Temporal Propositions as Vague Predicates

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Abstract. The idea that temporal propositions are vague predicates is examined with attention to the nature of the objects over which the predicates range. These objects should not, it is argued, be identified once and for all with points or intervals in the real line (or any fixed linear order). Context has an important role to play not only in sidestepping the Sorites paradox (Gaifman 2002) but also in shaping temporal moments/extent (Landman 1991). The Russell-Wiener construction of time from events (Kamp 1979) is related to a notion of context given by a string of observations, the vagueness in which is brought out by grounding the observations in the real line. With this notion of context, the context dependency functions in Gaifman 2002 are adapted to interpret temporal propositions.

1 Introduction

Fluents, as temporal propositions are commonly known in AI, have in recent years made headway in studies of events and temporality in natural language semantics (e.g. Steedman 2000, van Lambalgen and Hamm 2005). The present paper concerns the bounded precision implicit in sentences such as (1).

(1) Pat reached the summit of K2 at noon, and not a moment earlier.

Presumably, a moment in (1) is less than an hour but greater than a picosecond. Whether or not determining the exact size of a moment is necessary to interpret or generate (1), there are pitfalls well known to philosophers that lurk. One such danger is the Sorites paradox, which is commonly associated not so much with time as with vagueness. Focusing on time, Landman emphasizes the importance of context.

> It is not the abstract underlying time structure that is semantically crucial, but the system of temporal measurements. We shouldn't ask just 'what is a moment of time', because that is a context dependent question. We can assume that context determines how precisely we are measuring time: it chooses in the hierarchy of temporal measurements one measurement that is taken as 'time as finely grained as this context requires it to be.' The elements of that measurement are then regarded as moments *in that context.* (Landman 1991, page 138)

Different notions of context are explored below. We start in section 2 with the use of context in Gaifman 2002 to sidestep the Sorites paradox before returning

M. Aloni et al. (Eds.): Amsterdam Colloquium 2009, LNAI 6042, pp. 143–152, 2010.

in the succeeding sections to the special case of time. A basic aim is to critically examine the intuition that the temporal extent of an event is an interval — an intuition developed in Kamp 1979, Allen 1983 and Thomason 1989, among other works.

A concrete linguistic question concerning intervals is brought out by (2).

(2) John drank beer from nine to ten.

(2) is from Landman 1991, in page 137 of which we read

> What should this mean? At some time in ⟦*from nine to ten*⟧ John drank beer? At all times in ⟦*from nine to ten*⟧ John drank beer? At most times in ⟦*from nine to ten*⟧ John drank beer? This is just not clear ...

Framed as a choice between different quantificational readings, the problem in understanding (2) becomes one of ambiguity rather than vagueness. (Multiple non-equivalent logical forms is commonly understood to indicate ambiguity, in contrast to a single logical form with various divergent interpretations, marking vagueness.) The approach pursued below puts vagueness ahead of ambiguity in asking not about quantificational force, but about the domain of quantification (arguably a more fundamental question). What is at issue is a notion of time that, as previously mentioned, depends on context. Ignoring tense and aspect (for the sake of simplicity), a minimal context where (2) is true is the cartoon strip (3) describing three temporal stretches in which John drinks beer, the first at nine, and the third at ten.

(3)

nine, drink-beer(john)	drink-beer(john)	ten, drink-beer(john)

Is (3) consistent with times between nine and ten at which John does not drink beer? Although (3) describes no such time, there remains the possibility of refining or interpreting (3) to grant John a break between beers. Motivating and fleshing out that possibility is a large part of what this paper is about.

2 Sorites and Appropriate Contexts

The tolerance of a unary predicate P to small changes is expressed in Gaifman 2002 through conditionals of the form (4).

