Compositionality Inductively, Co-inductively and Contextually

Tim Fernando

To say that the meaning $[\![a]\!]$ of a term a is given by the meanings of a's parts and how these parts are combined is to state an equality

$$[\![a]\!] \;=\; \ldots [\![b]\!] \ldots \quad \text{for } b \text{ a part of } a \tag{1}$$

with the meaning function $[\![\cdot]\!]$ appearing on both sides. (1) is commonly construed as a prescription for computing the meaning of a based on the parts of a and their mode of combination. As equality is symmetric, however, we can also read (1) from right to left, as a constraint on the meaning $[\![b]\!]$ of a term b that brings in the wider context where b may occur, in accordance with what Dag Westerståhl has recently described as "one version of Frege's famous Context Principle"

> the meaning of a term is the contribution it makes to the meanings of complex terms of which it is a constituent. (Westerståhl, 2004, p.3)

That is, if reading (1) left-to-right suggests breaking a term apart (and delving inside it), then reading (1) right-to-left suggests merging it with other terms (and exploring its surroundings). These complementary perspectives on (1) underly inductive and co-inductive aspects of compositionality (respectively), contrasted below by

(i) reviewing the co-inductive approach to the *Fregean covers* of Hodges (2001) anticipated in Fernando (1997)

and by

(ii) inductively deriving a more recent theorem of (Westerståhl, 2004) on the extensibility of compositional semantics closed under subterms.

Choosing between inductive and co-inductive approaches to (1) does not, by itself, determine the meaning function $[\![\cdot]\!]$. The ellipsis in (1) points to a broader

Address for correspondence: Computer Science Department, Trinity College, Dublin 2, Ireland.
E-mail: Tim.Fernando@tcd.ie.

The Compositionality of Meaning and Content. Volume I: Foundational Issues.
Edited by Markus Werning, Edouard Machery, & Gerhard Schurz.

notion of context capturing background assumptions that shape $[\![\cdot]\!]$. To square (1) with "dynamic" conceptions of meaning as context change (e.g. Heim, 1983), we shall inject a certain notion of context c inside meanings, and not simply hang them outside $[\![\cdot]\!]$ as subscripts, $[\![\cdot]\!] = [\![\cdot]\!]_{\mathsf{c}}$.

We proceed below as follows. Section 1 provides some basic background for sections 2 and 3, where the aforementioned inductive and co-inductive applications to compositionality are then described. Section 4 turns to context change, before section 5 concludes.

1 Background: Congruences and Extensions

The present section records useful definitions and facts reducing meaning functions $[\![\cdot]\!]$ to synonymy relations. We begin by assuming that every element a of some fixed set T is assigned a meaning $[\![a]\!]$, before relaxing this assumption and considering the possibility of extending meaning assignments compositionally.

Given an n-ary function $f : T^n \to T$ on T, a function $[\![\cdot]\!] : T \to M$ is *f-compositional* if there is a function $[\![f]\!] : M^n \to M$ allowing us to push $[\![\cdot]\!]$ inward so that

$$[\![f(a_1,\ldots,a_n)]\!] \;=\; [\![f]\!]([\![a_1]\!],\ldots,[\![a_n]\!])$$

for all $a_1,\ldots,a_n \in T$. An *f-congruence* is an equivalence relation $\equiv$ on T such that $f(a_1,\ldots,a_n) \equiv f(b_1,\ldots,b_n)$ whenever $a_i \equiv b_i$ for $1 \le i \le n$ — that is,

$$\frac{a_1 \equiv b_1 \quad \cdots \quad a_n \equiv b_n}{f(a_1,\ldots,a_n) \equiv f(b_1,\ldots,b_n)}$$

for all $a_1,\ldots,a_n,b_1,\ldots,b_n \in T$.

