String Iconicity and Granularity*

Tim Fernando

Trinity College Dublin, Ireland

Abstract

Given any finite set A of temporal predicates, expressions iconically representing strings of subsets of A are defined that depend on whether the predicates in A are transitional or stative (i.e., homogeneous and to varying degrees, inertial, following Dowty). Subsets Σ of A represent granularities at which form-meaning resemblance is assessed (through transducers that approximate strings up to Σ). Complications to iconicity of order are investigated in a first-order fragment of MSO over strings, where experiential factors that trump time are expressible.

1 Introduction

A string s is an *iconic* regular expression inasmuch as the regular language $\{s\}$ it denotes consists of exactly one entity, an entity s that undeniably resembles the expression s. Not all regular expressions represent their denotations quite so faithfully. But pattern matching regular expressions is surely one reason regular expressions are far more popular than formulas of Monadic Second Order logic over strings (MSO; e.g., [Lib04]) for specifying languages accepted by finite automata. That said, conceptualising strings as models of predicate logic suggests useful notions of granularity for form-meaning likeness, supported by a logical apparatus for analysing such likeness. Our specific interest here is *iconicity of order*, linked in [New92] with the principle that "the order of elements in language parallels that in physical experience or the order of knowledge" [Gre63],[1] and understood more broadly below to cover pictorial narratives (e.g., [Abu14, AR17, MB19]). Granularity enters two ways (following [GB92] where it takes the form of *signatures*), syntactically as

(i) a tag Σ on a string s (qua expression) for a pair (Σ, s)

and semantically as

(ii) a subscript A on the interpretation of (Σ, s), for $\Sigma \subseteq A$, as the set

$$[\![(\Sigma, s)]\!]_A \ := \ \{s' \in \mathcal{L}_A \mid f_\Sigma(s') = s\}$$

of strings s' from a certain set $\mathcal{L}_A$ whose Σ-approximation $f_\Sigma(s')$ equals s

with the special case $\Sigma = A$ leading to the aforementioned instance of iconicity as identity

$$f_A(s') = s', \quad \text{implying} \quad [\![(A, s)]\!]_A = \{s\} \text{ for } s \in \mathcal{L}_A.$$

The precise definitions of $\mathcal{L}_A$ and f_Σ (from [Fer19b]) are given below, after pausing to consider if strings are fit for purpose.

*Thanks to three anonymous referees for constructive criticism and forbearance. I am grateful also for feedback from Robin Cooper, Simon Dobnik, Stergios Chatzikyriakidis and others at an initial airing in Gothenburg.

[1] The term in [PV14] is "iconicity of sequence" according to which "the sequence of forms conforms to the sequence of experience, as in the famous collocation *veni, vidi, vici*"

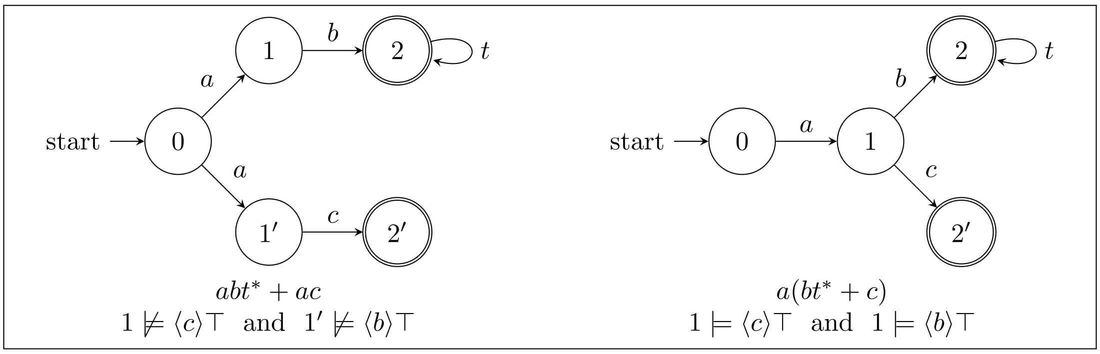

Figure 1: Two automata accepting $abt^* + ac$

Do strings have sufficient structure to encode the order-related notions of interest? What, for example, are we to do about the continuum $\mathbb{R}$ (employed in [KR93] and elsewhere as a model of the timeline)? Assuming granularity can be bounded to a finite subset A of $\mathbb{R}$, we can cast A as a string

$$a_1 a_2 \cdots a_n \qquad \text{where } A = \{a_1, a_2, \ldots, a_n\} \text{ and } a_1 < a_2 < \cdots < a_n$$

(and, if we wish, reconstruct $\mathbb{R}$ from a suitable inverse limit of such strings). But, of course, it is not points in $\mathbb{R}$ that are of linguistic concern, but rather events (and states) spanning intervals. These might — following Russell and Wiener (e.g., [KR93]) and [Ham71] — be deemed primitive, complicating matters in at least two ways. First, the 3 relations $<, =, >$ between points multiply into 13 (or more) interval relations (e.g., [All83]). And second, a predicate P that holds at an interval I immediately raises the question: must P also hold at a subinterval of I? The answer commonly given by linguists appeals to a typology of predicates, including *stative* predicates P, on which the aspectual calculus of [Dow79] is based, and for which

$$P \text{ holds at an interval } I \ \text{ iff } \ P \text{ holds at every subinterval of } I.$$

