String iconicity & granularity

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18 Dec 2019, Amsterdam

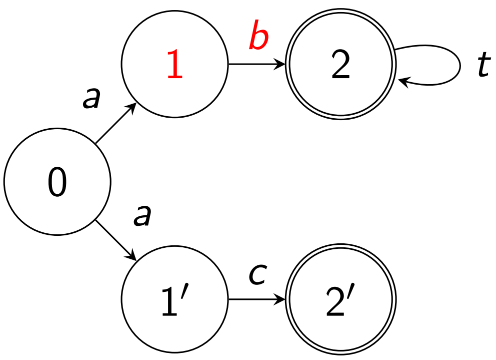
$$
\begin{aligned}
[\![\text{expression}]\!] &= \text{meaning} \\
[\![s]\!] &= \{s\}
\end{aligned}
$$

$$\boxed{q_0} \xrightarrow{a_1} \boxed{q_1} \xrightarrow{a_2} \cdots \xrightarrow{a_n} \boxed{q_n} \qquad (a_i \in \Sigma)$$

$$[\![(\Sigma, s)]\!]_A = \{s' \in \mathcal{L}_A \mid \underbrace{f_\Sigma(s') = s}_{s\ \Sigma\text{-approximates } s'}\} \qquad \text{for } \Sigma \subseteq A$$

Non-distributivity: process algebra Dynamic logic

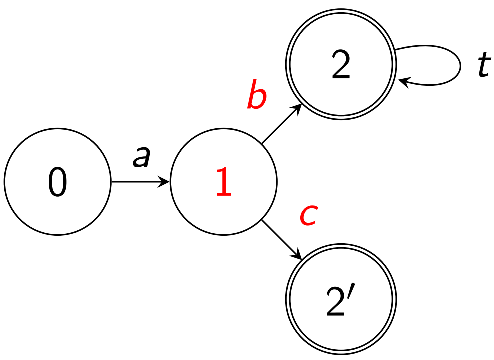
$abt^* + ac$



$q \models \langle l\rangle\top$ iff $\exists q'\ q \xrightarrow{l} q'$

$$\boxed{\langle a\rangle\top}\boxed{\langle b\rangle\top}\boxed{\langle t\rangle\top}^+ + \boxed{\langle a\rangle\top}\boxed{\langle c\rangle\top}\boxed{\ }$$

$a(bt^* + c)$



$$\boxed{\langle a\rangle\top}\overbrace{\boxed{\langle b\rangle\top, \langle c\rangle\top}}^{\text{symbol-as-set}}$$

$$(\boxed{\langle t\rangle\top}^+ + \boxed{\ })$$

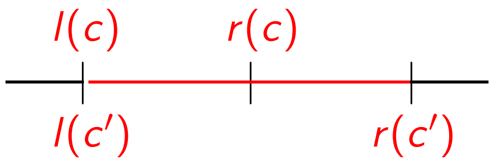
"disjunctions are conjunctive lists of epistemic possibilities"

Zimmermann 2000

Statives vs transitions

$$q \xrightarrow{a} q' \quad \left\{ \begin{array}{lll} \text{result } q' & \textit{break}/\text{result} & \text{BEC} \\ \text{action } a & \textit{hit}/\text{manner} & \text{DO} \end{array} \right. \qquad \begin{array}{c} \boxed{\neg\varphi}\boxed{\varphi} \\ \boxed{\mathrm{ap}(a)}\boxed{\mathrm{ef}(a)} \end{array}$$

Gärdenfors   Fillmore/LevinRH   Dowty



$c \approx (l(c), r(c)]$

Transition	$\boxed{l(c), l(c')}\boxed{r(c)}\boxed{r(c')}$	border	*Happens*
Stative	$\boxed{}\boxed{c, c'}\boxed{c'}\boxed{}$	interior	*Holds*

Strings as models and reducts

$\boxed{b, a}\boxed{} \models \exists x(P_b(x) \wedge \exists y(xSy))$ % b occurs at a non-final position