Given a family $\mathscr{F}$ of multi-ary functions (i.e. functions of various arities) on T, an *$\mathscr{F}$-congruence* is a binary relation on T that is an f-congruence for every $f \in \mathscr{F}$. Similarly, a function $[\![\cdot]\!] : T \to M$ is *$\mathscr{F}$-compositional* if $[\![\cdot]\!]$ is *f-compositional* for every $f \in \mathscr{F}$. The *kernel* of $[\![\cdot]\!]$ is the set

$$\kappa[\![\cdot]\!] \;=\; \{(a,b) \in T \times T \;:\; [\![a]\!] = [\![b]\!]\}$$

of $[\![\cdot]\!]$-synonymous pairs from T. It is well-known that

Fact 1. *$\kappa[\![\cdot]\!]$ is an equivalence relation on T, and moreover,*

$$[\![\cdot]\!] \text{ is } \mathscr{F}\text{-compositional} \quad \text{iff} \quad \kappa[\![\cdot]\!] \text{ is an } \mathscr{F}\text{-congruence.}$$

That is, the compositionality of a function $[\![\cdot]\!] : T \to M$ reduces to testing that $\kappa[\![\cdot]\!]$ is a congruence. Indeed, we may assume that meanings are simply subsets of T insofar as any binary relation $\equiv$ on T induces the "term[1] model" $\cdot^{\equiv} : T \to \text{Pow}(T)$ mapping $a \in T$ to its $\equiv$-equivalence class

$$a^{\equiv} \;=\; \{b \in T \;:\; a \equiv b\}$$

from which it follows that

$$\kappa[\![\cdot]\!] \;=\; \kappa(\cdot^{\kappa[\![\cdot]\!]}) \;.$$

We will make do in sections 2 and 3 with equivalences on T, returning to meanings in section 4.

But first, let us partialize the preceding notions as follows. Fix a partial n-ary map $\alpha : T^n \rightharpoonup T$. A partial map $[\cdot] : T \rightharpoonup M$ is *α-compositional* if there is a function $[\alpha] : M^n \to M$ such that for all $(a_1, \ldots, a_n) \in \text{domain}([\cdot])^n \cap \text{domain}(\alpha)$ for which $\alpha(a_1, \ldots, a_n) \in \text{domain}[\cdot]$,

$$[\alpha(a_1, \ldots, a_n)] \;=\; [\alpha]([a_1], \ldots, [a_n]) \;.$$

Given a subset $X \subseteq T$ and an n-tuple $\vec{a} \in T^n$, let

$$\mathsf{d}^X{}_\alpha(\vec{a}) \quad \text{iff} \quad \vec{a} \in X^n \cap \text{domain}(\alpha) \text{ and } \alpha(\vec{a}) \in X \;.$$

An (α, X)-*congruence* is an equivalence relation $\equiv$ on X such that

$$\frac{a_1 \equiv b_1 \quad \cdots \quad a_n \equiv b_n}{\alpha(a_1, \ldots, a_n) \equiv \alpha(b_1, \ldots, b_n)} \; \mathsf{d}^X{}_\alpha(\vec{a}),\; \mathsf{d}^X{}_\alpha(\vec{b})$$

for all $a_1, \ldots, a_n, b_1, \ldots, b_n \in T$, where $\vec{a} = (a_1, \ldots, a_n)$ and $\vec{b} = (b_1, \ldots, b_n)$. Given a set Σ of partial multi-ary maps on T, a (Σ, X)-*congruence* is an (α, X)-congruence for every $\alpha \in \Sigma$; and $[\cdot]$ is Σ-*compositional* if it is α-compositional for every $\alpha \in \Sigma$. Fact 1 generalizes to

Fact 2.

$$[\cdot] \textit{ is } \Sigma\textit{-compositional} \quad \text{iff} \quad \kappa[\cdot] \textit{ is a } (\Sigma, \textit{domain}[\cdot])\textit{-congruence.}$$

Henceforth, we write Σ-congruence for (Σ, T)-congruence, and d_α for $\mathsf{d}^T{}_\alpha$.