The present work builds on the Priorean perspective [Pri67], applied in [Fer16, Fer15b] to [Dow79] and other linguistic works. Beyond any particular linear order, there is the additional issue of branching or non-determinism, concerning which it has been argued a process is more than the set of its runs (recorded as strings). An instructive example, adapted from process algebra ([Fok07], pages 10,11), is the pair of automata in Figure 1 accepting the strings

$$abt^* + ac \;=\; a(bt^* + c). \tag{1}$$

An instance of the distributive law (true of languages, and a semiring axiom), the equality (1) fails to discriminate the left automaton from the right, even though a state *in*capable of a c-transition is a-accessible from state 0 in the left but not the right. Suppose, however, we pick out states q that can make an l-transition with the formula $\langle l \rangle \top$ from Dynamic Logic ([HKT00])

$$q \models \langle l \rangle \top \iff (\exists q')\ q \xrightarrow{l} q' \qquad (\text{i.e., } q \in \mathit{domain}(\xrightarrow{l}))$$

and turn say, the string ac of arc labels in $0 \xrightarrow{a} 1' \xrightarrow{c} 2'$ into the string $\boxed{\langle a\rangle\top}\boxed{\langle c\rangle\top}\boxed{}$ of boxes describing the states $0, 1', 2'$, respectively. Then equation (1) becomes the *in*equation

$$\boxed{\langle a\rangle\top}\boxed{\langle b\rangle\top}\boxed{\langle t\rangle\top}^{+} + \boxed{\langle a\rangle\top}\boxed{\langle c\rangle\top}\boxed{} \;\neq\; \boxed{\langle a\rangle\top}\boxed{\langle b\rangle\top, \langle c\rangle\top}\left(\boxed{\langle t\rangle\top}^{+} + \boxed{}\right) \tag{2}$$

with distributivity making (2)'s lefthand side equal to

$$\boxed{\langle a\rangle\top}(\boxed{\langle b\rangle\top}\boxed{\langle t\rangle\top}^{+} + \boxed{\langle c\rangle\top}\boxed{}).$$

The second box in (2)'s righthand side consists of two formulas, $\langle b\rangle\top$ and $\langle c\rangle\top$, representing state 1 of the right automaton in Figure 1, in accordance with the view that "disjunctions are conjunctive lists of epistemic possibilities" ([Zim00], page 255), perhaps rewriting $\langle l\rangle\top$ to may(l). Evidently, reducing automata to sets of strings need not conflate the pair in Figure 1, provided we are careful to encode the appropriate information into the strings. Exactly what information is appropriate determines the choice Σ of granularity such as

$$\Sigma \ = \ \{\langle l\rangle\top \mid l \in \{a,b,c,t\}\}$$

for Figure 1. The contrast between the loop label t in Figure 1 and the other transition labels a, b, c between distinct states is noteworthy. The label t can be derived in Dynamic Logic from a formula φ as a test that a state q satisfies φ, leaving q fixed if it does and aborting otherwise. More precisely, if the states satisfying φ are collected in the set $[\![\varphi]\!]$, then the input/output interpretation of the *test* $\varphi?$ is the binary relation

$$q \overset{\varphi?}{\to} q' \quad \iff \quad q \in [\![\varphi]\!] \text{ and } q = q'$$

testing φ on a state q without side-effects. Like all programs in Dynamic Logic, the test $\varphi?$ is interpreted as an input/output relation R, abstracting away all intermediate states, as in the *relational composition* $R;R'$ *of* R *with* R'

$$R;R' \ := \ \{(q,q') \mid (\exists\hat{q}) \ qR\hat{q} \text{ and } \hat{q}R'q'\}$$

with intermediate states $\hat{q}$ connecting R to R' quantified out, and tests $\varphi?$ idempotent relative to ;

$$\overset{\varphi?}{\to};\overset{\varphi?}{\to} \ = \ \overset{\varphi?}{\to}.$$

The strings $\alpha_1 \cdots \alpha_n$ of boxes α_i above keep not only the input α_1 and the output α_n, but also the intermediate boxes α_i, $1 < i < n$. Unlike strings $abt, abtt, \ldots$ of transition labels, a string $s = \alpha_1 \cdots \alpha_n$ of sets α_i can be intersected componentwise with any set Σ to form its Σ-*reduct*

$$\rho_\Sigma(\alpha_1 \cdots \alpha_n) \ := (\alpha_1 \cap \Sigma) \cdots (\alpha_n \cap \Sigma)$$

approximating s up to granularity Σ. [Fer19b] extracts a notion of time dependent on Σ, linked in [FWV19] to *guarded* strings (built with states given by subsets of a finite set B of Booleans) for *Kleene algebra with tests* [KS96]. Returning to the A-interpretation $[\![(\Sigma,s)]\!]_A$ mentioned above, the idea is that $f_\Sigma(s)$ is a compressed Σ-reduct of s

