19
EXISTENCE

Paal Antonsen
antonsp@tcd.ie
https://sites.google.com/site/paalantonsen/teaching/logic

Formal Logic
Arguments about existence

- Many philosophical debates are debates about the existence of some particular object or some particular kinds of objects.
Arguments about existence

Many philosophical debates are debates about the existence of some particular object or some particular kinds of objects.

According to the argument from evil, God doesn’t exist. God's existence is said to be incompatible with the fact that there is evil in the world.
Arguments about existence

Many philosophical debates are debates about the existence of some particular object or some particular kinds of objects.

According to the argument from evil, God doesn’t exist. God’s existence is said to be incompatible with the fact that there is evil in the world.

According to Descartes’s cogito argument, Descartes exists. His existence is implied (or presupposed) by the fact that he cannot doubt his own thinking.
Arguments about existence

▶ Many philosophical debates are debates about the existence of some particular object or some particular kinds of objects.

According to the argument from evil, God doesn’t exist. God’s existence is said to be incompatible with the fact that there is evil in the world.

According to Descartes’s cogito argument, Descartes exists. His existence is implied (or presupposed) by the fact that he cannot doubt his own thinking.

According to Mackie’s argument from queerness, moral facts don’t exist. Their existence is said to be incompatible with a naturalistic view of what facts there are.
Arguments about existence

Many philosophical debates are debates about the existence of some particular object or some particular kinds of objects.

According to the argument from evil, God doesn’t exist. God’s existence is said to be incompatible with the fact that there is evil in the world.

According to Descartes’s cogito argument, Descartes exists. His existence is implied (or presupposed) by the fact that he cannot doubt his own thinking.

According to Mackie’s argument from queerness, moral facts don’t exist. Their existence is said to be incompatible with a naturalistic view of what facts there are.

But what do we mean when we say that something does or doesn’t exist? If Jenny says about something that it exists and Jerry says that it doesn’t, what are they disagreeing about?
Do unicorns exist?

- Suppose two people are arguing about the existence of unicorns.
Do unicorns exist?

Suppose two people are arguing about the existence of unicorns.

(1.a) Unicorns exist
(2.a) Unicorns don’t exist
Do unicorns exist?

- Suppose two people are arguing about the existence of unicorns.

  (1.a) Unicorns exist
  (2.a) Unicorns don’t exist

- As a first step we can understand (1.a) and (2.a) as:

  (1.b) There is something that is a unicorn
  (2.b) There isn’t something that is a unicorn
Suppose two people are arguing about the existence of unicorns.

(1.a) Unicorns exist
(2.a) Unicorns don’t exist

As a first step we can understand (1.a) and (2.a) as:

(1.b) There is something that is a unicorn
(2.b) There isn’t something that is a unicorn

With the added interpretation

\[ Ux = x \text{ is a unicorn} \]

the logical form of the sentences involve existential quantification:

(1.c) \((\exists x) Ux\)
(2.c) \(\neg (\exists x) Ux\)
Does Achilles exist?

- Suppose two people are arguing about the existence of Achilles.
Suppose two people are arguing about the existence of Achilles.

(3.a) Achilles exists
(4.a) Achilles doesn’t exist
Does Achilles exist?

- Suppose two people are arguing about the existence of Achilles.

(3.a) Achilles exists
(4.a) Achilles doesn’t exist

- As a first step we can understand (3.a) and (4.a) as:

(3.b) There is something that is Achilles
(4.b) There isn’t something that is Achilles
Does Achilles exist?

▶ Suppose two people are arguing about the existence of Achilles.

(3.a) Achilles exists

(4.a) Achilles doesn’t exist

▶ As a first step we can understand (3.a) and (4.a) as:

(3.b) There is something that is Achilles

(4.b) There isn’t something that is Achilles

▶ With the added interpretation

\[
a = \text{Achilles}
\]

the logical form of the sentences involve existential quantification:

(3.c) \((\exists x)(x = a)\)

(4.c) \(\sim(\exists x)(x = a)\)
With that analysis in mind, we can try to identify the logical structure of Descarte’s *Cogito* argument.
With that analysis in mind, we can try to identify the logical structure of Descarte’s *Cogito* argument.

\[
\begin{align*}
a & = I \\
T_x & = x \text{ thinks}
\end{align*}
\]

\[
\begin{array}{c}
\text{I think} \\
\hline
\text{I am}
\end{array}
\]

\[
\begin{array}{c}
Ta \\
\sim (\exists x)(x = a) / a \\
\sim (a = a) \\
\times
\end{array}
\]

As we can see the argument is *valid*. 
With that analysis in mind, we can try to identify the logical structure of Descarte’s *Cogito* argument.

\[
\begin{align*}
a & = I \\
Tx & = x \text{ thinks}
\end{align*}
\]

\[
I \text{ think} \\
\underline{I \text{ am}}
\]

\[
\frac{Ta}{\sim(\exists x)(x = a) / a}
\]

\[
\frac{\sim(a = a)}{\times}
\]

As we can see the argument is valid.