Next, we introduce some terminology for comparing binary relations $\equiv$ and $\equiv'$ on T. We say $\equiv$ *refines* $\equiv'$ if $\equiv \subseteq \equiv'$, as the contrapositive

$$a \not\equiv b \quad \text{whenever} \quad a \not\equiv' b$$

states $\equiv$ respects all the distinctions $\equiv'$ makes, so that $\equiv$ is at least as *fine* as $\equiv'$, and $\equiv'$ at least as *coarse* as $\equiv$. For the term model $\cdot^{\equiv}$ of $\equiv$ to be a restriction of

[1] It is tempting to equate T with the set of terms generated by $\mathscr{F}$, although we will *not* need the assumption that the elements of T are terms before section 2.

the term model $\cdot^{\equiv'}$ of $\equiv'$, we need to strengthen the inclusion $\equiv \subseteq \equiv'$ a bit. Let us say $\equiv$ *extends to* $\equiv'$ if

$$\text{for all } a,b \in \text{domain}(\equiv), \quad a \equiv' b \text{ iff } a \equiv b$$

in which case we call $\equiv'$ an *extension of* $\equiv$. Clearly,

$$\equiv \text{ extends to } \equiv' \quad \text{iff} \quad \cdot^{\equiv} \subseteq \cdot^{\equiv'} .$$

Given $X \subseteq T$, let us call a (Σ, X)-congruence T-*extensible* if it extends to a Σ-congruence. In the next section, we consider the question: when is a (Σ, X)-congruence T-extensible?

2 Finest Extensions and Subterm Extensibility *Inductively*

Read from left to right, equation (1) in the introduction above suggests a subterm property that very roughly says:

> to decide if a and a' are synonymous (i.e., they have the same meaning), it suffices to consider subterms of a and a', and how they combine to yield a and a', respectively.

The present section makes this suggestion precise, fixing, as in the previous section, a family Σ of partial multi-ary functions α on T. Given a binary relation $\equiv$ on T, let $\equiv_\Sigma$ be the set of all pairs $(a,b) \in T \times T$ such that $a \doteq b$[2] is derivable from any finite number of applications of

(i) the $\equiv$-rule

$$\frac{}{a \doteq b}\; a \equiv b$$

guaranteeing that $\equiv_\Sigma$ contains $\equiv$

(ii) the $(\dagger)$-rule

$$\frac{a \doteq b \quad b \doteq c}{a \doteq c}$$

making $\equiv_\Sigma$ transitive, and

(iii) the α-rules

$$\frac{a_1 \doteq b_1 \quad \cdots \quad a_n \doteq b_n}{\alpha(a_1, \ldots, a_n) \doteq \alpha(b_1, \ldots, b_n)}\; \mathsf{d}_\alpha(\vec{a}),\ \mathsf{d}_\alpha(\vec{b})$$

for n-ary $\alpha \in \Sigma$ $(n \geq 0)$, formalizing the closure condition turning an equivalence relation into a Σ-congruence.

[2]We are borrowing here the dot notation used by Feferman to distinguish syntactic relations from the arithmetic relations they denote.

It is not difficult to see that

Lemma 3. *If $\equiv$ is an equivalence relation on T, then $\equiv_\Sigma$ is the finest Σ-congruence refined by $\equiv$ (that is, the $\subseteq$-least Σ-congruence containing $\equiv$).*

While $\equiv$ refines $\equiv_\Sigma$, we cannot assume $\equiv$ extends to $\equiv_\Sigma$. Nevertheless, the construction of $\equiv_\Sigma$ from $\equiv$ leads to a natural approach to answering the question:

when is a (Σ, X)-congruence T-extensible?

By Lemma 3, a (Σ, X)-congruence $\approx$ extends to some Σ-congruence iff $\approx$ extends to $\equiv_\Sigma$, where $\equiv$ is the union

$$\approx \;\cup\; \{(a,a) : a \in T\}$$

of $\approx$ with identity on T. But the question remains: when does $\approx$ extend to $\equiv_\Sigma$?

