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Formal Logic
Extending the language

Using the language that we have introduced so far, we know how to capture what is being expressed by the following sentence:

Bruce Wayne is a billionaire

\[
\begin{align*}
\text{Bruce Wayne} & = a \\
\text{x is a billionaire} & = Px
\end{align*}
\]

But we are not able to capture what is being expressed by:

Bruce Wayne is Batman

The use of “is” in this sentence is one of identity. In order to handle sentences that involve this special relation we need to extend our language.
Extending the language

Using the language that we have introduced so far, we know how to capture what is being expressed by the following sentence:

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\begin{align*}
\text{Bruce Wayne} &= a \\
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But we are not able to capture what is being expressed by:

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The use of "is" in this sentence is one of identity. In order to handle sentences that involve this special relation we need to extend our language.
We now expand our language by adding the identity predicate (=). We also complement our definition of what counts as a formula:

If $a$ and $b$ are names then $a = b$ is a formula.
We now expand our language by adding the identity predicate \((=)\). We also complement our definition of what counts as a formula:

If \(a\) and \(b\) are names then \(a = b\) is a formula.

Bruce Wayne is a billionaire

\[
\begin{align*}
\text{Bruce Wayne} & = a \\
x \text{ is a billionaire} & = Px
\end{align*}
\]

\(Pa\)

Bruce Wayne is Batman

\[
\begin{align*}
\text{Bruce Wayne} & = a \\
\text{Batman} & = b
\end{align*}
\]

\(a = b\)
Extending the language

The truth conditions for formulas with identity:

\[ a = b \text{ is true in } \mathcal{M} \iff \mathcal{I}(a) \text{ and } \mathcal{I}(b) \text{ pick out the same member of } D \]
Extending the language

- The truth conditions for formulas with identity:

\[ a = b \text{ is true in } \mathcal{M} \text{ iff } \mathcal{I}(a) \text{ and } \mathcal{I}(b) \text{ pick out the same member of } \mathcal{D} \]

- Identity is a logical notion. The extension of the identity relation varies from one domain to another, but it is always a set of pairs where every individual in the domain is paired with itself.
Extending the language

The truth conditions for formulas with identity:

\[ a = b \text{ is true in } \mathcal{M} \text{ iff } I(a) \text{ and } I(b) \text{ pick out the same member of } \mathcal{D} \]

Identity is a logical notion. The extension of the identity relation varies from one domain to another, but it is always a set of pairs where every individual in the domain is paired with itself.

For example, suppose that the domain of a model \( \mathcal{M} \) is

\[ \mathcal{D} = \{1, 2, 3\} \]

then the extension of "=" in \( \mathcal{M} \) is the set

\[ \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle\} \]
Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

Only logicians can dance

- $x$ can dance $= Px$
- $x$ is a logician $= Qx$
Examples of translations with identity

Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

Only logicians can dance

\[
\begin{align*}
x \text{ can dance} &= Px \\
x \text{ is a logician} &= Qx
\end{align*}
\]

\[(\forall x)(Px \supset Qx)\]
Examples of translations with identity

Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

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\begin{align*}
x \text{ can dance} & = Px \\
x \text{ is a logician} & = Qx \\
(\forall x)(Px \supset Qx)
\end{align*}
\]

Only Ruth can dance
Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

**Only logicians can dance**

\[
\begin{align*}
\text{x can dance} &= P_x \\
\text{x is a logician} &= Q_x
\end{align*}
\]

\[(\forall x)(P_x \supset Q_x)\]

**Only Ruth can dance**

\[
\begin{align*}
\text{Ruth} &= a \\
\text{x can dance} &= P_x
\end{align*}
\]
Examples of translations with identity

Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

Only logicians can dance

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x \text{ can dance} = Px \\
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(\forall x)(Px \supset Qx)
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Only Ruth can dance

\[
\text{Ruth} = a \\
x \text{ can dance} = Px \\
(\forall x)(Px \supset x = a)
\]
Examples of translations with identity

Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

Juliet loves no one but Romeo
Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

Juliet loves no one but Romeo

<table>
<thead>
<tr>
<th>Juliet</th>
<th>$=\ a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Romeo</td>
<td>$=\ b$</td>
</tr>
<tr>
<td>$x$ loves $y$</td>
<td>$=\ Rx$</td>
</tr>
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Examples of translations with identity

Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

Juliet loves no one but Romeo

| Juliet   | = | a |
| Romeo    | = | b |
| x loves y | = | Rx |

\[ Rab \land (\forall x)(Rax \supset x = b) \]
Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

### Juliet loves no one but Romeo

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\[ Rab \land (\forall x)(Rax \supset x = b) \]

### All children, except Peter Pan, grow up

\[ (\forall x)(Px \supset (\neg Qx \supset x = a)) \]
Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

**Juliet loves no one but Romeo**

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\[ Rab \land (\forall x)(Rax \supset x = b) \]

**All children, except Peter Pan, grow up**

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\[(\forall x)((Px \land \neg Qx) \supset x = a)\]
Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

Juliet loves no one but Romeo

\[
\begin{align*}
\text{Juliet} &= a \\
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\end{align*}
\]

\[Rab \land (\forall x)(Rax \supset x = b)\]

All children, except Peter Pan, grow up

\[
\begin{align*}
\text{Peter Pan} &= a \\
\text{x is a child} &= Px \\
\text{x is grow up} &= Qx
\end{align*}
\]

\[\forall x((Px \land \neg Qx) \supset x = a)\]

or

\[\forall x((Px \land x \neq a) \supset Qx)\]
Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

Jenny is smarter than all the other philosophers
Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

Jenny is smarter than all the other philosophers

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\[
\begin{align*}
\text{Jenny} &= a \\
\text{x is a philosopher} &= P_x \\
\text{x is smarter than y} &= R_{xy}
\end{align*}
\]

\[Pa \land (\forall x)((P_x \land x \neq a) \supset R_{ax})\]
Now that we have added identity to our language, we can handle many ordinary English sentence that we previously could not.

Jenny is smarter than all the other philosophers

\[
\begin{array}{|l|c|}
\hline
\text{Jenny} & a \\
\text{x is a philosopher} & Px \\
\text{x is smarter than y} & Rxy \\
\hline
\end{array}
\]

\(Pa \land (\forall x)((Px \land x \neq a) \supset Rax)\)

There is a human that is the ancestor of all other humans
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\[ Pa \land (\forall x)((Px \land x \neq a) \supset Rax) \]

There is a human that is the ancestor of all other humans

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\[ Pa \land (\forall x)((Px \land x \neq a) \supset Rax) \]

There is a human that is the ancestor of all other humans

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\[ (\exists x)(Px \land (\forall y)((Py \land x \neq y) \supset Rxy)) \]
How many jedi are there?

Before we introduced the identity predicate we could express that

There is a jedi

\[ x \text{ is a jedi} = Px \]  \quad (\exists x)Px
How many jedi are there?

Before we introduced the identity predicate we could express that there is a jedi:

\[ x \text{ is a jedi} = Px \]

\[ (\exists x)Px \]

But we were not able to capture what is being expressed by:

There is exactly one jedi

This sentence is saying that there is one, and only one, individual that instantites the property of *being a jedi*. Now that we have the identity predicate we are able to handle this sentence.
How many jedi are there?

To capture the truth conditions of the claim that there is exactly one jedi, we can analyze this as saying two things:
How many jedi are there?

To capture the truth conditions of the claim that there is exactly one jedi, we can analyze this as saying two things:

\[
\text{x is a jedi } = Px
\]

There is at least one jedi

There is at most one jedi
To capture the truth conditions of the claim that there is exactly one jedi, we can analyze this as saying two things:

\[
x \text{ is a jedi} = Px
\]

There is at least one jedi \((\exists x)Px\)

There is at most one jedi
How many jedi are there?

