Negation and events as truthmakers

Tim Fernando

Trinity College Dublin, Ireland
Tim.Fernando@tcd.ie

Abstract

Kit Fine's conception of truthmakers as exact verifiers and falsifiers provides an approach to negation that can be tested against the notion that events are truthmakers. Temporally locating events presents complications that call for an account of indices at bounded but refinable granularity. A proposal is outlined that links grain with the choice of a finite alphabet, from which strings are formed representing change under forces. Fusing strings is noted to lead to sets of strings, raising the prospects of fusion as a non-deterministic operation against the prospects of truthmakers as string sets.

1 Introduction

As a logical connective, negation is an operation on propositions that are commonly understood as abstract types, in contrast to concrete particulars such as events that according to Davidson 1967, make action sentences true (page 91). Without restricting truthmakers to concrete particulars, Fine 2015 outlines a "highly general and abstract approach" to truthmaking under which a statement φ is interpreted as a pair $(V(\varphi), F(\varphi))$ of sets $V(\varphi)$ of φ's verifiers and $F(\varphi)$ of φ's falsifiers related by negation $\neg$ according to (1), subject to the intuition (2).

(1) $V(\neg\varphi) = F(\varphi)$ and $F(\neg\varphi) = V(\varphi)$

(2) verification is "exact" — i.e., a verifier of φ is "wholly relevant" to φ

What (1) and (2) might mean for events as truthmakers is explored below through examples of the form

$$A(t) \ := \ \text{Amundsen flew to the North Pole in } t$$

for different times t (e.g., 1926, May 1926). The basic claim of the present work is that exactness in (2) must be relativized to a bounded granularity that is refinable, and that negation must be viewed more broadly than what (1) suggests, encompassing forces for change. (1) reduces verification and falsification of $\neg\neg\varphi$ to φ

$$V(\neg\neg\varphi) = F(\neg\varphi) = V(\varphi) \quad \text{and} \quad F(\neg\neg\varphi) = V(\neg\varphi) = F(\varphi)$$

supporting the law of double negation described in Horn 1989 as a "betrayal of natural language for logical elegance and simplicity" (page 2), in direct opposition to an "asymmetricalist position" that "negative statements are about the positive statements, while affirmatives are directly about the world" (page 3). This latter position becomes tenable once the interpretation of φ is allowed to be more than the pair $(V(\varphi), F(\varphi))$. Indeed, Fine makes such allowances in expanding a *state space* $(S, \leq)$ to a *modalized state space* $(S, S^\diamond, \leq)$, where $V(\varphi)$ and $F(\varphi)$ are defined in $(S, \leq)$ as subsets of a set S of *states*[1] partially ordered by $\leq$, and constraints are encoded as the set $S^\diamond \subseteq S$ of *possible* states satisfying them. We start in the next section with a constraint concerning what Fine refers to as "truth-value gluts" which we ban through inexact truthmaking under classical negation.

[1] The term "state" is used in Fine 2015 with the understanding that it "is a mere term of art and need not be a state in any intuitive sense of the term." That said, the thrust of the present paper is to flesh out how it might be interpreted in linguistic applications appealing to events.

2 No gluts: inexact truthmaking and falsification

Given a partial order $\leq$ on a set S of states, including verifiers of a statement φ, let the set $V(\varphi_{\leq})$ of *inexact verifiers of* φ consist of states with $\leq$-parts that verify φ

$$V(\varphi_{\leq}) \ := \ \{s \in S \mid (\exists s' \in V(\varphi))\ s' \leq s\}.$$

A φ-*glut* is an inexact verifier of both φ and $\neg\varphi$ — i.e., a state in $V(\varphi_{\leq}) \cap V((\neg\varphi)_{\leq})$. To express φ-gluts in terms of $\varphi_{\leq} \wedge (\neg\varphi)_{\leq}$, let us recall Fine's interpretation of conjunction $\wedge$, assuming $\leq$ has least upper bounds given by "fusion" $\sqcup$, used also for disjunction $\vee$, (3).

(3) $V(\varphi \wedge \psi) := \{s \sqcup s' \mid (s, s') \in V(\varphi) \times V(\psi)\}$ and $F(\varphi \wedge \psi) := F(\varphi) \cup F(\psi)$
$V(\varphi \vee \psi) := V(\varphi) \cup V(\psi)$ and $F(\varphi \vee \psi) := \{s \sqcup s' \mid (s, s') \in F(\varphi) \times F(\psi)\}$

(4) $V(\varphi_{\leq} \wedge \psi_{\leq}) = V(\varphi_{\leq}) \cap V(\psi_{\leq})$

From (3), we can derive (4) and equate φ-gluts with verifiers of $\varphi_{\leq} \wedge (\neg\varphi)_{\leq}$

$$V(\varphi_{\leq}) \cap V((\neg\varphi)_{\leq}) \ = \ V(\varphi_{\leq} \wedge (\neg\varphi)_{\leq}).$$

If we try to capture the the complement

$$S - (V(\varphi_{\leq}) \cap V((\neg\varphi)_{\leq}))$$

of φ-gluts by negating $\varphi_{\leq} \wedge (\neg\varphi)_{\leq}$, we find

$$V(\neg(\varphi_{\leq} \wedge (\neg\varphi)_{\leq})) \ = \ F(\varphi_{\leq} \wedge (\neg\varphi)_{\leq}) \ = \ F(\varphi_{\leq}) \cup F((\neg\varphi)_{\leq})$$

and are led to asking what the set $F(\varphi_{\leq})$ of inexact falsifiers of φ is. Fine 2015 appears to be silent on this question. One possible answer, recorded as (A1) below, is that an inexact falsifier of φ is a state with a $\leq$-part that falsifies φ.