Additional assumptions on T and Σ will prove useful. We assume a distinct symbol $\dot{\alpha}$ can be associated with each $\alpha \in \Sigma$ such that T is the set of $\{\dot{\alpha} : \alpha \in \Sigma\}$-terms[3] and each α is a restriction of the map $(t_1, \ldots, t_n) \mapsto$ '$\dot{\alpha}(t_1, \ldots, t_n)$', allowing us to confuse $\alpha(t_1, \ldots, t_n)$ with '$\dot{\alpha}(t_1, \ldots, t_n)$' whenever $(t_1, \ldots, t_n) \in$ domain(α). The main result of (Westerståhl, 2004) is

Theorem W. *A (Σ, X)-congruence is T-extensible if X is closed under subterms (that is, $t_i \in X$ for $1 \le i \le n$, whenever $\alpha(t_1, \ldots, t_n) \in X$).*

For the remainder of this section, let us assume X is closed under subterms, $\approx$ is a (Σ, X)-congruence, and $\equiv$ is $\approx \cup \{(a,a) : a \in T\}$. Westerståhl (2004) extends $\approx$ to a Σ-congruence different from $\equiv_\Sigma$. In view of Lemma 3, however, Theorem W says no more and no less than:

$$\text{for all } a, b \in X, \qquad a \approx b \;\text{ iff }\; a \equiv_\Sigma b \tag{2}$$

(under the aforementioned assumptions on $\approx$ and X). (2) formulates Theorem W as a conservative extension claim about the formal system defining $\equiv_\Sigma$ above. Observe that the transitivity rule $(\dagger)$ is the only rule in the system whose premises may include terms which are subterms of *neither* terms in the conclusion. That is, a $(\dagger)$-free derivation of $a \dot{=} b$ can only employ subterms of a or of b. Eliminating $(\dagger)$ is the key to (2), just as eliminating Cut is to many conservative extension arguments in proof theory.

The left-to-right direction $\Rightarrow$ of (2) is an immediate consequence of the $\equiv$-rule and the inclusion $\approx \subseteq \equiv$. To establish the converse, $\Leftarrow$, let us define

$$a \dot{=}_k b \quad \text{iff} \quad \text{'}a \dot{=} b\text{' can be derived in } \le k \text{ steps.}$$

[3]That is, T is generated inductively from Σ by the rule: for n-ary $\alpha \in \Sigma$ and $t_1, \ldots, t_n \in T$, the term '$\dot{\alpha}(t_1, \ldots, t_n)$' belongs to T (beginning with $n = 0$, treating atoms as 0-ary maps in Σ).

The plan is to derive a contradiction from a k-minimal counter-example to $\Leftarrow$. Accordingly, fix a k-length derivation $\mathscr{D}$ of $a \doteq b$ with $a,b \in X$, $a \not\approx b$, and for all $a',b' \in X$ and $k' < k$

$$a' \doteq_{k'} b' \quad \text{implies} \quad a' \approx b' \; .$$

By the minimality of k, the last step of $\mathscr{D}$ must be (†) — say, $a \doteq_{k-1} x$ and $x \doteq_{k-1} b$. Expanding out uses of (†) within $\mathscr{D}$, we can convert the sequence a,x,b to a sequence $t_1 \ldots t_l$ of terms occurring in $\mathscr{D}$ such that $t_1 = a$, $t_l = b$ and for $1 \leq j < l$, $\mathscr{D}$ contains a derivation of $t_j \doteq t_{j+1}$ ending with an instance of the $\equiv$-rule or of an α-rule (for some $\alpha \in \Sigma$). We can rule out the $\equiv$-rule, appealing to k's minimality. As T consists of $\{\dot{\alpha} : \alpha \in \Sigma\}$-terms, it follows that $a = \alpha(a_1 \ldots a_n)$ and $b = \alpha(b_1 \ldots b_n)$ for the same $\alpha \in \Sigma$ and for some $a_1 \ldots a_n, b_1 \ldots b_n \in T$. But X is closed under subterms, so by k's minimality (again), $a_i \approx b_i$. We then obtain the contradiction $\alpha(a_1 \ldots a_n) \approx \alpha(b_1 \ldots b_n)$ from the assumption that $\approx$ is an (α, X)-congruence.