$$f_\Sigma(s) = \begin{cases} \mathit{bc}(\rho_\Sigma(s)) & \text{for stative } \Sigma \\ \mathsf{d}_\square(\rho_\Sigma(s)) & \text{for transitional } \Sigma \end{cases}$$

where *block compression* bc compresses substrings $\alpha\alpha$ to α, as in

$$\mathit{bc}(\boxed{\langle a\rangle\top}\boxed{\langle b\rangle\top}\boxed{\langle t\rangle\top}^{n}) \ = \ \boxed{\langle a\rangle\top}\boxed{\langle b\rangle\top}\boxed{\langle t\rangle\top} \qquad \text{for } n \geq 1$$

while *depadding* $\mathsf{d}_\square$ eliminates all occurrences of $\boxed{}$, as in

$$\mathsf{d}_\square(\boxed{\langle a\rangle\top}\boxed{\langle c\rangle\top}\boxed{}) \ = \ \boxed{\langle a\rangle\top}\boxed{\langle c\rangle\top}$$

and the line between stative and transitional Σ rests on whether or not Σ consists of tests. Full definitions are given in the next section, which is followed by a delineation of star-free languages for string iconicity and some applications to temporal interpretation.

As the material below can be off-puttingly dry and technical, a few words about its linguistic motivation are in order. The basic challenge is to determine how events and states described by a sequence of sentences or pictures are temporally related. A starting point (suggested by order iconicity) is the order in which expressions are described, refined so that events move the time of a narrative forward, but states do not (e.g., [Dow86]). This refinement coincides with the refinement of concatenation to *coalesced product* $\diamond$ in *Kleene algebra with tests* ([KS96]), provided statives are, as in [FWV19], understood as tests. The coalesced product $s \diamond s'$ of strings s and s' attaches the right end of s to the left end of s', assuming these match (in which case they are fused). A more flexible alternative associates sets Σ and Σ' with s and s', respectively, and interprets the combination $\{(\Sigma, s), (\Sigma', s')\}$ at A as the intersection

$$[\![\{(\Sigma, s), (\Sigma', s')\}]\!]_A \;=\; [\![(\Sigma, s)]\!]_A \cap [\![(\Sigma', s')]\!]_A$$

of the A-interpretations of (Σ, s) and (Σ', s'). The remainder of this paper investigates string expressions built and interpreted along these lines, related to iconicity of order by granularities Σ, Σ' based on contextual factors other than time that shape "physical experience or the order of knowledge." Space is an obvious factor, as are an agent's cognitive limitations, leading to distinct (if not disjoint) perspectives Σ and Σ'. For a concrete illustration, take two football matches kicking off at the same time in different cities, for distinct views Σ and Σ', and narratives s and s' (respectively) that may temporally overlap here and there.

2 Stative and transitional projections

Just as automata transitions $q \xrightarrow{a} q'$ are formed from states q, q' and actions a,

(i) events are analysed in [GJW18] in terms of results and actions

(ii) non-stative verbs are classified in [LRH13] as result or manner, exemplified in [Fil70] by *break* and *hit*

(iii) telic and iterable transitions are expressed in [Dow79] through operators BEC(ome) and DO, leading in [Fer15a] to strings $\boxed{\neg\varphi}\boxed{\varphi}$ and $\boxed{\text{ap}(a)}\boxed{\text{ef}(a)}$ respectively, with result described by a stative φ and expressions ef(a) and ap(a) of the effect and application of a.

$q \xrightarrow{a} q'$	result q'	*break*/result	BEC	$\boxed{\neg\varphi}\boxed{\varphi}$
	action a	*hit*/manner	DO	$\boxed{\text{ap}(a)}\boxed{\text{ef}(a)}$
	Gärdenfors	Fillmore/LevinRH	Dowty	

[LRH13] distinguishes between a result being specified lexically, φ, and supplied contextually, ef(a), which we can understand as a context of use supplying MSO-formulas that constrain ef(a) and ap(a). An MSO-formula is built from a binary (successor) relation S, and unary relations P_σ labeled by elements σ of some finite set Σ (including, for example, $\varphi, \neg\varphi$, ap(a) and ef(a)). A string $s = \alpha_1 \cdots \alpha_n$ of subsets $\alpha_i \subseteq \Sigma$ is identified with the MSO$_\Sigma$-model

$$\text{Mod}_\Sigma(s) \;:=\; \langle [n], S_n, \{U_\sigma\}_{\sigma \in \Sigma} \rangle$$

with string positions forming its universe/domain

$$[n] \;:=\; \{1, 2, \ldots, n\},$$

to which the $+1$ function is restricted to interpret S as

$$S_n \ := \ \{(i, i+1) \mid i \in [n-1]\} \ = \ \{(1,2), (2,3), \ldots, (n-1, n)\}$$

and each P_σ interpreted as the subset

$$U_\sigma \ := \ \{i \in [n] \mid \sigma \in \alpha_i\} \qquad (\sigma \in \Sigma)$$

of string positions where σ occurs. Each box α_i of s can be recovered by the equation

$$\alpha_i \ = \ \{\sigma \in \Sigma \mid i \in U_\sigma\} \qquad (i \in [n])$$

establishing an isomorphism between $(2^\Sigma)^+$ and a family of MSO_Σ-models. That family of MSO_Σ-models is large enough to accommodate for any finite set $A \supseteq \Sigma$, the Σ-*reduct* $M \restriction \Sigma$ *of an* MSO_A*-model* $M = \langle [n], S_n, \{U_a\}_{a \in A}\rangle$ approximating M by restricting $\{U_a\}_{a\in A}$ to $\{U_\sigma\}_{\sigma \in \Sigma}$

$$M \restriction \Sigma \ := \langle [n], S_n, \{U_\sigma\}_{\sigma\in\Sigma}\rangle.$$