(A1) $F(\varphi_{\leq}) \ := \ \{s \in S \mid (\exists s' \in F(\varphi))\ s' \leq s\} \ = \ V((\neg\varphi)_{\leq})$

(A1) allows us to drop the parentheses in $\neg(\varphi_{\leq})$ and $(\neg\varphi)_{\leq}$

$$V(\neg(\varphi_{\leq})) = F(\varphi_{\leq}) = V((\neg\varphi)_{\leq}) \ \text{ and } \ F(\neg(\varphi_{\leq})) = V(\varphi_{\leq}) = F((\neg\varphi)_{\leq})$$

from which it follows that $\neg(\varphi_{\leq} \wedge (\neg\varphi)_{\leq})$ and $\varphi_{\leq} \vee \neg\varphi_{\leq}$ have the same verifiers

$$\begin{aligned} V(\neg(\varphi_{\leq} \wedge \neg\varphi_{\leq})) \ &= \ F(\varphi_{\leq}) \cup F(\neg\varphi_{\leq}) \ = \ F(\varphi_{\leq}) \cup V(\varphi_{\leq}) \\ &= \ V(\neg\varphi_{\leq}) \cup V(\varphi_{\leq}) \ = \ V(\varphi_{\leq}) \cup V(\neg\varphi_{\leq}) \ = \ V(\varphi_{\leq} \vee \neg\varphi_{\leq}) \end{aligned}$$

and falsifiers (the φ-gluts)

$$\begin{aligned} F(\varphi_{\leq} \vee \neg\varphi_{\leq}) \ &= \ \{s \sqcup s' \mid s \in F(\varphi_{\leq}) \text{ and } s' \in F(\neg\varphi_{\leq})\} \\ &= \ \{s \sqcup s' \mid s \in V(\neg\varphi_{\leq}) \text{ and } s' \in V(\varphi_{\leq})\} \\ &= \ V(\neg\varphi_{\leq} \wedge \varphi_{\leq}) \ = \ F(\neg(\varphi_{\leq} \wedge \neg\varphi_{\leq})). \end{aligned}$$

The equivalence between $\neg(\varphi_{\leq} \wedge \neg\varphi_{\leq})$ and $\varphi_{\leq} \vee \neg\varphi_{\leq}$ conflates "no gluts" with "no gaps" (states verifying or falsifying $\varphi_{\leq}$). While we might tolerate the silence of gaps (in the interest of partiality), the noise of φ-gluts is another matter.

For an alternative to (A1) that keeps gluts separate from gaps, it is instructive to consider the example of events. Let us suppose for the sake of the argument the sequence of events described in (5), (6).

(5) Amundsen flew to the North Pole in May 1926.

(6) Amundsen stayed home in July 1926.

Given (5), we might assert (7), interpreting "in" as "within" (following, for example, Pratt-Hartmann 2005 and Beaver & Condoravdi 2007).

(7) Amundsen flew to the North Pole in 1926.

The step from (5) to (7) points to a satisfaction relation $\models$ between temporal intervals t and statements φ validating the implication (U) for the subinterval relation $\sqsubseteq$.

(U) $t \models \varphi$ and $t \sqsubseteq t' \implies t' \models \varphi$

A sufficient condition for (U) is that $\models$ be inexact verification, (8), for some relation $\leq$ closed under $\sqsubseteq$ according to (U′).

(8) $t \models \varphi \iff t \in V(\varphi_{\leq})$

(U′) $e \leq t$ and $t \sqsubseteq t' \implies e \leq t'$

Adding (6) into the mix would then lead to Amundsen flying to the North Pole and staying home the same year — a somewhat dubious conclusion that only becomes worse when we replace *same year* by *same time*. Talk of *the same time* suggests some notion of temporal extent $\tau(e)$, through which the temporal fit tolerated by (U′) can be tightened as in (E).

(E) $e \leq t \iff \tau(e) = t$

With (E) in place of (U′), (U) no longer falls out of (8). And indeed, the step from (6) to (9) illustrates an inference not upward (U) but downward (D), derivable from a pointwise notion $V_\circ$ of verification according to the account (H) of homogeneity in Taylor 1977 and Dowty 1979.

(9) Amundsen stayed home in the second week of July 1926.

(D) $t \models \varphi$ and $t' \sqsubseteq t \implies t' \models \varphi$

(H) $t \models \varphi \iff t \subseteq V_\circ(\varphi)$ (where $V_\circ(\varphi)$ is a set of time points)

Together, (U) and (D) have the catastrophic consequence that time collapses under the triviality

$$(\exists t)\ t \models \varphi \iff (\forall t)\ t \models \varphi.$$

This collapse can be avoided by treating (5)/(7) separately from (6)/(9) on the basis of a difference in aspect: (5) describes an event that culminates, while (6) describes a state that holds (e.g., Parsons 1990). This separation is threatened by negation; a small step away from (6) is (10).