String $\alpha_1 \cdots \alpha_n \in (2^A)^*$ as the model with universe/domain

$$[n] := \{1, \ldots, n\}$$

interpreting P_a and S as

$$\begin{array}{rl} [\![P_a]\!] & := \{i \in [n] \mid a \in \alpha_i\} \qquad (\text{for each } a \in A) \\ [\![S]\!] & := \{(1,2), \ldots, (n-1, n)\}. \end{array}$$

$$B\text{-reduct} \qquad \rho_B(\alpha_1 \cdots \alpha_n) := (\alpha_1 \cap B) \cdots (\alpha_n \cap B)$$

$$\rho_{\{b\}}(\boxed{b, a}\boxed{}) = \boxed{b}\boxed{}$$

$$M \models \varphi \iff M \restriction \text{vocabulary}(\varphi) \models \varphi$$

Compression two ways

stative (homogeneous)	transition (punctual)
$bc : s\alpha\alpha s' \rightsquigarrow s\alpha s'$	$\mathrm{d}_\square : s\square s' \rightsquigarrow ss'$

$$\{bc(s) \mid s \in (2^C)^*\} = \{s \in (2^C)^* \mid bc(s) = s\}$$
$$= [\![\forall x \forall y (xSy \supset \neg \bigwedge_{c \in C} (P_c(x) \equiv P_c(y)))]\!]_C \quad \text{“steps}_S \text{ implies change}_C\text{”}$$

$$\{\mathrm{d}_\square(s) \mid s \in (2^\Sigma)^*\} = \{s \in (2^\Sigma)^* \mid \mathrm{d}_\square(s) = s\} = (2^\Sigma - \{\square\})^*$$
$$= [\![\forall x \bigvee_{a \in \Sigma} P_a(x)]\!]_\Sigma \quad \text{“no time without change}_\Sigma\text{”}$$

$$[\![(\Sigma, s)]\!]_A \ := \ \{s' \in \mathcal{L}_A \mid f_\Sigma(s') = s\}$$

	stative	transitional
$\mathcal{L}_A$	$\{bc(s) \mid s \in (2^A)^*\}$	$\{\mathrm{d}_\square(s) \mid s \in (2^A)^*\}$
f_Σ	$\rho_\Sigma; bc$	$\rho_\Sigma; \mathrm{d}_\square$

Border translation

	stative (interior)	transition (border)
c meets c'	$\boxed{\ }\boxed{c}\boxed{c'}\boxed{\ }$	$\boxed{l(c)}\boxed{r(c), l(c')}\boxed{r(c')}$

$$C_\bullet \ := \ \{l(c) \mid c \in C\} \cup \{r(c) \mid c \in C\}$$

$\mathbf{b}_C : (2^C)^* \to (2^{C_\bullet})^*, \ \ \alpha_1 \cdots \alpha_n \mapsto \beta_1 \cdots \beta_n$ where

$$\beta_i := \{l(c) \mid c \in \alpha_{i+1} - \alpha_i\} \ \cup \ \{r(c) \mid c \in \alpha_i - \alpha_{i+1}\} \quad \text{for } i < n$$
$$\beta_n := \{r(c) \mid c \in \alpha_n\}$$

e.g. $\mathbf{b}_{\{c,c'\}}(\boxed{c'}\boxed{c}\boxed{c}) \ = \ \boxed{l(c), r(c')}\boxed{\ }\boxed{r(c)}$

$\boxed{c'}\boxed{c} \qquad \boxed{l(c), r(c')}\boxed{r(c)}$

$$\mathbf{b}_C(bc(s)) = \mathrm{d}_\square(\mathbf{b}_C(s)) \ \text{ for } s \in (2^C)^* \ \textit{not} \text{ ending in } \square$$

Some iconic string expressions

$$\varphi \ ::= s \mid \langle\Sigma\rangle\varphi \mid \varphi \wedge \varphi' \qquad (\dagger)$$

$$\begin{aligned} [\![s]\!]_A &:= \{s\} \qquad \text{for } s \in \mathcal{L}_A \\ [\![\langle\Sigma\rangle\varphi]\!]_A &:= \mathcal{L}_A \cap f_\Sigma{}^{-1}[\![\varphi]\!]_A \\ [\![\varphi \wedge \varphi']\!]_A &:= [\![\varphi]\!]_A \cap [\![\varphi']\!]_A \end{aligned}$$