Proceeding from a string s of subsets of A, the Σ-reduct of s picks out the Σ-reduct of $\text{Mod}_A(s)$

$$\text{Mod}_A(s) \restriction \Sigma \ = \text{Mod}_\Sigma(\rho_\Sigma(s)).$$

For $f_\Sigma(s)$, we compress the Σ-reduct of s, removing stutters $\alpha\alpha$ through bc

$$\mathit{bc}(s) := s \quad \text{if length}(s) < 2 \qquad \text{and} \qquad \mathit{bc}(\alpha\alpha' s) := \begin{cases} \mathit{bc}(\alpha' s) & \text{if } \alpha = \alpha' \\ \alpha\, \mathit{bc}(\alpha' s) & \text{otherwise} \end{cases}$$

or occurrences of the empty box $\square$ through $\mathsf{d}_\square$

$$\mathsf{d}_\square(\epsilon) := \epsilon \qquad \text{and} \qquad \mathsf{d}_\square(\alpha s) := \begin{cases} \mathsf{d}_\square(s) & \text{if } \alpha = \square \\ \alpha\, \mathsf{d}_\square(s) & \text{otherwise.} \end{cases}$$

The functions bc and $\mathsf{d}_\square$ are related by a certain border translation ([Fer19b]) that illuminates the difference between stative and transitional Σ, determining whether $f_\Sigma = \rho_\Sigma; \mathit{bc}$ or $f_\Sigma = \rho_\Sigma; \mathsf{d}_\square$. More precisely, let l and r be 1-1 functions with domain A, whose images form $A_\bullet$

$$A_\bullet := \ \{l(a) \mid a \in A\} \cup \{r(a) \mid a \in A\}$$

on the assumption that A, $\{l(a) | a \in A\}$ and $\{r(a) | a \in A\}$ are pairwise disjoint. The border translation $\mathbf{b} : (2^A)^* \to (2^{A_\bullet})^*$ marks a's left borders $l(a)$ and a's right borders $r(a)$, mapping $\alpha_1 \cdots \alpha_n \in (2^A)^*$ to $\beta_1 \cdots \beta_n$ where

$$\begin{aligned} \beta_i &:= \{l(a) \mid a \in \alpha_{i+1} - \alpha_i\} \ \cup \ \{r(a) \mid a \in \alpha_i - \alpha_{i+1}\} \quad \text{for } i < n \\ \beta_n &:= \{r(a) \mid a \in \alpha_n\}. \end{aligned}$$

For example,

$$\mathbf{b}(\boxed{a'}\boxed{a}\boxed{a}) = \boxed{l(a), r(a')}\boxed{}\boxed{r(a)} \qquad \text{and} \qquad \mathsf{d}_\square(\mathbf{b}(\boxed{a'}\boxed{a}\boxed{a})) = \mathbf{b}(\mathit{bc}(\boxed{a'}\boxed{a}\boxed{a}))$$

and in general,

$$\mathbf{b}(\mathit{bc}(s)) \ = \ \mathsf{d}_\square(\mathbf{b}(s)) \qquad \text{for any } s \in (2^A)^+ \textit{ not} \text{ ending in } \square.$$

The border symbols $l(a)$ and $r(a)$ specify their effects lexically through the $\text{MSO}_{\{a\}}$-formulas

$$\begin{aligned} \chi_{l(a)}(x) &:= \ \exists y(xSy \wedge P_a(y)) \wedge \neg P_a(x) && \text{"a occurs at x's successor but not at x"} \\ \chi_{r(a)}(x) &:= \ P_a(x) \wedge \neg\exists y(xSy \wedge P_a(y)) && \text{"a occurs at x but not at any successor of x"} \end{aligned}$$

characterising $\mathbf{b}$ in that for any $s \in (2^{A \cup A_\bullet})^+$,

$$\mathbf{b}(\rho_A(s)) = \rho_{A_\bullet}(s) \iff \text{for all } b \in A_\bullet, \ s \models \forall x(P_b(x) \equiv \chi_b(x)).$$

	stative (result)	transition (action)
compression	$\mathit{bc} : s\alpha\alpha s' \rightsquigarrow s\alpha s'$	$\mathsf{d}_\Box : s\Box s' \rightsquigarrow ss'$
[All83] a starts a'	$\boxed{}\boxed{a, a'}\boxed{a'}\boxed{}$ interior	$\boxed{l(a), l(a')}\boxed{r(a)}\boxed{r(a')}$ border
[vLH05]	fluent (*HoldsAt*)	event type (*Happens*)
MSO_A	$\forall x \forall y (xSy \supset \neg \bigwedge_{a \in A} (P_a(x) \equiv P_a(y)))$	$\forall x \bigvee_{a \in A} P_a(x)$

Table 1: Statives vs transitions

More concretely, Table 1 represents an interval relation in [All83] two ways ([Fer19b]): statively in terms of an interval's interior, and transitionally in terms of an interval's borders ([DS08], page 3288). Following section 3, chapter 7 of [vLH05], Table 1 also aligns statives with fluents (comparable to Dynamic logic formulas φ that induce tests $\varphi?$ which are idempotent relative to relational composition ;), and transitions with event types (which may, like the program $x := x + 1$ incrementing x, change with iteration). Idempotence relative to ; is tied in [FWV19] to the homogeneity of statives assumed in [Dow79].