3 Coarsest Refinements and Fregean Covers *Co-inductively*

Lemma 3 is the dual of (the proof of) Theorem 6 in (Fernando, 1997, pp. 592–594), which we briefly sketch below as Lemma 4. This will take us from the subterm property made precise by Theorem W to what Westerståhl (2004) calls "the Contribution Principle" (arguably "one version of Frege's famous Context Principle"). For orientation, let us tabulate the dualities to be fleshed out presently.

subterm property	$\equiv_\Sigma$ is least $\supseteq \equiv$	derivation	bottom-up $\bigcup$
contribution principle	$\equiv^{\mathscr{F}}$ is greatest $\subseteq \equiv$	constraint	top-down $\bigcap$

Given a family $\mathscr{F}$ of multi-ary functions on T and a binary relation $\equiv$ on T, we will define a binary relation $\equiv^{\mathscr{F}}$ on T satisfying

Lemma 4. *If $\equiv$ is an equivalence relation $\equiv$ on T, then $\equiv^{\mathscr{F}}$ is the coarsest $\mathscr{F}$-congruence refining $\equiv$ (that is, the $\subseteq$-largest $\mathscr{F}$-congruence contained in $\equiv$).*

To define $\equiv^{\mathscr{F}}$, a bit of notation is handy. Given a function $g : T \to T$ on T and a binary relation $R \subseteq T \times T$ on T, let R^g be the subset

$$R^g = \{(a,b) \in R \; : \; g(a) \; R \; g(b)\}$$

of R preserved by g. Notice that if $\equiv$ is an equivalence relation,

$$\equiv \text{ is a } g\text{-congruence} \quad \text{iff} \quad \equiv \subseteq \equiv^g$$

and the intersection

$$\equiv \;\cap\; \equiv^{g} \;\cap\; (\equiv^{g})^{g} \;\cap\; ((\equiv^{g})^{g})^{g} \;\cap\; \cdots$$

is the coarsest g-congruence refining $\equiv$. But what do we do if instead of a unary function g, we have an $(n+1)$-ary function $f : T^{n+1} \to T$ with $n > 0$? In that case, we form f's unary projections: given $1 \leq i \leq n+1$ and $\vec{a} \in T^n$, let $f_{i,\tilde{a}} : T \to T$ map $a \in T$ to

$$f_{i,\tilde{a}}(a) \;=\; f((a,\vec{a})_i)$$

where $(a,\vec{a})_i$ is $\vec{a}$ with a inserted at the ith position. (For example, $(a,b)_1 = (a,b)$ and $(a,(b,c))_2 = (b,a,c)$.) Let us collect f's unary projections in

$$\mathscr{U}(f) \;=\; \{f_{i,\tilde{a}} \;:\; 1 \leq i \leq n+1 \text{ and } \vec{a} \in T^n\}.$$

Now, to satisfy Lemma 4, set $\equiv^{\mathscr{F}} = \bigcap_{k \geq 0} \equiv^{\mathscr{F}}{}_k$ where $\equiv^{\mathscr{F}}{}_0$ is $\equiv$ and for $k \geq 0$,

$$\equiv^{\mathscr{F}}{}_{k+1} \;=\; \bigcap_{f \in \mathscr{F}} \bigcap_{g \in \mathscr{U}(f)} (\equiv^{\mathscr{F}}{}_k)^{g} \,.$$

Whereas $\equiv_\Sigma$ (from the previous section) $\bigcup$-collects the conclusions of derivations from a system of rules, $\equiv^{\mathscr{F}}$ $\bigcap$-filters $\equiv$ through constraints given by $\mathscr{F}$. (We can say $\equiv$ is *g-constrained* if $\equiv \subseteq \equiv^{g}$.)