(4) $N_P(x, y) \;\rightarrow\; (P(x) \;\rightarrow\; P(y))$

P is asserted in (4) to be tolerant insofar as P holds of y whenever P holds of an x that is N_p-near y. Repeatedly applying (4), we conclude $P(z)$, given any finite sequence $y_1, \ldots, y_n$ such that $y_n = z$, $P(y_1)$ and $N_P(y_i, y_{i+1})$ for $1 \leq i < n$. A *Sorites chain* is a sequence $y_1, \ldots, y_n$ such that P holds of y_1 but not y_n, even though $N_P(y_i, y_{i+1})$ for $1 \leq i < n$. Gaifman's way out of the Sorites paradox is to interpret P against a *context dependency function* f mapping a finite set C

(of objects in a first-order model) to a subset $f(C)$ of C, understood to be the extension of P at "context" C. (In effect, the predication $P(x)$ becomes $P(x, C)$, for some comparison class C that contains x.) The idea then is to pick out finite sets C that do *not* contain a Sorites chain. Such sets are called *feasible contexts*.[1] Formally, Gaifman sets up a *Contextual Logic* preserving classical logic in which tolerance conditionals (4) can be sharpened to (5), using a construct $[C]$ to constrain the contexts relative to which $P(x)$ and $P(y)$ are interpreted.

(5) $[C]\ (N_P(x, y) \rightarrow (P(x) \rightarrow P(y)))$

As contexts in Contextual Logic need not be feasible, (5) must be refined further to restrict C to feasible contexts

$$\text{feasible}(C) \rightarrow [C]\ (N_P(x, y) \rightarrow (P(x) \rightarrow P(y))).$$

The formal notation gets quite heavy, but the point is simple enough:

> sentences and proofs have associated contexts. Those whose contexts are feasible form the feasible portion of the language; and it is within this portion that a tolerant predicate is meant to be used. The proof of the Sorites contradiction fails, because it requires an unfeasible context and in unfeasible contexts a tolerant predicate looses [*sic*] its tolerance; it has some sharp cutoff. Unfeasible contexts do not arise in practice. (Gaifman 2002, pages 23, 24)

The obvious question is why not build into Contextual Logic only contexts that *do* "arise in practice" — viz. the feasible ones? For tolerant predicates in general, such a restriction may, as Gaifman claims, well result in a "cumbersome system." Fluents are, however, a very particular case of vague predicates, and surely practice is all that matters. That said, Contextual Logic leaves open the question of what the stuff of time is, demanding only that for every fluent P, finite sets C of times be picked out that validate (5), for a suitable interpretation of N_P. Such feasible contexts C avoid the sharp cutoffs characteristic of unfeasible contexts, and allow us to sidestep the difficulty of pinning down the precise moment of change by bounding granularity. Bounded granularity is crucial for making sense of talk about the first (or last) moment a fluent is true (or of claims that a fluent true at an interval is true at every non-null part of that interval).

3 Contexts for Temporal Extent

Where might a feasible context for a fluent come from? The present section traces context back to some (given) set E of events (of interest), making time just fine grained enough to compare events in E. The comparisons are, following Kamp 1979, given by binary relations on E of temporal overlap $\bigcirc$ and complete

[1] Unless I am mistaken, for C to be feasible, it suffices that there be *no* pairs a, b in C such that $N_P(a, b)$ and P holds (at C) of a but not b.

precedence $\prec$ constituting an *event structure* $\langle E, \bigcirc, \prec\rangle$ that satisfies (A_1) to (A_5).[2]

- (A_1) $e \bigcirc e$ (i.e. $\bigcirc$ is reflexive)
- (A_2) $e \bigcirc e'$ implies $e' \bigcirc e$
- (A_3) $e \prec e'$ implies not $e \bigcirc e'$
- (A_4) $e \prec e' \bigcirc e'' \prec e'''$ implies $e \prec e'''$
- (A_5) $e \prec e'$ or $e \bigcirc e'$ or $e' \prec e$

Before extracting temporal moments from $\langle E, \bigcirc, \prec\rangle$, it is useful for orientation to proceed in the opposite direction, forming an event structure from a linear order $\langle T, <\rangle$ and a relation $s \subseteq T \times E$ associating a time $t \in T$ with an event $e \in E$ according to the intuition that

$$s(t, e) \text{ says `} e \ s\text{-occurs at } t\text{'}.$$

It is natural to view s as a schedule, with temporal overlap $ov(s)$ holding between events e and e' that s-occur at some time in common

$$e \ ov(s) \ e' \overset{\text{def}}{\Leftrightarrow} (\exists t) \ s(t, e) \text{ and } s(t, e')$$

and an event e completely preceding another e' if e s-occurs only $<$-before e

$$e <_s e' \overset{\text{def}}{\Leftrightarrow} (\forall t, t' \text{ such that } s(t, e) \text{ and } s(t', e')) \ t < t' .$$