Under the present approach, the $\mathit{bc}/\mathsf{d}_\Box$ divide between statives and transitions translates to a contrast between MSO_A-formulas χ_A^{bc} and $\chi_A^{\mathsf{d}_\Box}$ prescribing "no steps$_S$ without change$_A$"

$$\chi_A^{\mathit{bc}} \ := \ \forall x \forall y (xSy \supset \Delta_A(x, y)) \quad \text{ where } \Delta_A(x, y) := \neg \bigwedge_{a \in A} (P_a(x) \equiv P_a(y))$$

and "no time without change$_A$"

$$\chi_A^{\mathsf{d}_\Box} \ := \ \forall x \bigvee_{a \in A} P_a(x) \quad \text{(each } a \in A \text{ specifying a transition)}$$

(respectively) in that for any $s \in (2^A)^*$,

$$s = \mathit{bc}(s) \iff \text{Mod}_A(s) \models \chi_A^{\mathit{bc}} \qquad \text{and} \qquad s = \mathsf{d}_\Box(s) \iff \text{Mod}_A(s) \models \chi_A^{\mathsf{d}_\Box}$$

[Fer19b]. Coming back to the A-interpretation $[\![(\Sigma, s)]\!]_A \subseteq \mathcal{L}_A$ above, for stative Σ, we let $\mathcal{L}_A$ be the image of $(2^A)^*$ under bc for the stutterless strings in $(2^A)^*$

$$\mathcal{L}_A \ := \ \{\mathit{bc}(s) \mid s \in (2^A)^*\} \ = \ \{s \in (2^A)^* \mid s = \mathit{bc}(s)\} \ = \ \{s \in (2^A)^* \mid \text{Mod}_\Sigma(s) \models \chi_A^{\mathit{bc}}\}$$

and similarly for transitional Σ, let $\mathcal{L}_A$ be the image of $(2^A)^*$ under $\mathsf{d}_\Box$ for the *S-words* of [DS08]

$$\mathcal{L}_A \ := \ \{\mathsf{d}_\Box(s) \mid s \in (2^A)^*\} \ = \ (2^A - \{\Box\})^* \ = \ \{s \in (2^A)^* \mid \text{Mod}_\Sigma(s) \models \chi_A^{\mathsf{d}_\Box}\}.$$

In either case, f_A is the identity function on $\mathcal{L}_A$

$$f_A(s) = s \qquad \text{for any } s \in \mathcal{L}_A$$

and the family $\{f_\Sigma\}_{\Sigma \subseteq A}$ of functions indexed by subsets of A forms a projective system with composition amounting to intersection

$$f_{\Sigma'}; f_\Sigma \ = \ f_{\Sigma \cap \Sigma'}. \tag{3}$$

As we will see presently, (3) allows us to simplify sets $\mathsf{r} \subseteq 2^A \times \mathcal{L}_A$ of pairs (Σ, s) interpreted at A by intersection

$$[\![\mathsf{r}]\!]_A := \bigcap_{(\Sigma, s) \in \mathsf{r}} [\![(\Sigma, s)]\!]_A \quad \text{ where } \quad [\![(\Sigma, s)]\!]_A := \{s' \in \mathcal{L}_A \mid f_\Sigma(s') = s\}.$$

3 First-order fragments for string iconicity

Throughout this section, a choice between stative and transitional Σ is assumed that fixes f_Σ and $\mathcal{L}_A$ accordingly (using bc for the stative case, and $\mathsf{d}_\Box$ for the transitional). That choice is made explicit only as needed, to simplify the notation (suppressing the relativization of the notions of L-traits and A-profiles defined below to bc or $\mathsf{d}_\Box$). Given a non-empty language $L \subseteq (2^A)^*$, an *L-trait* is a pair (Σ, s) of a subset Σ of A and a string $s \in \mathcal{L}_\Sigma$ such that

$$f_\Sigma(s') = s \quad \text{for every } s' \in L.$$

The *trait record of* L, $\mathrm{tr}_A(L)$, is the set of L-traits. This subset of $2^A \times \mathcal{L}_A$ is related to any $\mathsf{r} \subseteq 2^A \times \mathcal{L}_A$ by the (anti-tone) Galois connection

$$\mathsf{r} \subseteq \mathrm{tr}_A(L) \iff L \subseteq [\![\mathsf{r}]\!]_A$$

recalling from the previous section that $[\![\mathsf{r}]\!]_A$ is defined as $\bigcap_{(\Sigma,s)\in\mathsf{r}}\{s' \in \mathcal{L}_A \mid f_\Sigma(s') = s\}$. As the indexed family $\{f_\Sigma\}_{\Sigma\subseteq A}$ forms a projective system, we can reduce any $\mathsf{r} \subseteq 2^A \times \mathcal{L}_A$ such that $[\![\mathsf{r}]\!]_A \neq \emptyset$ as follows, without changing its interpretation $[\![\mathsf{r}]\!]_A$. Let us call r an *A-profile* if