But hang on! It is true that I exists without the premise. In fact, it’s a logical truth.
Everything exists

- With our standard predicate logic, we can prove that everything exists.

Everything exists
$(\forall x)(\exists y)(y = x)$
With our standard predicate logic, we can prove that everything exists.

Everything exists

$(\forall x)(\exists y)(y = x)$

$\sim (\forall x)(\exists y)(y = x)$
With our standard predicate logic, we can prove that everything exists.

Everything exists
\((\forall x)(\exists y)(y = x)\)

\(\checkmark \sim (\forall x)(\exists y)(y = x) / a\)

\(\sim (\exists y)(y = a)\)
With our standard predicate logic, we can prove that everything exists.

Everything exists

\((\forall x)(\exists y)(y = x)\)

\(\vdash \sim(\forall x)(\exists y)(y = x) \vdash a\)

\(\vdash (\exists y)(y = a) \vdash a\)

\(\vdash (a = a)\)
Everything exists

With our standard predicate logic, we can prove that everything exists.

Everything exists

$\forall x (\exists y (y = x))$

$\checkmark \sim (\forall x (\exists y (y = x)) / a$

$\sim (\exists y (y = a) / a$

$\sim (a = a)$

$\times$
With our standard predicate logic, we can prove that everything exists.

Everything exists

\((\forall x)(\exists y)(y = x)\)

\(\checkmark \sim (\forall x)(\exists y)(y = x) / a\)

\(\sim (\exists y)(y = a) / a\)

\(\sim (a = a)\)

\(\times\)

Why is it a problem that everything exists?

Because it trivializes arguments about existence. It is now a logical truth that Achilles and God exists. The deniers of their existence lose the debate before it’s even started.
A curious thing about the ontological problem is its simplicity. It can be put in three Anglo-Saxon monosyllables: 'what is there?' It can be answered, moreover, in a word - 'everything' – and everyone will accept this answer is true. However, this is merely to say that there is what there is. There remains room for disagreement over cases; and so the issue has stayed alive down the centuries.
– Willard van Orman Quine, *From a Logical Point of View*
Bertrand Russell
Definite Descriptions
Definite Descriptions

Names in English are covert definite descriptions – i.e. a set of predicates that uniquely pick a particular individual.
Definite Descriptions
Names in English are covert definite descriptions – i.e. a set of predicates that uniquely pick a particular individual.

Achilles is brave is analyzed as saying
(i) There is at least one thing that is a Greek invulnerable hero
(ii) There is at most one thing that is a Greek invulnerable hero
(iii) That thing is brave
Definite descriptions: basic idea

Definite Descriptions
Names in English are covert definite descriptions – i.e. a set of predicates that uniquely pick a particular individual.

Achilles is brave is analyzed as saying
(i) There is at least one thing that is a Greek invulnerable hero
(ii) There is at most one thing that is a Greek invulnerable hero
(iii) That thing is brave

\[
\begin{align*}
Px & = x \text{ is Greek} \\
Qx & = x \text{ is invulnerable} \\
Rx & = x \text{ is a hero} \\
Sx & = x \text{ is brave}
\end{align*}
\]
Definite descriptions: basic idea

**Definite Descriptions**
Names in English are covert definite descriptions – i.e. a set of predicates that uniquely pick a particular individual.

**Achilles is brave** is analyzed as saying

(i) There is at least one thing that is a Greek invulnerable hero
(ii) There is at most one thing that is a Greek invulnerable hero
(iii) That thing is brave

\[
\begin{align*}
Px &= x \text{ is Greek} \\
Qx &= x \text{ is invulnerable} \\
Rx &= x \text{ is a hero} \\
Sx &= x \text{ is brave}
\end{align*}
\]

\[
(\exists x)((Px \land (Qx \land Rx)) \land (\forall y)((Py \land (Qy \land Ry)) \supset x = y) \land Sx)
\]
Definite Descriptions
Names in English are covert definite descriptions – i.e. a set of predicates that uniquely pick a particular individual.

God exists is analyzed as saying
(i) There is at least one thing that is allgood, allpowerful and allknowing.
(ii) There is at most one thing that is allgood, allpowerful and allknowing.
Definite Descriptions
Names in English are covert definite descriptions – i.e. a set of predicates that uniquely pick a particular individual.

God exists is analyzed as saying
(i) There is at least one thing that is is allgood, allpowerful and allknowing.
(ii) There is at most one thing that is is allgood, allpowerful and allknowing.

