particular}} \approx_\Sigma \frac{\text{generic}}{\text{episodic}}$$

See: Tense & aspect chapter of Lappin & Fox's Semantics Handbk
ESSLLI course: Finite-state methods for subatomic semantics

Strings & mechanisms

months in a year	|Jan|Feb| $\cdots$ |Dec|		
+ d1,d2,… d31	|Jan,d1|Jan,d2| $\cdots$ |Jan,d31|Feb,d1| $\cdots$ |Dec,d31|		
		tense	aspect
`it rained`	|E,R| |S|	|R| |S|	|E,R|
`it has rained`	|E| |R,S|	|R,S|	|E| |R|
`soup cool in an hour`	| $x, d \leq sDg$ | $d \leq sDg$ | $\mathrm{hour}(x), sDg < d$ |		
`soup cool for an hour`	| x | $[\sqsupseteq]sDg_\downarrow$ | $\mathrm{hour}(x), [\sqsupseteq]sDg_\downarrow$ |		

Barsalou 1999 (2008: situated simulation)

two levels of structure are proposed: a deep set of generating mechanisms produces an infinite set of surface images. . . . Mental models tend not to address the underlying generative mechanisms that produce a family of related simulations.