Proposition 1. $\langle E, ov(s), <_s\rangle$ *is an event structure provided*

(i) $<$ *linearly orders* T
(ii) $(\forall e \in E)(\exists t \in T) \ s(t, e)$, *and*
(iii) $s(t, e)$ *whenever* $s(t_0, e)$ *and* $s(t_1, e)$ *for some* $t_0 < t$ *and* $t_1 > t$.

Let us call $\langle s, T, <\rangle$ an *interval schedule* if it satisfies (i) – (iii).

Now for the Russell-Wiener construction in Kamp 1979 of time from an event structure $\langle E, \bigcirc, \prec\rangle$. We collect subsets of E any two in which $\bigcirc$-overlap in

$$O_{\bigcirc} \overset{\text{def}}{=} \{t \subseteq E \mid (\forall e, e' \in t) \ e \bigcirc e'\}$$

and equate temporal moments with $\subseteq$-maximal elements of $O_{\bigcirc}$

$$T_{\bigcirc} \overset{\text{def}}{=} \{t \in O_{\bigcirc} \mid (\forall t' \in O_{\bigcirc}) \ t \subseteq t' \text{ implies } t = t'\} .$$

[2] Seven postulates are given in Kamp 1979, but two are superfluous. In Thomason 1984, event structures are called event orderings, with $\bigcirc$ the complement of the union $\prec \cup \succ$ of $\prec$ and its converse $\succ$, and $\prec$ an irreflexive interval ordering

$$\text{not} \quad e \prec e$$
$$e_1 \prec e_2 \text{ and } e_1' \prec e_2' \quad \text{imply} \quad e_1 \prec e_2' \text{ or } e_1' \prec e_2 .$$

We then lift $\prec$ to $T_{\bigcirc}$ existentially

$$t \prec_{\bigcirc} t' \overset{\text{def}}{\Leftrightarrow} (\exists e \in t)(\exists e' \in t')\ e \prec e'$$

and define $sched_{\bigcirc} \subseteq T_{\bigcirc} \times E$ as the converse of membership

$$sched_{\bigcirc}(t, e) \overset{\text{def}}{\Leftrightarrow} e \in t$$

for all $t \in T_{\bigcirc}$ and $e \in E$.

Theorem 2. (Russell, Wiener, Kamp) $\langle sched_{\bigcirc}, T_{\bigcirc}, \prec_{\bigcirc}\rangle$ *is an interval schedule if* $\langle E, \bigcirc, \prec\rangle$ *is an event structure.*

Applying the transformations in Theorem 2 and Proposition 1 in sequence to an event structure yields the same event structure, but applying the transformations in reverse to an interval schedule may result in a different (reduced) interval schedule. The notion of time obtained from events is fine enough just to determine overlap $\bigcirc$ and complete precedence $\prec$ between events. But are there not other temporal relations to preserve?

Thirteen different relations between intervals are catalogued in Allen 1983, strung out below as sequences of snapshots (enclosed in boxes).

e equals e'	$\boxed{e, e'}$		
e meets e'	$\boxed{e}\boxed{e'}$	e met-by e'	$\boxed{e'}\boxed{e}$
e overlaps e'	$\boxed{e}\boxed{e, e'}\boxed{e'}$	e overlapped-by e'	$\boxed{e'}\boxed{e, e'}\boxed{e}$
e before e'	$\boxed{e}\square\boxed{e'}$	e after e'	$\boxed{e'}\square\boxed{e}$
e contains e'	$\boxed{e}\boxed{e, e'}\boxed{e}$	e during e'	$\boxed{e'}\boxed{e, e'}\boxed{e'}$
e finishes e'	$\boxed{e'}\boxed{e, e'}$	e finished-by e'	$\boxed{e}\boxed{e, e'}$
e starts e'	$\boxed{e, e'}\boxed{e'}$	e started-by e'	$\boxed{e, e'}\boxed{e}$