<table>
<thead>
<tr>
<th>Predicate</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Px$</td>
<td>$x$ is allgood</td>
</tr>
<tr>
<td>$Qx$</td>
<td>$x$ is allpowerful</td>
</tr>
<tr>
<td>$Rx$</td>
<td>$x$ is allknowing</td>
</tr>
</tbody>
</table>

Theism: $(\exists x)(Px \& Qx \& Rx) \land (\forall y)(Py \& Qy \& Ry) \supset x = y)$

Atheism: $\sim(\exists x)(Px \& Qx \& Rx) \land (\forall y)(Py \& Qy \& Ry) \supset x = y)$
Definite descriptions: basic idea

**Definite Descriptions**
Names in English are covert definite descriptions – i.e. a set of predicates that uniquely pick a particular individual.

**God exists** is analyzed as saying

(i) There is at least one thing that is is allgood, allpowerful and allknowing.

(ii) There is at most one thing that is is allgood, allpowerful and allknowing.

\[
P_x = x \text{ is allgood} \\
Q_x = x \text{ is allpowerful} \\
R_x = x \text{ is allknowing}
\]

**Theism** \((\exists x)((P_x \& (Q_x \& R_x)) \& (\forall y)(P_y \& (Q_y \& R_y)))) \supset x = y\)
Definite descriptions: basic idea

**Definite Descriptions**
Names in English are covert definite descriptions – i.e. a set of predicates that uniquely pick a particular individual.

---

**God exists** is analyzed as saying

(i) There is at least one thing that is is allgood, allpowerful and allknowing.

(ii) There is at most one thing that is is allgood, allpowerful and allknowing.

---

\[
Px = x \text{ is allgood} \\
Qx = x \text{ is allpowerful} \\
Rx = x \text{ is allknowing}
\]

**Theism** \((\exists x)((Px & (Qx & Rx)) & (\forall y)(Py & (Qy & Ry)) \supset x = y)\)

**Atheism** \(\sim(\exists x)((Px & (Qx & Rx)) & (\forall y)(Py & (Qy & Ry)) \supset x = y)\)
Definite descriptions: problem

- A problem with **Definite Descriptions** is that names and definite descriptions behave differently in modal contexts.

  Let Washington be a covert description for the 1st US president.

  - If Washington is true then Washington should be substitutable with the 1st US president in every sentence in which it occurs.

  (5) Had Washington died during the war of independence he wouldn’t have been the 1st US president.

  (6) Had Washington died during the war of independence he wouldn’t have been Washington.

  (7) Washington could have failed to be the 1st US president.

  (8) Washington could have failed to be Washington.

  (9) The 1st US president could have failed to be the 1st US president.
Definite descriptions: problem

A problem with **Definite Descriptions** is that names and definite descriptions behave differently in modal contexts.

**Washington**
Let *Washington* be a covert description for *the 1st US president.*
A problem with **Definite Descriptions** is that names and definite descriptions behave differently in modal contexts.

**Washington**

Let *Washington* be a covert description for *the 1st US president*.

- If *Washington* is true then *Washington* should be substitutable with *the 1st US president* in every sentence in which it occurs.
A problem with **Definite Descriptions** is that names and definite descriptions behave differently in modal contexts.

Let *Washington* be a covert description for the *1st US president*.

If *Washington* is true then *Washington* should be substitutable with the *1st US president* in every sentence in which it occurs.

(5) Had *Washington* died during the war of independence he wouldn’t have been the *1st US president*. 

A problem with **Definite Descriptions** is that names and definite descriptions behave differently in modal contexts.

**Washington**

Let *Washington* be a covert description for **the 1st US president**.

- If *Washington* is true then *Washington* should be substitutable with **the 1st US president** in every sentence in which it occurs.

(5) Had *Washington* died during the war of independence he wouldn’t have been **the 1st US president**.

(6) Had *Washington* died during the war of independence he wouldn’t have been *Washington*. 
A problem with **Definite Descriptions** is that names and definite descriptions behave differently in modal contexts.

**Washington**
Let Washington be a covert description for the 1st US president.

If Washington is true then Washington should be substitutable with the 1st US president in every sentence in which it occurs.

(5) Had Washington died during the war of independence he wouldn’t have been the 1st US president.

(6) Had Washington died during the war of independence he wouldn’t have been Washington.

(7) Washington could have failed to be the 1st US president.
A problem with **Definite Descriptions** is that names and definite descriptions behave differently in modal contexts.

**Washington**
Let **Washington** be a covert description for the 1st US president.

If **Washington** is true then **Washington** should be substitutable with the 1st US president in every sentence in which it occurs.

(5) Had **Washington** died during the war of independence he wouldn’t have been the 1st US president.
(6) Had **Washington** died during the war of independence he wouldn’t have been **Washington**.