(i) r is functional (deterministic): for all $(\Sigma, s), (\Sigma, s') \in \mathsf{r}$, $s = s'$

(ii) $\mathit{domain}(\mathsf{r})$ is an anti-chain:

$$(\forall \Sigma, \Sigma' \in \mathit{domain}(\mathsf{r}))\ \Sigma \subseteq \Sigma' \text{ implies } \Sigma = \Sigma'$$

(iii) for all $(\Sigma, s) \in \mathsf{r}$, $s \in \mathcal{L}_\Sigma$.

For example, we can reduce the trait record $\mathrm{tr}_A(L)$ to two A-profiles by restricting its domain to subsets of A with exactly two elements

$$\mathrm{tr2}_A(L) := \ \{(\Sigma, s) \in \mathrm{tr}_A(L) \mid \text{cardinality}(\Sigma) = 2\}$$

or to $\subseteq$-maximal subsets

$$\mathrm{mtr}_A(L) := \ \{(\Sigma, s) \in \mathrm{tr}_A(L) \mid (\forall (\Sigma', s') \in \mathrm{tr}_A(L))\ \Sigma \subseteq \Sigma' \text{ implies } \Sigma' = \Sigma\}$$

without straying from the A-interpretation of $\mathrm{tr}_A(L)$

$$[\![\mathrm{tr2}_A(L)]\!]_A \ = \ [\![\mathrm{tr}_A(L)]\!]_A \ = \ [\![\mathrm{mtr}_A(L)]\!]_A.$$

Restricting $\mathrm{tr}_A(L)$ to singletons may alter its interpretation, as $L = \{\boxed{a}\boxed{a'}\}$ shows

$$[\![\mathrm{tr}_A(L)]\!]_A \ \neq \ [\![\{(\Sigma, s) \in \mathrm{tr}_A(L) \mid \text{cardinality}(\Sigma) = 1\}]\!]_A \ \ni \ \boxed{a}\boxed{a, a'}\boxed{a'} \quad \text{for stative } a, a'.$$

For slightly more complicated languages L, we can delete a pair from $\mathrm{tr2}_A(L)$ without changing its interpretation, as is clear from the implication

$$\mathit{bc}_{\{a,a'\}}(s) = \boxed{}\boxed{a}\boxed{a'}\boxed{} \text{ and } \mathit{bc}_{\{a,a'\}}(s) = \boxed{}\boxed{a'}\boxed{a''}\boxed{} \quad \text{imply} \quad \mathit{bc}_{\{a,a''\}}(s) = \boxed{}\boxed{a}\boxed{}\boxed{a''}\boxed{}$$

behind the meets-meets entry for the transitivity table of [All83] (Figure 4, page 836) Redundancies in $\mathrm{tr2}_A(L)$ are an argument for preferring the A-profile $\mathrm{mtr}_A(L)$, although there is no denying the popularity of binary relations on intervals labeling interval networks.

A simple notational variant of A-profiles is provided by formulas φ generated by

$$\varphi \ ::= \ s \mid \langle\Sigma\rangle\varphi \mid \varphi \wedge \varphi' \qquad (\Sigma \subseteq A) \tag{4}$$

driving the semantic interpretations

$$[\![s]\!]_A := \{s\} \cap \mathcal{L}_A \qquad [\![\langle\Sigma\rangle\varphi]\!]_A := \{s \in \mathcal{L}_A \mid f_\Sigma(s) \in [\![\varphi]\!]_A\} \qquad [\![\varphi \wedge \varphi']\!]_A := [\![\varphi]\!]_A \cap [\![\varphi']\!]_A$$

matching A-profiles

$$\{[\![\varphi]\!]_A \mid \varphi \text{ from (4)}\} \ = \ \{[\![\mathsf{r}]\!]_A \mid \mathsf{r} \text{ is an } A\text{-profile}\} \cup \{\emptyset\}.$$

A larger family of languages is expressed by *first-order* formulas φ generated by

$$\varphi \ ::= \ P_a(x) \mid x < y \mid x = y \mid \neg\varphi \mid \varphi \wedge \varphi' \mid \exists x\varphi \qquad (a \in A) \tag{5}$$

with $<$ in place of S, to match *star-free* expressions (such as strings s), obtained from regular expressions by substituting complementation for Kleene star $\cdot^*$ (e.g., [Lib04], §7.5). The clause for $\langle\Sigma\rangle\varphi$ in (4) is mirrored by relativizing a formula φ from (5) to φ_Σ by induction

$$\varphi_\Sigma := \varphi \ \text{ for atomic } \varphi \qquad (\neg\varphi)_\Sigma := \neg(\varphi_\Sigma) \qquad (\varphi \wedge \psi)_\Sigma := \varphi_\Sigma \wedge \psi_\Sigma$$