If a box must be $\subseteq$-maximal (as required by $T_{\bigcirc}$), only three of the thirteen strings above survive

$$\boxed{e}\boxed{e'} + \boxed{e'}\boxed{e} + \boxed{e, e'}\ .$$

To recover the ten other strings, it is useful to equip an event e with a pre-event $pre(e)$ and a post-event $post(e)$, enriching a schedule s to

$$s^{<} \overset{\text{def}}{=} s \ \cup\ s_{-}^{\leq}\ \cup\ s_{+}^{<}$$

where

$$s_{-}^{\leq} \overset{\text{def}}{=} \{\langle t, pre(e)\rangle \mid (\exists t' > t)\ s(t', e) \text{ and } (\forall t' \leq t) \text{ not } s(t, e)\}$$
$$s_{+}^{<} \overset{\text{def}}{=} \{\langle t, post(e)\rangle \mid (\exists t' < t)\ s(t', e) \text{ and } (\forall t' \geq t) \text{ not } s(t, e)\}$$

so that, for example, $\fbox{$e$}\Box\fbox{$e'$}$ becomes $\fbox{$e, pre(e')$}\fbox{$post(e), pre(e')$}\fbox{$post(e), e'$}$.[3] It is easy to see that if $\langle s, T, <\rangle$ is an interval schedule on E, then so is $\langle s^<, T, <\rangle$ on the extended set

$$E_s^< \stackrel{\text{def}}{=} \{y \mid (\exists t)\ s^<(t, y)\}$$

of events, and moreover each of the thirteen Allen relations between e and $e' \in E$ can be determined from the overlap relation $ov(s^<)$ induced by $s^<$ — e.g.

$$\begin{array}{rll} e \text{ before } e' & \text{iff} & post(e)\ ov(s^<)\ pre(e') \\ e \text{ meets } e' & \text{iff} & post(e)\ ov(s^<)\ e' \text{ but neither} \\ & & e\ ov(s^<)\ e' \text{ nor } post(e)\ ov(s^<)\ pre(e')\ . \end{array}$$

For the record,

Proposition 3. *For every interval schedule* $\langle s, T, <\rangle$, $\langle E_s^<, ov(s^<), \prec\rangle$ *is an event structure where* $\prec$ *is the precedence* $<_{s^<}$ *induced by* $<$ *and* $s^<$, *and for* $\bigcirc \stackrel{\text{def}}{=} ov(s^<)$,

$$T_\bigcirc \quad = \quad \{\{y \mid s^<(t, y)\} \mid t \in T\}\ .$$

According to the last equation, $T_\bigcirc$ does *not* discard a time $t \in T$ (as may be the case were $\bigcirc = ov(s)$) but merely identifies it with $t' \in T$ such that for all y,

$$s^<(t, y) \quad \text{iff} \quad s^<(t', y)\ .$$

This equivalence is, in general, stronger than one with s in place of $s^<$. Note also that for $T_\bigcirc$ to be finite, it suffices that E be finite (as $T_\bigcirc$ consists of subsets of $E_s^<$ for any schedule s, no matter how large T is). Even if E were not finite, we could apply the transformation described in Proposition 3 to the restriction

$$s \restriction X \stackrel{\text{def}}{=} \{(t, e) \in s \mid e \in X\}$$

of s to a finite subset X of E for a finitary approximation that, as we shall see next, can be pictured as a string.

4 Contexts as Strings

In this section, we equate contexts with strings $\in Pow(\Phi)^*$ over the alphabet $Pow(\Phi)$ of subsets of some fixed set Φ. Any relation $s \subseteq T \times E$ over a finite linear order $\langle T, <\rangle$ can be formulated as a string in $Pow(\Phi)^*$ with $\Phi = E \cup T$. For example, the schedule $\{(0, e), (1, e), (1, e'), (2, e), (2, e'), (3, e), (3, e'), (4, e')\}$ can be represented as (6), assuming the usual order on $\{0, 1, 2, 3, 4\}$.