(10) Amundsen did not fly to the North Pole in July 1926.

(10) lends plausibility to the view that the negation of an event is a state. The view, however, is controversial (see, for instance, Condoravdi 2008, Landman 2006). We can sidestep the controversy by avoiding the connective $\neg$ on φ in (8), and instead negating the full statement "$t \models \varphi$" (over time and $\models$) classically.[2] Complementing (8), we introduce a counter-satisfaction

[2] This is essentially the move Champollion 2015 makes, applying negation not on an event predicate φ but on a set of event predicates. Champollion's interpretation $\lambda V \lambda f. \neg V(f)$ of *not* in (28), page 47, is approximated here (without the adjustment of his closure, (15), page 40) by $\lambda t \lambda \varphi$. not$(t \models \varphi)$, where his $V : \langle v\mathrm{t}, \mathrm{t}\rangle$ is recast as the temporal interval t (understood as the set of event predicates that happen in t), and his event predicate $f : v\mathrm{t}$ as φ (understood as the set of events verifying φ). This is in line with Davidson 1967, where (5) above appears as (7), and Reichenbach's use of an existential quantifier is adopted in page 91

> When we were tempted into thinking a sentence like (7) describes a single event we were misled: it does not describe any event at all. But if (7) is true, then there is an event that makes it true. This unrecognized element of generality in action sentences is, I think, of the utmost importance in understanding the relation between actions and desires.

predicate in (11), replacing (A1) by (A2) to (re)define inexact falsifiers of φ as states that fail to inexactly verify φ.

(11) $\quad t \dashv \varphi \iff t \in F(\varphi_{\leq})$

(A2) $\quad F(\varphi_{\leq}) \;:=\; S - V(\varphi_{\leq}) \;=\; \{s \in S \mid (\forall s' \leq s)\ s' \notin V(\varphi)\}$

(A2) breaks the equivalence between $\neg(\varphi_{\leq})$ and $(\neg\varphi)_{\leq}$, for two distinct negations

(i) the classical form, $S - V(\varphi_{\leq})$, universally quantifying over $\leq$-parts, and

(ii) the internal form, $V((\neg\varphi)_{\leq})$ existentially quantifying over $\leq$-parts.

Under (A2), "no gluts" is expressed by $\neg(\varphi_{\leq} \wedge (\neg\varphi)_{\leq})$

$$\begin{aligned}
V(\neg(\varphi_{\leq} \wedge (\neg\varphi)_{\leq})) &= F(\varphi_{\leq}) \cup F((\neg\varphi)_{\leq}) \\
&= (S - V(\varphi_{\leq})) \cup (S - V((\neg\varphi)_{\leq})) \;=\; S - (V(\varphi_{\leq}) \cap V((\neg\varphi)_{\leq})) \\
F(\neg(\varphi_{\leq} \wedge (\neg\varphi)_{\leq})) &= V(\varphi_{\leq} \wedge (\neg\varphi)_{\leq}) \;=\; V(\varphi_{\leq}) \cap V((\neg\varphi)_{\leq})
\end{aligned}$$

which is not to be confused with the expression $\varphi_{\leq} \vee (\neg\varphi)_{\leq}$ of "no gaps"

$$\begin{aligned}
V(\varphi_{\leq} \vee (\neg\varphi)_{\leq}) &= V(\varphi_{\leq}) \cup V((\neg\varphi)_{\leq}) \\
F(\varphi_{\leq} \vee (\neg\varphi)_{\leq}) &= \{s \sqcup s' \mid s \in S - V(\varphi_{\leq}) \text{ and } s' \in S - V((\neg\varphi)_{\leq})\} \\
&= (S - V(\varphi_{\leq})) \cap (S - V((\neg\varphi)_{\leq})) \;=\; S - (V(\varphi_{\leq}) \cup V((\neg\varphi)_{\leq})).
\end{aligned}$$

The question for events above is what is $\leq$, and, in particular, what are the states to the left and right of $\leq$?

3 Temporal grain: strings fusing into languages

The previous section adopts the common assumption that an event e has a temporal extent $\tau(e)$ that, for instance, determines when $e \leq t$ in line (E). Notorious difficulties in pinning down the precise moment of change (e.g., Kamp 1979) and concerns about "minimalitis" (Fine 2015, page 9) ought, however, to give us pause before fixing an event's temporal extent *once and for all*. As widely accepted a condition as (H) might be on statives φ, note that in practice, "July 1926" in (8) is chosen from a limited set of alternatives (constituting the answers to some question under discussion), sufficient for the purpose at hand (say, to rule out Amundsen from flying to the North Pole within that period), making (8) compatible with (12).

(12) $\quad$ Amundsen was away from home for a couple of hours in July 1926.