(7) **Washington** could have failed to be the 1st US president.
(8) **Washington** could have failed to be **Washington**.
A problem with **Definite Descriptions** is that names and definite descriptions behave differently in modal contexts.

**Washington**
Let *Washington* be a covert description for *the 1st US president*.

- If *Washington* is true then *Washington* should be substitutable with *the 1st US president* in every sentence in which it occurs.

(5) Had *Washington* died during the war of independence he wouldn’t have been *the 1st US president*.

(6) Had *Washington* died during the war of independence he wouldn’t have been *Washington*.

(7) *Washington* could have failed to be *the 1st US president*.

(8) *Washington* could have failed to be *Washington*.

(9) *The 1st US president* could have failed to be *the 1st US president*.
Another tempting solution is to accept

Reference failure

Some predicates have empty extensions, so they fail to refer, e.g. is an elf or is a unicorn. Maybe names for things that don’t exist fail to refer.
Another tempting solution is to accept

**Reference failure**
Some predicates have empty extensions, so they fail to refer, e.g. is an elf or is a unicorn. Maybe names for things that don’t exist fail to refer.

The sentence

Prince Charming is handsome

is neither true nor false, but is a reference failure. There is no such thing as Prince Charming for the name to refer to. It is a mock-refernece.
Another tempting solution is to accept

**Reference failure**
Some predicates have empty extensions, so they fail to refer, e.g. is an elf or is a unicorn. Maybe names for things that don’t exist fail to refer.

The sentence

**Prince Charming is handsome**

is neither true nor false, but is a reference failure. There is no such thing as Prince Charming for the name to refer to. It is a mock-reference.

Here is a problem with **Reference failure**.
Second Solution: Names that fail to refer

- Another tempting solution is to accept

**Reference failure**
Some predicates have empty extensions, so they fail to refer, e.g. is an elf or is a unicorn. Maybe names for things that don’t exist fail to refer.

- The sentence
  
  Prince Charming is handsome

  is neither true nor false, but is a reference failure. There is no such thing as Prince Charming for the name to refer to. It is a mock-reference.

- Here is a problem with **Reference failure**.

  (10) Prince Charming exists
  \[(\exists x)(\text{Prince Charming} = x)\]

  (11) Prince Charming doesn’t exist
  \[\sim(\exists x)(\text{Prince Charming} = x)\]

  Both (10) and (11) lack truth-value. However, it seems very intuitive that (11) at least should be true.
Third Solution: existence is a predicate

- Perhaps we shouldn’t be using quantifiers to designated existence.

![Existence predicate](image-url)
Third Solution: existence is a predicate

▶ Perhaps we shouldn’t be using quantifiers to designated existence.

Existence predicate
We use predicates for words that express properties. Maybe existence is a property and so should be introduced as a predicate.

God exists
$$E!a$$

Here is a problem with Existence predicate:
Intuitively, the sentence (12) There is something that doesn’t exist $$(\exists x) \neg E!x$$ should come out as true on this solution. But what does it mean to say (12)? What kind of thing are these non-existent things?
Third Solution: existence is a predicate

Perhaps we shouldn’t be using quantifiers to designated existence.

Existence predicate
We use predicates for words that express properties. Maybe existence is a property and so should be introduced as a predicate.

\[
\begin{align*}
a & = \text{God} \\
E!x & = x \text{ exists}
\end{align*}
\]
Third Solution: existence is a predicate

- Perhaps we shouldn’t be using quantifiers to designated existence.

**Existence predicate**
We use predicates for words that express properties. Maybe existence is a property and so should be introduced as a predicate.

\[
\begin{array}{c|c}
\text{a} & \text{God} \\
\text{E!x} & \text{x exists}
\end{array}
\]

God exists

\[E!a\]
Third Solution: existence is a predicate

Perhaps we shouldn’t be using quantifiers to designated existence.

**Existence predicate**
We use predicates for words that express properties. Maybe existence is a property and so should be introduced as a predicate.

\[
\begin{align*}
a &= \text{God} \\
E!x &= x \text{ exists}
\end{align*}
\]

God exists

\[E!a\]

Here is a problem with **Existence predicate**:
Perhaps we shouldn’t be using quantifiers to designated existence.

**Existence predicate**
We use predicates for words that express properties. Maybe existence is a property and so should be introduced as a predicate.

\[
a = \text{God} \\
E!x = x \text{ exists}
\]

God exists

\[E!a\]

Here is a problem with **Existence predicate**:

Intuitively, the sentence

(12) There is something that doesn’t exists

\[(\exists x) \sim E!x\]

should come out as true on this solution. But what *does it mean* to say (12)? What kind of thing are these non-existent things?