[3] This construction marries and mangles ideas from Allen and Ferguson 1994 and Walker instants (Thomason 1984 , van Lambalgen and Hamm 2005). It goes without saying that for all $e, e', e'' \in E$, the set $\{e, pre(e'), post(e'')\}$ has cardinality 3.

(6) $\boxed{0, e}\boxed{1, e, e'}\boxed{2, e, e'}\boxed{3, e, e'}\boxed{4, e'}$

To restrict Φ to some subset X, we project a string in $Pow(\Phi)^*$ to one in $Pow(X)^*$ by componentwise intersection r_X with X

$$r_X(\alpha_1 \cdots \alpha_n) \stackrel{\text{def}}{=} (\alpha_1 \cap X) \cdots (\alpha_n \cap X)$$

so that for $X = E \cup \{0, 4\}$, r_X maps (6) to

$$\boxed{0, e}\boxed{e, e'}\boxed{e, e'}\boxed{e, e'}\boxed{4, e'}.$$

Another useful projection on $Pow(\Phi)^*$ is *block compression* π reducing adjacent identical boxes $\alpha\alpha^n$ in a string σ to one α

$$\pi(\sigma) \stackrel{\text{def}}{=} \begin{cases} \pi(\alpha\sigma') & \text{if } \sigma = \alpha\alpha\sigma' \\ \alpha\pi(\beta\sigma') & \text{if } \sigma = \alpha\beta\sigma' \text{ with } \alpha \neq \beta \\ \sigma & \text{otherwise} \end{cases}$$

so that, for example,

$$\pi(\boxed{0, e}\boxed{e, e'}\boxed{e, e'}\boxed{e, e'}\boxed{4, e'}) = \boxed{0, e}\boxed{e, e'}\boxed{4, e'}\ .$$

Underlying the projection π is the slogan "no time without change" (Kamp and Reyle 1993, page 674), reducing any string

$$\boxed{\text{nine, drink-beer(john)}}\boxed{\text{drink-beer(john)}}^n\boxed{\text{ten, drink-beer(john)}}$$

of length $n + 2$, for $n \geq 1$, to (3).

(3) $\boxed{\text{nine, drink-beer(john)}}\boxed{\text{drink-beer(john)}}\boxed{\text{ten, drink-beer(john)}}$

Next, to form the schedule induced by the set $T_{\bigcirc}$ in Proposition 3 as an inverse limit over finite subsets X of E, let the function $\pi_X : Pow(E)^* \rightarrow Pow(X)^*$ restrict the events to X before applying π

$$\pi_X(\sigma) \stackrel{\text{def}}{=} \pi(r_X(\sigma))\ .$$

An instructive example is provided by the real line $\langle \mathbb{R}, < \rangle$; for $X = \{-1, 2, 7\}$, the π_X-approximation of the schedule $\{(r, r) \mid r \in \mathbb{R}\}$ is

$$\Box\boxed{-1}\Box\boxed{2}\Box\boxed{7}\Box$$

with empty boxes $\Box$ from pre- and post-events. In general, the π_X-approximation of s is the string corresponding to the result of the construction in Proposition 3 applied to the restriction $s \lceil X$ of s to X.

Why bother turning interval schedules into strings? One reason is that we can relax the interval requirement (condition (iii) in Proposition 1) on strings, as π is well-behaved without such an assumption (whereas Russell-Wiener-Kamp relies on it). In particular, we can construe an element of Φ as a point-wise fluent, and reconceive the string $\boxed{pre(e)}\boxed{e}\boxed{post(e)}$ as $\alpha(e)\boxed{\text{oc}(e)}\omega(e)$ where the negation $\neg\mathsf{oc}(e)$ of an occurrence of e is split between

$$\alpha(e) \stackrel{\text{def}}{=} \boxed{\neg\mathsf{oc}(e), \neg Past(\mathsf{oc}(e)), Future(\mathsf{oc}(e))}$$
$$\omega(e) \stackrel{\text{def}}{=} \boxed{\neg\mathsf{oc}(e), Past(\mathsf{oc}(e)), \neg Future(\mathsf{oc}(e))}$$