We can describe any $L \subseteq \mathcal{L}_A$ by

$$\bigwedge\{\langle\Sigma\rangle s \mid \Sigma \subseteq A,\ s \in \mathcal{L}_A \text{ and } f_\Sigma L = \{s\}\}$$

and reformulate each φ from $(\dagger)$ as a record

$$\{(\Sigma_1, s_1), \ldots, (\Sigma_n, s_n)\}$$

A first-order fragment of MSO

$$\varphi \ ::= \ P_a(x) \mid x < y \mid x = y \mid \neg\varphi \mid \varphi \wedge \varphi' \mid \exists x\varphi \qquad (a \in A)$$

For $\langle\Sigma\rangle\varphi$, restrict $\exists x$ to

$$V_\Sigma(x) := \begin{cases} \forall y(xSy \supset \neg \bigwedge_{a\in\Sigma}(P_a(x) \equiv P_a(y))) & \text{for stative } \Sigma \\ \bigvee_{a\in\Sigma} P_a(x) & \text{for transitional } \Sigma \end{cases}$$

relativizing φ to φ_Σ

$$(\exists x\varphi)_\Sigma := \exists x(V_\Sigma(x) \wedge \varphi_\Sigma) \qquad (\neg\varphi)_\Sigma := \neg(\varphi_\Sigma) \quad \cdots$$

for $\qquad s \models \varphi_\Sigma \iff f_\Sigma(s) \models \varphi$

Satisfaction condition (Goguen & Burstall's *institution*)

$\vee$ is inimical to iconicity, adding spurious possibilities

One picture followed by another

$\boxed{p}\boxed{p'}$ (*and then*: default progression)

$\boxed{p, p'}\boxed{p, p'}$ (inertial flow, stutter)

$\boxed{p, p'}$ (*no* temporal progression)

No change without force - INERTIA

No time without change - ARISTOTLE, bc

- forces may intervene + pictures are not wholly stative

  A picture's worth a thousand words – some stative, some not

- model-theoretic interpretation (MSO)
  + projective system sensitive to stative - transitional divide

Iconicity of order (Newmeyer)

order of elements in language parallels that in physical experience
J. Greenberg 1966

Experience in time, from a $\underbrace{\text{place, by a bounded experiencer}}_{\text{viewpoint (Hagen ...)}}$
$+$

Δ in any contextual dimension $\rightsquigarrow$ multiple pictures at same time

$$\cdots \boxed{p, p', \cdots} \cdots$$

$$\cdots \boxed{\text{from}(p, v), \text{from}(p', v'), \cdots} \cdots$$

$$P_p(x) \equiv \bigvee_{v \in V} P_{\text{from}(p,v)}(x)$$

finite classification V of viewpoints (bounded granularity)

From strings to languages

> *it seems downright wrong to insist that everything that happens in a possible history, let alone separate possible histories, be mappable onto a single time line.*
>
> E. Bach 1986

Simultaneous kick-offs: Sweden vs Mexico $\parallel$ Germany vs sKorea

$$(\Sigma_{swedenVmexico}, s), \ (\Sigma_{germanyVskorea}, s')$$

- structure around experiencer (viewpt trumps global clock)
- don't interleave s and s' unless viewing is simultaneous

$$\begin{array}{ccc} \boxed{a_1}\boxed{b_1}\boxed{} & & \\ \vdots & \text{vs} & \boxed{a_1, \ldots, a_n}\, L\boxed{} \\ \boxed{a_n}\boxed{b_n}\boxed{} & & \\ O(n) & & \Omega(n! \cdot n) \end{array}$$

Take-away

$$s \ \rightsquigarrow \ \{(\Sigma_1, s_1), \ldots, (\Sigma_n, s_n)\} \ \rightsquigarrow_A \ \bigcap_{i=1}^{n} f_{\Sigma_i}{}^{-1}\{s_i\}$$

$$f_\Sigma \ = \ \Sigma\text{-reduct; compress (2 ways)}$$

T h a n k   Y o u

☺