Bounded granularity calls for tolerating temporal imprecision or, when necessary, adjustments to that granularity. Granularity is analyzed below in terms of temporal propositions which express statives as well as the application of forces that bring about changes constituting events. Henceforth, we refer to temporal propositions as *fluents* (for short), which we assume to belong to some fixed infinite set Φ. Against a linear order $(T, <)$ (such as the real line), a fluent a is interpreted as a subset $\tau(a)$ of T subject to an assumption (BV) of bounded variation saying $\tau(a)$ has a finite boundary under the order topology.

(BV) $\quad$ there are finitely many intervals $I_1, \ldots I_k$ of T such that $\tau(a) = \bigcup_{i=1}^{k} I_i$

Given (BV), we form the function $\tau_a : T \to \mathbb{N}$ from T to the set $\mathbb{N} := \{0, 1, \ldots\}$ of non-negative integers such that for each $t \in T$, $\tau_a(t)$ is the smallest integer in $\mathbb{N}$ for which

$$\tau_a(t) \text{ is odd} \quad \iff \quad t \in \tau(a) \tag{†}$$

and for all $t' \leq t$, $\tau_a(t') \leq \tau_a(t)$. Next, given any finite subset Σ of Φ, we approximate τ by a string s_Σ over the alphabet 2^Σ of subsets of Σ as follows. For any $t \in T$, let τ_t^Σ be the function $\{(a, \tau_a(t)) \mid a \in \Sigma\}$ from Σ to the set $\mathbb{N}$ of non-negative integers. Because Σ is finite, so is the image

$$T_\tau^\Sigma \;:=\; \{\tau_t^\Sigma \mid t \in T\}$$

of T under the projection $t \mapsto \tau_t^\Sigma$, and there is a string $f_1 \cdots f_n$ of functions from Σ to $\mathbb{N}$ such that

$$T_\tau^\Sigma = \{f_i \mid 1 \leq i \leq n\} \;\text{ and }\; f_1 < f_2 < \cdots < f_n$$

where $\leq$ is defined componentwise on functions $f, f' : \Sigma \rightarrow \mathbb{N}$

$$f \leq f' \iff (\forall a \in \Sigma)\ f(a) \leq f'(a).$$

Since the function space $\Sigma \rightarrow \mathbb{N}$ is a subset of $2^{\Sigma \times \mathbb{N}}$, the string $f_1 \cdots f_n$ belongs to $(2^{\Sigma \times \mathbb{N}})^+$, and it natural to construe a pair $(a, i) \in \Sigma \times \mathbb{N}$ as a fluent. Under the biconditional (†) above, the fluent a is essentially the disjunction $\bigvee\{(a, i) \mid i \in \mathbb{N} \text{ and } i \text{ is odd}\}$. We can turn $f_1 \cdots f_n$ into a string s_Σ over the alphabet 2^Σ by deleting (a, i) when i is even and simplifying all remaining pairs (a, i) to a. It is easy to reconstruct the string $f_1 \cdots f_n$ from s_Σ.

Behind the reduction of T to s_Σ above is the intuition that time advances only through change, which we capture by working with strings $\alpha_1\alpha_2 \cdots \alpha_j$ that are *stutter-free* in that $\alpha_i \neq \alpha_{i+1}$ for i from 1 to $j - 1$. An equivalent way of characterizing stutter-free strings is through the biconditional

$$s \text{ is stutter-free} \iff s = \mathit{bc}(s)$$

where the *block compression* $\mathit{bc}(s)$ *of* s compresses blocks α^i of $i > 1$ consecutive occurrences in s of the same symbol α to a single α, leaving s otherwise unchanged

$$\mathit{bc}(s) \;:=\; \begin{cases} \mathit{bc}(\alpha s') & \text{if } s = \alpha\alpha s' \\ \alpha\ \mathit{bc}(\beta s') & \text{if } s = \alpha\beta s' \text{ with } \alpha \neq \beta \\ s & \text{otherwise.} \end{cases}$$

For example,

$$\mathit{bc}(\boxed{}\boxed{}\boxed{e}\boxed{e}\boxed{e}\boxed{e,y}\boxed{e,y}\boxed{e}) \;=\; \boxed{}\boxed{e}\boxed{e,y}\boxed{e}$$

where we draw boxes (as in Kamp and Reyle 1993) instead of curly braces $\{,\}$ for sets construed as symbols in a string (to be read much like a film/cartoon strip). Apart from applying bc, we can make a string stutter-free by introducing a fresh fluent, say tic, to turn, for instance, $\boxed{a}\boxed{a}\boxed{a}$ into $\boxed{a}\boxed{a,\text{tic}}\boxed{a}$. Similarly, to extend the string

$$s_\Sigma \;:=\; \boxed{\text{Jan}}\boxed{\text{Feb}}\cdots\boxed{\text{Dec}}$$

of length 12 (listing the months in a year), we add days d1,…, d31 to $\Sigma := \{\text{Jan},\ldots,\text{Dec}\}$ for

$$s_{\Sigma\cup\{\text{d1},\ldots,\text{d31}\}} \;:=\; \boxed{\text{Jan,d1}}\boxed{\text{Jan,d2}}\cdots\boxed{\text{Jan,d31}}\boxed{\text{Feb,d1}}\cdots\boxed{\text{Dec,d31}}.$$