not to mention

$$\text{hole}(e) \stackrel{\text{def}}{=} \boxed{\neg\mathsf{oc}(e), Past(\mathsf{oc}(e)), Future(\mathsf{oc}(e))}$$
$$\text{never}(e) \stackrel{\text{def}}{=} \boxed{\neg\mathsf{oc}(e), \neg Past(\mathsf{oc}(e)), \neg Future(\mathsf{oc}(e))} \; .$$

For a model-theoretic interpretation based on observations that take time greater than 0, we shall use open intervals in the real line that have length greater than some fixed real number $\epsilon > 0$. Let

$$\mathcal{O}_\epsilon \stackrel{\text{def}}{=} \{(a,b) \mid a,b \in \mathbb{R}_{\pm\infty} \text{ and } b > a + \epsilon\}$$

where

$$(a,b) \stackrel{\text{def}}{=} \{r \in \mathbb{R} \mid a < r < b\} \; .$$

Complete precedence in $\mathcal{O}_\epsilon$ is given by

$$o \prec_\epsilon o' \stackrel{\text{def}}{\Leftrightarrow} o, o' \in \mathcal{O}_\epsilon \text{ and } (\forall r \in o)(\forall r' \in o')\; r < r'$$

and abutment (or successors) by

$$o \; s_\epsilon \; o' \stackrel{\text{def}}{\Leftrightarrow} o \prec_\epsilon o' \text{ and not } (\exists o'' \prec_\epsilon o')\; o \prec_\epsilon o'' \; .$$

An ϵ-*chain* $\mathbf{c}$ is a sequence $o_1 \ldots o_n$ in $\mathcal{O}_\epsilon^*$ such that

$$o_1 \; s_\epsilon \; o_2 \; s_\epsilon \; o_3 \; \cdots \; s_\epsilon \; o_n \; .$$

A string $\alpha_1 \cdots \alpha_n \in Pow(\Phi)^*$ *holds at* an ϵ-chain $\mathbf{c} = o_1 \ldots o_n$ if for $1 \leq i \leq n$,

$$\mathbf{c}, o_i \models \varphi \;\text{ for every } \varphi \in \alpha_i$$

where the satisfaction relation $\models$ is based on a function $v : \Phi_0 \to Pow(\mathcal{O}_\epsilon)$, kept fixed in the background, mapping an *atomic* fluent $\varphi \in \Phi_0 \subseteq \Phi$ to a set $v(\varphi)$ of open sets in $\mathcal{O}_\epsilon$

$$\mathbf{c}, o_i \models \varphi \stackrel{\text{def}}{\Leftrightarrow} o_i \in v(\varphi) \qquad \text{for } \varphi \in \Phi_0$$

extended in the classical manner — e.g.

$$\mathbf{c}, o_i \models \neg A \stackrel{\text{def}}{\Leftrightarrow} \text{not } \mathbf{c}, o_i \models A$$
$$\mathbf{c}, o_i \models Past\; A \stackrel{\text{def}}{\Leftrightarrow} (\exists j < i)\; \mathbf{c}, o_j \models A$$
$$\mathbf{c}, o_i \models Future\; A \stackrel{\text{def}}{\Leftrightarrow} (\exists j > i)\; \mathbf{c}, o_j \models A \; .$$

But what is gained by introducing a positive real number $\epsilon > 0$? Very briefly, ϵ gives us a handle on vagueness, if following Gaifman 2002 (where tolerance is distinguished from vagueness), we identify borderline cases via a modal logic with

> possible worlds as representations of semantic views: ways of applying the vague predicates when "yes/no" decisions are required. (page 10)