In practice, we will want to apply Lemma 4 to an equivalence relation $\approx$ on a subset X of T. To do so, we let $\equiv$ be the union

$$\approx \;\;\cup\;\; ((T-X) \times (T-X))$$

of $\approx$ *not* with identity on T (as in the previous section) but with $(T-X) \times (T-X)$.[4] As it turns out, $\equiv^{\mathscr{F}}$ exemplifies what Hodges (2001) calls a Fregean cover of $\equiv$. More precisely, let us write $t(a|x)$ with the understanding that t is an $(\mathscr{F} \cup \{x\})$-term, $a \in T$, and $t(a|x) \in T$ is t with x replaced by a.

Definition (Hodges). Given equivalence relations $\approx$ and $\approx'$ on subsets of T, $\approx'$ is a *Fregean cover of* $\approx$ if conditions F(a)-F(c) below hold for $X = \text{domain}(\approx)$.

F(a): if $a \approx' b$ and $t(a|x) \in X$ then $t(b|x) \in X$

[4] If $\approx$ is the kernel of $[\cdot] : X \to M$, this union is the kernel of the 1-point extension $[\cdot]_\perp : T \to M \cup \{\perp\}$ of $[\cdot]$ mapping $a \in T$ to

$$[a]_\perp \;=\; \begin{cases} [a] & \text{if } a \in X \\ \perp & \text{for } a \in T - X \end{cases}$$

for a fixed object $\perp \notin M$.

F(b): if $a \approx' b$ and $t(a|x), t(b|x) \in X$ then $t(a|x) \approx t(b|x)$
F(c): if $a \not\approx' b$ then for some t, $t(a|x) \not\approx t(b|x)$.

As an analogue to Theorem W, we have

Theorem 5. *Given an equivalence relation $\approx$ on a subset X of T, $\equiv^{\mathscr{F}}$ is a Fregean cover of $\approx$, where $\equiv$ is $\approx \cup ((T-X) \times (T-X))$. Moreover, every Fregean cover of $\approx$ extends to $\equiv^{\mathscr{F}}$.*

To prove Theorem 5, let $\approx, X$ and $\equiv$ be as given in the theorem. First, we verify that F(a)-F(c) hold for $\approx'$ equal to $\equiv^{\mathscr{F}}$. Indeed, we can show by induction on the number of occurrences of $\mathscr{F}$-symbols in t that

(i) if $a \equiv^{\mathscr{F}} b$ then $t(a|x) \equiv^{\mathscr{F}} t(b|x)$

and by induction on k, that

(ii) if $a \not\equiv^{\mathscr{F}}{}_k b$ then for some t, $t(a|x) \not\equiv t(b|x)$.

(The inductions bring out the encoding of $t(a|x)$ by iterations of $\cdot^g$ for $g \in \bigcup_{f \in \mathscr{F}} \mathscr{U}(f)$.) Then, given a Fregean cover $\approx'$ of $\approx$, we deduce

$$\text{for all } a, b \in \text{domain}(\approx'), \quad a \not\approx' b \text{ implies not } a \equiv^{\mathscr{F}} b \tag{3}$$

from F(c) and (i), and derive the converse of (3) from F(a), F(b) and (ii).

4 Changing the *Context*

The extensions in Theorems W and 5 of synonymies $\equiv$ to an arbitrary term $a \in T$ fall short of determining *the* meaning $[\![a]\!]$ of a. Term models $a^{\equiv}$ fail to connect language with the reality it describes. Talk of *the* meaning of a presupposes a notion of context, such as in model-theoretic semantics, that given by a model M, underlying the meaning $[a] = [a]_M$ of a. For a concrete example, suppose a_1 were a well-formed formula saying *Pat's spouse is lucky*, and a_2 were a well-formed formula saying *Pat is married*. Relative to a model M where a_2 is false, it is tempting to take a_1 to be meaningless — that is, to leave $[a_1]_M$ undefined. Abstracting over M, however, we might build context into a richer meaning