In general, a finite set $\Sigma \subseteq \Phi$ of fluents fixes a level of granularity, or grain (for short), that gets finer the larger Σ is. Given a string s over the alphabet 2^Φ, its componentwise intersection with Σ yields a string over the alphabet 2^Σ, denoted by $\rho_\Sigma(s)$

$$\rho_\Sigma(\alpha_1 \cdots \alpha_n) \;:=\; (\alpha_1 \cap \Sigma) \cdots (\alpha_n \cap \Sigma).$$

In the calendar example above, $\rho_\Sigma(s_{\Sigma\cup\{\mathrm{d1},\ldots,\mathrm{d31}\}})$ is $\boxed{\mathrm{Jan}}^{31}\cdots\boxed{\mathrm{Dec}}^{31}$ which bc maps back to s_Σ. Stutter-freeness keeps a string with a particular grain Σ from stretching beyond the distinctions that can be expressed in Σ.

Given a grain Σ, an obvious candidate for a state in Fine's state space is a stutter-free string over the alphabet 2^Σ. As for $\leq$, it is instructive to consider what the fusion $\boxed{\,}\boxed{e}\boxed{\,}\sqcup\boxed{\,}\boxed{e'}\boxed{\,}$ of the strings $\boxed{\,}\boxed{e}\boxed{\,}$ and $\boxed{\,}\boxed{e'}\boxed{\,}$ might be. The string $\boxed{\,}\boxed{e,e'}\boxed{\,}$ comes immediately to mind. But if a stutter-free string s is to represent all strings whose block compression is s, then there is also $\boxed{\,}\boxed{e}\boxed{e'}\boxed{\,}$ and the other strings in the set

$$\{\mathit{bc}(s) \mid s \in (2^{\{e,e'\}})^+,\ \mathit{bc}(\rho_{\{e\}}(s)) = \boxed{\,}\boxed{e}\boxed{\,} \ \text{ and } \ \mathit{bc}(\rho_{\{e'\}}(s)) = \boxed{\,}\boxed{e'}\boxed{\,}\}$$

which can be shown to equal

$$\mathrm{Allen}(e,e') \ := \ \mathcal{L}(e \bigcirc e') \cup \mathcal{L}(e \prec e') \cup \mathcal{L}(e' \prec e)$$

where $\mathcal{L}(e \bigcirc e')$ is the set of 9 strings $\boxed{\,}\boxed{s}\boxed{e,e'}\boxed{s'}\boxed{\,}$, for $s,s' \in \{\epsilon, \boxed{e}, \boxed{e'}\}$, in which e overlaps with e', while $\mathcal{L}(e \prec e')$ consists of the two strings $\boxed{\,}\boxed{e}\boxed{e'}\boxed{\,}$ and $\boxed{\,}\boxed{e}\boxed{\,}\boxed{e'}\boxed{\,}$ in which e precedes e' (putting 13 strings into $\mathrm{Allen}(e,e')$, one for each interval relation in Allen 1983). The set $\mathrm{Allen}(e,e')$ can be obtained from $\boxed{\,}\boxed{e}\boxed{\,}$ and $\boxed{\,}\boxed{e'}\boxed{\,}$ by applying a certain binary operation $\&_{\mathit{bc}}$ defined as follows. The *superposition of* two strings over 2^Φ of the same length is their componentwise union

$$\alpha_1\cdots\alpha_n \ \& \ \beta_1\cdots\beta_n \ := \ (\alpha_1\cup\beta_1)\cdots(\alpha_n\cup\beta_n)$$

while the *superposition of* two languages L and L' over 2^Φ is the set of superpositions of strings of the same length from L and L'

$$L \ \& \ L' \ := \ \{s\&s' \mid (s,s') \in L \times L' \text{ and } \mathrm{length}(s) = \mathrm{length}(s')\}.$$

Next, we collect all strings bc-equivalent to a string in L in

$$L^{\mathit{bc}} \ := \ \{s \in (2^\Phi)^* \mid (\exists s' \in L)\ \mathit{bc}(s) = \mathit{bc}(s')\}$$

and take the image under bc of the superposition of L^{bc} and L'^{bc} for

$$L \ \&_{\mathit{bc}} \ L' \ := \ \{\mathit{bc}(s) \mid s \in L^{\mathit{bc}} \ \& \ L'^{\mathit{bc}}\}.$$

Conflating strings with their singletons, we have, as promised,

$$\mathrm{Allen}(e,e') \ = \ \boxed{\,}\boxed{e}\boxed{\,} \ \&_{\mathit{bc}} \ \boxed{\,}\boxed{e'}\boxed{\,}.$$

Three relations on strings that may or may not be stutter-free are useful in defining a partial order on stutter-free strings:

(i) the *factor* relation, formulated as the set-valued function

$$\mathrm{factor}(s) \ := \ \{s' \mid (\exists u,v)\ s = us'v\}$$

returning the factors of s (obtained by removing a prefix u and suffix v from s)