In the present set-up, we can take a possible world to be an ϵ-chain, over which an accessibility relation $\sim_\epsilon$ is defined by putting

$$(a,b) \sim_\epsilon (a',b') \overset{\text{def}}{\Leftrightarrow} |a-a'| < \epsilon \text{ and } |b-b'| < \epsilon$$

for all $(a,b),(a',b') \in \mathcal{O}_\epsilon$ and

$$o_1 \ldots o_n \sim_\epsilon o'_1 \ldots o'_m \overset{\text{def}}{\Leftrightarrow} n = m \text{ and } o_i \sim_\epsilon o'_i \text{ for } 1 \leq i \leq n$$

for all ϵ-chains $o_1 \ldots o_n$ and $o'_1 \ldots o'_m$. The idea is that an ϵ-chain $\mathbf{c}$ is a borderline of a string $\sigma \in Pow(\Phi)^*$ if

$$(\exists \mathbf{c}_1 \sim_\epsilon \mathbf{c})\ \sigma \text{ holds at } \mathbf{c}_1 \text{ and } (\exists \mathbf{c}_2 \sim_\epsilon \mathbf{c})\ \sigma \text{ does not hold at } \mathbf{c}_2.$$

As required by the modal logic KTB in Gaifman 2002, $\sim_\epsilon$ is reflexive and symmetric. Clearly, $\sim_\epsilon$ is not transitive, and different strings may hold at $\sim_\epsilon$-related ϵ-chains (making temporal relations possibly vary with the ϵ-chain chosen).[4]

5 Conclusion

Three notions of context were considered above:

- feasible contexts in §2 for Sorites (Gaifman 2002) amounting to comparison classes
- selected events in §3 that induce temporal moments (applying the Russell-Wiener-Kamp construction on event structures with pre- and post-events)
- strings in §4 that generalize event occurrences to event types, and are interpretable as incomplete samples from open intervals in the real line $\mathbb{R}$ of bounded granularity $\epsilon > 0$.

The incompleteness of the sequences of observations constituting strings gives rise to vagueness, with borderline cases analyzable in a modal logic, following Gaifman 2002. Focusing on the contexts that "arise in practice," recall that Gaifman interprets a tolerant unary predicate P via a *context dependency function* f_P mapping a context C to the extension $f_P(C) \subseteq C$ of P at C. Similarly, we might analyze a temporal proposition P as a function mapping C to the set $f_P(C)$ of *parts of* C that *make* P *true* such that

$$P \text{ is true at } C \quad \text{iff} \quad f_P(C) \neq \emptyset \ .$$

[4] Hence, (3) can be made consistent with John *not* drinking at 9:30. Also, note that the same conclusions can be drawn were $\prec_\epsilon$ defined with $r + \epsilon < r'$ in place of $r < r'$.

That is, if $\sqsubseteq$ is the obvious part-of relation between strings with, for example,

$$\boxed{\text{a,b}}\boxed{\text{c}}\boxed{\text{d}} \quad \sqsubseteq \quad s\,\boxed{\text{a,b,c}}\boxed{\text{a,c,d}}\boxed{\text{d}}\,s' ,$$

and $\mathcal{L}(P)$ is the set $\bigcup_C f_P(C)$ of strings s that make P true, with for instance,

$$\mathcal{L}(\text{rain from dawn to dusk}) \quad = \quad \boxed{\text{dawn, rain}}\boxed{\text{rain}}^{+}\boxed{\text{dusk, rain}}$$

(Fernando 2009a), we can assert (7).

(7) $\quad f_P(C) \quad = \quad \{s \sqsubseteq C \mid s \in \mathcal{L}(P)\}$

The step up to a language $\mathcal{L}(P) \subseteq Pow(\Phi)^*$ from a single string $\sigma \in Pow(\Phi)^*$ accommodates the different ways a fluent P can be true, as well as multiple granularities. For instance, we might expect $\mathcal{L}(P)$ to be closed under block compression π (from §4)

$$\sigma \in \mathcal{L}(P) \;\text{ implies }\; \pi(\sigma) \in \mathcal{L}(P) .$$

The proposed variant (7) of a context dependency function f_P enlists strings as situations to serve as indices C and elements s of denotations alike (Fernando 2009b). The domain of f_P can be given by a system of interpretations in the real line as outlined in §4, although further work is required to clarify just what variations in the indices C such a system allows.

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