$$(\exists x\varphi)_\Sigma := \exists x(V_\Sigma(x) \wedge \varphi_\Sigma) \ \text{ where } \ V_\Sigma(x) := \begin{cases} \forall y(xSy \supset \Delta_\Sigma(x,y)) & \text{for stative } \Sigma \\ \bigvee_{a\in\Sigma} P_a(x) & \text{for transitional } \Sigma \end{cases}$$

supporting the induction hypothesis (on φ) that

$$\mathrm{Mod}(s), g \models \varphi_\Sigma \iff \mathrm{Mod}(f_\Sigma(s)), \hat{g} \models \varphi$$

for all $s \in (2^A)^*$ and assignments $g : \mathit{Var} \to [\mathrm{length}(s)]$ of variables x to $g(x) \in [\![V_\Sigma]\!]_s$

$$\mathrm{Mod}(s), g \models V_\Sigma(x) \quad \text{for each } x \in \mathit{Var}$$

(where $\hat{g}$ is the obvious f_Σ-adjustment of g). Missing from (4) above is a clause for negation, yielding disjunction (by conjunction and De Morgan). It is questionable whether negation or disjunction preserves form-meaning resemblance, which negation denies and spurious disjuncts spoil. (Hence, *no* $\neg$ or $\vee$ in (4).) While any non-empty subset of Allen interval relations may label an arc in an interval network, a move away from arbitrary disjunctions towards neighborhoods of similar configurations is advocated in [Fre92], §2.2. Forms (Σ, s) of strings with Σ-projection s represent such neighborhoods. Beyond (4), we can generalize f_Σ to mereological relations R (e.g., subsumption $\trianglerighteq$; [Fer15b]) to interpret accessibility in iconic forms $\Diamond_R\varphi$, producing non-determinism via underspecification (as opposed to overspecified disjunction).

4 Conclusion

The significance of iconicity, resemblance of form to meaning, is an issue open to dispute even in pictorial representation, positions about which differ based on how the issue is framed (e.g., [Gre13]). Broadly construed, iconicity points to transparent representations that facilitate the communication and inferential processing of information up to a limited but useful extent. The present work applies that broad construal to the experience of change, proposing representations around string iconicity. For any non-empty set L of strings of sets, an iconic star-free approximation, $\mathrm{mtr}_A(L)$, of L is formed from maximal L-traits, eschewing arbitrary disjunctions (inimical to iconicity and underspecification). I close by drawing attention to two tensions running through the work above.

The first is between iconicity and abstraction. A-profiles are iconic expressions carving out a small fragment of first-order formulas in MSO, as the grammars (4) and (5) bring out. Exactly what the Σ-projections f_Σ (behind A-profiles) come to, for stative and transitional Σ, are described above using first-order formulas $V_\Sigma(x)$ that fall outside the fragment generated by (4). That is, decidedly non-iconic MSO formulas are employed to explain iconic expressions. Similarly, pictures (exemplifying iconicity) can be described, when paired with viewpoints, by predicates that are (in some cases) stative (in accordance with [Abu14]'s argument that "all pictures have stative informational content") and (in other cases) non-stative, accounting for temporal progression under an inertial constraint of change requiring force/action ([Fer19a]). The basic claim of [Fer19a] is that for all its abstractness (non-iconicity), MSO is a helpful tool in breaking down and piecing together the meaning of pictorial narratives.

The second tension is between the temporal ontology expressed by the successor predicate S and the vocabulary Σ for articulating a particular perspective. S and Σ are linked by $V_\Sigma(x)$, transparently in the stative case ("no step$_S$ without change$_\Sigma$") and less overtly in the transitional case ("no time without change$_\Sigma$"), S being buried in what it means for an element of Σ to be transitional. With respect to iconicity of order, Σ may represent contextual dimensions other than time, constituting, for example, viewpoints in viewpoint-centered propositions for pictorial narratives. It is not unreasonable to assume that "the order of elements in language" (natural or formal) is significant, even if that is complicated by a wealth of factors behind "physical experience or the order of knowledge." These factors presumably shape discourse structure, used in [AL03] to explain *push-fall* temporal reversals and other challenges to order iconicity, as well as types recording the interaction between speakers and listeners around questions under discussion ([CG15]). [Ste05]'s proposal that there is more to temporality in natural language than temporal order suggests an understanding of iconicity of order that subordinates time given by S to notions of causality and contingency expressed through Σ.

References

[Abu14] D. Abusch. Temporal succession and aspectual type in visual narrative. In L. Crnič and U. Sauerland, editors, *The Art and Craft of Semantics: A Festschrift for Irene Heim*, volume 1, pages 9–29. MIT Working Papers in Linguistics 70, 2014.

[AR17] D. Abusch and M. Rooth. The formal semantics of free perception in pictorial narratives. In *Proc. 21st Amsterdam Colloquium*, pages 85–95. University of Amsterdam, 2017.

[All83] J. Allen. Maintaining knowledge about temporal intervals. *Communications of the ACM*, 26(11):832–843, 1983.