(ii) *subsumption* $\unrhd$, componentwise containment $\supseteq$ between strings over 2^Φ of the same length

$$\begin{aligned}\alpha_1\cdots\alpha_n \unrhd \beta_1\cdots\beta_m &\iff n=m \text{ and } \alpha_i \supseteq \beta_i \text{ for } 1\le i\le n\\ &\iff \alpha_1\cdots\alpha_n = \alpha_1\cdots\alpha_n \ \&\ \beta_1\cdots\beta_m\end{aligned}$$

so that $s \unrhd \rho_\Sigma(s)$ for all $s \in (2^\Phi)^*$, and

(iii) equivalence $=_{bc}$ up to block compression

$$s =_{bc} s' \iff bc(s) = bc(s').$$

Now, we define $s' \le s$ to mean that some factor of s subsumes some string bc-equivalent to s'

$$s' \le s \iff (\exists s_1 \in \text{factor}(s))(\exists s_2 =_{bc} s')\ s_1 \unrhd s_2.$$

One can show $\le$ is a partial order on stutter-free strings, and that for stutter-free s,

$$s' \le s \iff s \in s \ \&_{bc}\ (\boxed{}+\epsilon)s'(\boxed{}+\epsilon)\ . \qquad (\ddagger)$$

Insofar as $\&_{bc}$ is a well-defined binary operation on languages (rather than strings) over 2^Φ, it is natural to step from states as strings up to states as languages, generalizing (‡) to a set L of stutter-free strings for

$$L' \le L \iff L \subseteq L \ \&_{bc}\ (\boxed{}+\epsilon)L'(\boxed{}+\epsilon).$$

Alternatively, keeping stutter-free strings s, s' as states, we might treat $\&_{bc}$ as a non-deterministic operation in which the fusion $s \ \&_{bc}\ s'$ is a set of stutter-free strings, adjusting clause (3) above by taking unions of string sets

$$\begin{aligned}V(\varphi\wedge\psi) &:= \bigcup\{s \ \&_{bc}\ s' \mid (s,s') \in V(\varphi)\times V(\psi)\}\\ F(\varphi\vee\psi) &:= \bigcup\{s \ \&_{bc}\ s' \mid (s,s') \in F(\varphi)\times F(\psi)\}.\end{aligned}$$

Block compression aside, let us lift subsumption $\unrhd$ to languages L and L' over the alphabet 2^Φ, and agree L *subsumes* L' if L is a subset of its superposition with L'

$$L \unrhd L' \iff L \subseteq L\&L'$$

so that conflating a string s as usual with the singleton $\{s\}$,

$$s \unrhd L \iff (\exists s' \in L)\ s \unrhd s'\ .$$

Now, given a representation of an event e at grain Σ as a string $str_\Sigma(e) \in (2^\Sigma)^+$, we define e to be

- Σ-*telic* if $str_\Sigma(e) \unrhd \boxed{\neg\varphi}^+\boxed{\varphi}$ for some fluent $\varphi \in \Sigma$ (marking the culmination of e), and
- Σ-*durative* if $bc(str_\Sigma(e))$ has length ≥ 3

(Fernando 2015). For representations $str(e)$ of events that fall into the Vendler classes described in Dowty 1979 and modified in Moens and Steedman 1988, we form not only transitions $\boxed{\neg\varphi}\boxed{\varphi}$ in which a fluent φ representing a stative becomes true, but also transitions $\boxed{ap(f)}\boxed{ef(f)}$ recording an effectual application of a force f, with the intention that

$$\begin{aligned}&ap(f) \ \text{ says } \ \text{“force } f \text{ is applied”}\\ &ef(f) \ \text{ says } \ \text{“a previous application of } f \text{ is effectual.”}\end{aligned}$$

The transitions $\boxed{ap(f)}\boxed{ef(f)}$ and $\boxed{\neg\varphi}\boxed{\varphi}$ describe semelfactives and achievements, respectively, together forming the non-durative column in Table 1.

	$-$durative	+durative
$-$telic	semelfactive $\boxed{ap(f)}\boxed{ef(f)}$	activity $\boxed{ap(f)}\boxed{ap(f), ef(f)}\boxed{ef(f)}$
+telic	achievement $\boxed{\neg\varphi}\boxed{\varphi}$	accomplishment $\boxed{\neg\varphi, ap(f)}\boxed{\neg\varphi, ap(f), ef(f)}\boxed{\varphi, ef(f)}$

Table 1. *A minimal picture of $\pm$durativity and φ-telicity*

Iterating the transitions $\boxed{ap(f)}\boxed{ef(f)}$ yields the language

$$\mathcal{L}(f) \;:=\; \boxed{ap(f)}\boxed{ap(f), ef(f)}^{*}\boxed{ef(f)}$$

which we superpose with $\boxed{\;}\boxed{\;}\boxed{\;}^{+}$ and block compress for Table 1's activity entry ($-$telic, +durative), and superpose further with $\boxed{\neg\varphi}^{+}\boxed{\varphi}$ for Table 1's accomplishment entry (+telic, +durative). The four strings in Table 1 can be obtained from $\mathcal{L}(f)$ and $\boxed{\;}\boxed{\;}$ using block compression and the three operators