$$\{[a]\} \;=\; \{(M, [a]_M) \;:\; a \in \text{domain}([\cdot]_M)\}$$

consisting of pairs $(M, [a]_M)$ such that $[a]_M$ is defined. The step from $[a]_M$ to $\{[a]\}$ goes beyond the extensions in the previous sections inasmuch as it may involve different meanings $[\cdot]_M, [\cdot]_{M'}, \ldots$ That said, not only would $\{[a]\}$ always be defined, but following Karttunen (1974), we would have

$$\text{domain}\{[a]\} \;=\; \text{set of contexts satisfying } a\text{'s presuppositions.}$$

A fuller-blooded relational semantics would as in Heim (1983) formulate the meaning $([a])$ of a as a *context change potential*

$$\mathsf{c}_{in}\ ([a])\ \mathsf{c}_{out}$$

between an input context c_{in} and an output context c_{out} incorporating c_{in}. The contexts here can be formulated as in (Martin-Löf, 1984) to implement the presupposition filtering in (a) and (b) below[5] (Ranta, 1994).

(a) *If Pat is married, then Pat's spouse is lucky.*

(b) *Pat is married, and Pat's spouse is lucky.*

Moreover, the type-theoretic approach can be adapted model-theoretically so that the same mechanism for presupposition filtering accounts for the conservativity of generalized quantifiers (Keenan and Stavi, 1986) illustrated by the equivalences in (c) and (d), as well as the binding of the donkey pronoun *it* in (e).

(c) *Every ant bites.*
Every ant is an ant that bites.

(d) *Some ant bites.*
Some ant is an ant that bites.

(e) *Every farmer who owns a donkey beats it.*

The interested reader is referred to (Fernando, 2001) for details, the essential point for the present discussion being that the contextual shift from c to c' in

$$[f(a,b)]_{\mathsf{c}} \;=\; [f]([a]_{\mathsf{c}},[b]_{\mathsf{c}'})$$

can be formulated compositionally by enriching the notions of meaning and context in $[\cdot]_{\mathsf{c}}$, $[\cdot]_{\mathsf{c}'}$.

5 Conclusion

Compositionality can be approached inductively from below (as in section 2) or co-inductively from above (as in section 3). Although meaning may under certain assumptions be preserved by extensions, some applications call for an enrichment of meaning reflecting differences in contexts lying behind different meanings (section 4).

[5]That is, neither (a) nor (b) presupposes *Pat is married*, which is locally presupposed by the constituent *Pat's spouse is lucky*.

References

Fernando, T. (1997). Ambiguity under changing contexts. *Linguistics and Philosophy*, *20*(6), 575–606.

Fernando, T. (2001). Conservative generalized quantifiers and presupposition. In *Proceedings of Semantics and Linguistic Theory XI* (pp. 172–191). Cornell Linguistics Circle Publications.

Heim, I. (1983). On the projection problem for presuppositions. In M. Barlow, D. Flickinger, & M. Westcoat (Eds.), *Proceedings of the West Coast Conference on Formal Linguistics* (Vol. 2, pp. 114–125). Stanford Linguistics Association.

Hodges, W. (2001). Formal features of compositionality. *Journal of Logic, Language and Information*, *10*(1), 7–28.

Karttunen, L. (1974). Presupposition and linguistic context. *Theoretical Linguistics*, 181–194.

Keenan, E., & Stavi, J. (1986). A semantic characterization of natural language determiners. *Linguistics and Philosophy*, *9*(3), 252–326.

Martin-Löf, P. (1984). *Intuitionistic type theory.* Napoli: Bibliopolis.

Ranta, A. (1994). *Type-theoretical grammar.* Oxford University Press.

Westerståhl, D. (2004). On the compositional extension problem. *Journal of Philosophical Logic*, *33*(6), 549–582.