[AL03] N. Asher and A. Lascarides. *Logics of Conversation*. Cambridge University Press, 2003.

[CG15] R. Cooper and J. Ginzburg. TTR for natural language semantics. In S. Lappin and C. Fox, editors, *Handbook of Contemporary Semantic Theory*, pages 375–407. Wiley-Blackwell, second edition, 2015.

[Dow79] D.R. Dowty. *Word Meaning and Montague Grammar*. Reidel, Dordrecht, 1979.

[Dow86] D.R. Dowty. The effects of aspectual class on the temporal structure of discourse: Semantics or pragmatics? *Linguistics and Philosophy*, 9(1):37–61, 1986.

[DS08] I.A. Durand and S.R. Schwer. A tool for reasoning about qualitative temporal information: the theory of S-languages with a Lisp implementation. *Journal of Universal Computer Science*, 14(20):3282–3306, 2008.

[Fer15a] T. Fernando. Negation and events as truthmakers. In *Proc. 20th Amsterdam Colloquium*, pages 109–118. University of Amsterdam, 2015.

[Fer15b] T. Fernando. The semantics of tense and aspect: A finite-state perspective. In S. Lappin and C. Fox, editors, *Handbook of Contemporary Semantic Theory*, pages 203–236. Wiley-Blackwell, second edition, 2015.

[Fer16] T. Fernando. Prior and temporal sequences for natural language. *Synthese*, 193(11):3625–3637, 2016.

[Fer19a] T. Fernando. Pictorial narratives and temporal refinement. *Proc. 29th Semantics and Linguistic Theory*, 2019. Draft available at www.scss.tcd.ie/Tim.Fernando/salt29.pdf.

[Fer19b] T. Fernando. Projecting temporal properties, events and actions. In *Proc. 13th International Conference on Computational Semantics*, pages 1–12. www.aclweb.org/anthology/W19-0401, 2019.

[FWV19] T. Fernando, D. Woods, and C. Vogel. MSO with tests and reducts. *Proc. 14th International Conference on Finite-State Methods and Natural Language Processing*, pages 27–36, 2019. www.aclweb.org/anthology/W19-3106/.

[Fil70] C.J. Fillmore. The grammar of hitting and breaking. In R.A. Jacobs and P.S. Rosenbaum, editors, *Readings in English Transformational Grammar*, pages 120–133. Ginn, 1970.

[Fok07] W. Fokkink. *Introduction to Process Algebra*. Springer, 2nd edition, 2007. www.cs.vu.nl/∼wanf/BOOKS/procalg.pdf.

[Fre92] C. Freksa. Temporal reasoning based on semi-intervals. *Artificial Intelligence*, 54:199–227, 1992.

[GJW18] P. Gärdenfors, J. Jost, and M. Warglien. From actions to effects: Three constraints on event mappings. *Frontiers in Psychology*, 9:1391, 2018.

[GB92] J.A. Goguen and R.M. Burstall. Institutions: abstract model theory for specification and programming. *Journal of the ACM*, 39(1):95–146, 1992.

[Gre13] G. Greenberg. Beyond resemblance. *Philosophical Review* 122(2), 215–287, 2013.

[Gre63] J. Greenberg. *Universals of Language*. MIT Press, 1963.

[Ham71] C.L. Hamblin. Instants and intervals. *Studium generale*, 24:127–134, 1971.

[HKT00] D. Harel, D. Kozen, and J. Tiuryn. *Dynamic Logic*. MIT Press, 2000.

[KR93] H. Kamp and U. Reyle. *From Discourse to Logic*. Kluwer, 1993.

[KS96] D. Kozen and F. Smith. Kleene algebra with tests: Completeness and decidability. In *Proc. 10th Int. Workshop Computer Science Logic*, LNCS 1258, pages 244–259. Springer, 1996.

[LRH13] B. Levin and M. Rappaport Hovav. Lexicalized meaning and manner/result complementarity. In B. Arsenijević, B. Gehrke, and R. Marín, editors, *Subatomic Semantics of Event Predicates*, pages 49–70. Springer, 2013.

[Lib04] L. Libkin. *Elements of Finite Model Theory*. Springer, 2004.

[MB19] E. Maier and S. Bimpikou. Shifting perspectives in pictorial narratives. In *Proc. Sinn und Bedeutung 23(2)*, pages 91–106. Barcelona, 2019.

[New92] F.J. Newmeyer. Iconicity and generative grammar. *Language*, 68(4):756–96, 1992.

[PV14] P. Perniss and G. Vigliocco. The bridge of iconicity: from a world of experience to the experience of language. *Phil. Trans. R. Soc. B*, 369: 20130300, 2014.

[Pri67] A.N. Prior. *Past, Present and Future*. Clarendon Press, Oxford, 1967.

[Ste05] M. Steedman. The Productions of Time, 2005. Draft tutorial notes about temporal semantics, http://homepages.inf.ed.ac.uk/steedman/papers.html.

[vLH05] M. van Lambalgen and F. Hamm. *The Proper Treatment of Events*. Blackwell, 2005.

[Zim00] T.E. Zimmermann. Free choice disjunction and epistemic possibility. *Natural Language Semantics*, 8:255–290, 2000.