$$\begin{aligned} dur(L) &:= L \,\&\, \boxed{\;}\boxed{\;}\boxed{\;}^{+} \\ non\text{-}dur(L) &:= L - dur(L) \\ cul(L, \varphi) &:= L \,\&\, \boxed{\neg\varphi}^{+}\boxed{\varphi} \end{aligned}$$

that pick out the durative, non-durative and φ-telic strings in L, respectively. These operators bring to mind Dowty's hypothesis that "the different aspectual properties of the various kinds of verbs can be explained by postulating a single homogeneous class of predicates – stative predicates – plus three or four sentential operators and connectives" (Dowty 1979, page 71). The obvious question is: do we need the fluents $ap(f)$ and $ef(f)$, and all the fuss about block compression? Our answer to this question in the next section implicates an event $\boxed{ap(f)}\boxed{ef(f)}$ in the negation $\neg\varphi$ of a stative fluent φ (and in extending stutter-free strings beyond length 1).

4 Change: negation by force

The notion of force mentioned in the fluents $ap(f)$ and $ef(f)$ links up with fluents φ representing statives through a law of inertia decreeing that a stative will continue to hold "unless something happens to change" it (Comrie 1976, page 49). To make this precise, more notation is helpful. Given languages L and L' over the alphabet 2^{Φ}, let us collect all strings over 2^{Φ} whose factors subsuming L also subsume L' in the language

$$L \Rightarrow L' \;\;:=\;\; \{s \in (2^{\Phi})^{*} \mid (\forall s' \in \text{factor}(s)) \text{ if } s' \trianglerighteq L \text{ then } s' \trianglerighteq L'\}.$$

For example, $\boxed{\varphi}\boxed{\;} \Rightarrow \boxed{\;}\boxed{\varphi}$ is the set of strings over 2^{Φ} such that whenever φ appears at a position, it appears at all later positions. The idea now is to let $force(\varphi)$ be the fluent $ap(f)$ for some force f with $ef(f) := \varphi$ so that the constraint

$$\text{Inr}(\varphi) \;:=\; \boxed{\varphi}\boxed{\;} \;\Rightarrow\; \boxed{\;}\boxed{\varphi} + \boxed{force(\neg\varphi)}\boxed{\;}$$

says φ persists (forward) unless a force is applied opposing φ. Similarly, the constraint

$$\mathrm{Inr}_b(\varphi) \ := \ \boxed{}\boxed{\varphi} \ \Rightarrow \ \boxed{\varphi}\boxed{} + \boxed{\mathit{force}(\varphi)}\boxed{}$$

says φ persists backward unless a force was applied making it true, while

$$\mathrm{Suo}(\varphi) \ := \ \boxed{\mathit{force}(\varphi)}\boxed{} \ \Rightarrow \ \boxed{\varphi}\boxed{} + \boxed{\mathit{force}(\neg\varphi)}\boxed{}$$

says an application of a force for φ succeeds unless (as in $\mathrm{Inr}(\varphi)$) opposed. We do *not* assume that for every force f, there is a fluent φ with $\mathit{ef}(f) := \varphi$ that is subject to the three constraints above. The constraints $\mathrm{Inr}(\varphi)$ and $\mathrm{Inr}_b(\varphi)$ may fail to apply when the change described is incremental; for example, an increase in the degree $\mathit{deg}[\psi]$ associated with a claim ψ

$$\uparrow\mathit{deg}[\psi] \ := \ \bigvee_{d \in D[\psi]} (d \leq \mathit{deg}[\psi]) \wedge \mathit{Previous}(\mathit{deg}[\psi] < d))$$

over some set $D[\psi]$ of ψ-degrees (such as temperatures, for the claim ψ that "the soup is hot"). It is understood above that *Previous* is the obvious temporal operator which, in the case of $\uparrow\mathit{deg}[\psi]$, unwinds to the language

$$\boxed{}\boxed{\uparrow\mathit{deg}[\psi]} \quad \Leftrightarrow \quad \sum_{d \in D[\varphi]} \boxed{\mathit{deg}[\psi] < d}\boxed{d \leq \mathit{deg}[\psi]}$$

where $L \Leftrightarrow L'$ abbreviates the intersection of $L \Rightarrow L'$ and $L' \Rightarrow L$. To keep the alphabet of the language finite, the set $D[\varphi]$ must be assumed finite, and indefinitely refinable, as any finite set chosen for $D[\varphi]$ can be expanded to a larger vocabulary $\Sigma \subset \Phi$.

For fluents φ to which $\mathrm{Inr}(\varphi)$ and $\mathrm{Inr}_b(\varphi)$ apply, let us be clear about what is said about negating φ. Any string over an alphabet 2^{Σ} in which each $\varphi \in \Sigma$ is subject to $\mathrm{Inr}(\varphi)$ and $\mathrm{Inr}_b(\varphi)$ block compresses to a string of length at most 1

$$(\forall s \in (2^{\Sigma})^* \cap \bigcap_{\varphi \in \Sigma} \mathrm{Inr}(\varphi) \cap \mathrm{Inr}_b(\varphi)) \ \ \mathrm{length}(\mathit{bc}(s)) \leq 1$$

(assuming none of the fluents $\mathit{force}(\varphi)$ belong to Σ). Of course, we can always neutralize $\mathrm{Inr}(\varphi)$, $\mathrm{Inr}_b(\varphi)$ and $\mathrm{Suo}(\varphi)$ by applying forces (with an expanded alphabet), but the challenge is to do so in a principled manner. It is worth quoting from page 52 of Dowty 1986 at length

> This principle of "inertia" in the interpretation of statives in discourse applies to many kinds of statives but of course not to all of them ... there must be a graded hierarchy of the likelihood that various statives will have this kind of implicature, depending on the nature of the state, the agent, and our knowledge of which states are long-lasting and which decay or reappear rapidly. Clearly, an enormous amount of real-world knowledge and expectation must be built into any system which mimics the understanding that humans bring to the temporal interpretations of statives in discourse, so no simple non-pragmatic theory of discourse interpretation is going to handle them very effectively.

There is no "graded hierarchy" of inertia in $\mathrm{Inr}(\varphi)$ and $\mathrm{Inr}_b(\varphi)$. The defeasibility lies in the forces understood to be at play and in the resolution of opposing forces, on which $\mathrm{Inr}(\varphi)$, $\mathrm{Inr}_b(\varphi)$ and $\mathrm{Suo}(\varphi)$ are silent. Inasmuch as evidence for these forces is to be found in discourse (conceived broadly), the negation $\neg\varphi$ of a fluent φ representing an inertial stative is asymmetricalist in the sense of Horn 1989. It is tempting to formulate these forces as attribute value structures or frames (Fillmore 1982, Cooper 2015, Fernando 2015a), but it remains to be seen how much of that theory can be encoded as constraints (like $\mathrm{Inr}(\varphi)$, $\mathrm{Inr}_b(\varphi)$ and $\mathrm{Suo}(\varphi)$) satisfied by some set $S^{\diamond}$ of possible states in the sense of Fine 2015.

References

James F. Allen. Maintaining knowledge about temporal intervals. *Commun. ACM* 26(11):832–843. 1983.

David Beaver and Cleo Condoravdi. On the logic of verbal modification. In M. Aloni, P. Dekker, & F. Roelofsen (eds.), *Proceedings of the 16th Amsterdam Colloquium*, pp. 3–9. University of Amsterdam. 2007.

Lucas Champollion. The interaction of compositional semantics and event semantics. *Linguistics and Philosophy* 38:31–66. 2015.

Bernard Comrie. *Aspect*. Cambridge University Press, 1976.

Cleo Condoravdi. Punctual until as a scalar NPI. In K. Hanson & S. Inkelas (eds.), *The Nature of the Word: Studies in Honor of Paul Kiparsky*, pp. 631–654. MIT. 2008.

Robin Cooper. Frames as records. In A. Foret, G. Morrill, R. Muskens & R. Osswald (eds.), *Preproceedings of the 20th Conference on Formal Grammar*, pp. 1–17. Barcelona. 2015.

Donald Davidson. The logical form of action sentences. In N. Rescher (ed.), *The Logic of Decision and Action*, pp. 81–95. University of Pittsburgh Press. 1967.

David R. Dowty. *Word Meaning and Montague Grammar.* Reidel. 1979.

David R. Dowty. The effects of aspectual class on the temporal structure of discourse: semantics or pragmatics? *Linguistics and Philosophy* 9(1):37–61. 1986.

Tim Fernando. The semantics of tense and aspect: a finite-state perspective. In S. Lappin & C. Fox (eds.), *The Handbook of Contemporary Semantic Theory*, second edition, pp. 203–236. Wiley. 2015.

Tim Fernando. Types from frames as finite automata. In A. Foret, G. Morrill, R. Muskens & R. Osswald (eds.), *Preproceedings of the 20th Conference on Formal Grammar*, pp. 18–38. Barcelona. 2015a.

Charles J. Fillmore. Frame semantics. In *Linguistics in the Morning Calm*, pp. 111–137. Hanshin Publishing Co. (Seoul). 1982.

Kit Fine. Truthmaker semantics. Chapter for the Blackwell Philosophy of Language Handbook. 22 page draft available online. 2015.

Laurence R. Horn. *A Natural History of Negation*. University of Chicago Press. 1989.

Hans Kamp and Uwe Reyle. *From Discourse to Logic.* Kluwer. 1993.

Hans Kamp. Events, instants and temporal reference. In Rainer Bäuerle, Urs Egli & Arnim von Stechow (eds.), *Semantics from Different Points of View*, pp. 376–417. Springer-Verlag. 1979.

Fred Landman. Stativity operators in 1066. Handout, Tel Aviv University. 2006.

Marc Moens and Mark Steedman. Temporal ontology and temporal reference. *Computational Linguistics* 14(2):15–28. 1988.

Terence Parsons. *Events in the Semantics of English: A Study in Subatomic Semantics*. MIT Press. 1990.

Ian Pratt-Hartmann. Temporal prepositions and their logic. *Artificial Intelligence* 166:1–36. 2005.

Barry Taylor. Tense and continuity. *Linguistics and Philosophy* 1:199–220. 1977.