To capture the truth conditions of the claim that there is exactly one jedi, we can analyze this as saying two things:

\[ x \text{ is a jedi } = P_x \]

There is at least one jedi

\[ (\exists x) P_x \]

There is at most one jedi

\[ (\forall x)(\forall y)((P_x \& P_y) \supset x = y) \]
To capture the truth conditions of the claim that there is exactly one jedi, we can analyze this as saying two things:

\[
x \text{ is a jedi } = Px
\]

There is at least one jedi \((\exists x)Px\)

There is at most one jedi \((\forall x)(\forall y)((Px \& Py) \supset x = y)\)

So, we translate the sentence

There is exactly one jedi
How many jedi are there?

To capture the truth conditions of the claim that there is exactly one jedi, we can analyze this as saying two things:

\[ \text{x is a jedi} = P_x \]

There is at least one jedi \( (\exists x)P_x \)

There is at most one jedi \( (\forall x)(\forall y)((P_x \& P_y) \supset x = y) \)

So, we translate the sentence

There is exactly one jedi

as

\( (\exists x)P_x \& (\forall x)(\forall y)((P_x \& P_y) \supset x = y) \)
How many jedi are there?

- Using a similar technique we can also analyze other precise claims about how many jedi there are.
How many jedi are there?

- Using a similar technique we can also analyze other precise claims about how many jedi there are.

\[
x \text{ is a jedi} = Px
\]

We translate the sentence

There are exactly two jedi
How many jedi are there?

- Using a similar technique we can also analyze other precise claims about how many jedi there are.

\[
\text{x is a jedi} = P_x
\]

We translate the sentence

There are exactly two jedi

as

\[
(\exists x)(\exists y)(((P_x \land P_y) \land x \neq y) \land \forall z(P_z \supset (z = x \lor z = y)))
\]
How many jedi are there?

Using a similar technique we can also analyze other precise claims about how many jedi there are.

\[ x \text{ is a jedi} = P_x \]

We translate the sentence

There are exactly two jedi

as

\[ (\exists x)(\exists y)(((P_x \land P_y) \land x \neq y) \land \forall z(P_z \supset (z = x \lor z = y))) \]

We can go on for there being exactly three, four, five, etc. jedi.
The principle of extensionality

Intuitively, if $a = b$ then $Pa$ is true if and only if $Pb$ is true. More generally, this can be couched as a principle of extensionality.

**Principle of extensionality**
If $a = b$ and $A(a)$ is true, then $A(b:= a)$ is true, where $A(b:= a)$ is the result of substituting all occurrences of $a$ with $b$. 
The principle of extensionality

Intuitively, if $a = b$ then $Pa$ is true if and only if $Pb$ is true. More generally, this can be couched as a principle of extensionality.

**Principle of extensionality**
If $a = b$ and $A(a)$ is true, then $A(b:= a)$ is true, where $A(b:= a)$ is the result of substituting all occurrences of $a$ with $b$.

We can see how this principle is applied to account for the validity of the following arguments.

<table>
<thead>
<tr>
<th>Bruce Wayne = Batman</th>
<th>Damian = Godzilla</th>
</tr>
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<tbody>
<tr>
<td>Bruce Wayne is a billionaire</td>
<td>Billy kills Damian</td>
</tr>
<tr>
<td>___________________</td>
<td>_________________</td>
</tr>
<tr>
<td>Batman is a billionaire</td>
<td>Billy kills Godzilla</td>
</tr>
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</table>
To handle formulas with identity, we add two further rules:

- $a = b$
- $A(b := a)$

This means that if both $A$ and $a = b$ occur in the branch (no matter where), you can extend the branch with a new formula $A(b/a)$ that replaces all occurrences of $a$ with $b$.

Another closure rule, which says that if $a \neq a$ occur in the branch, that branch closes.
Trees with identity

Example: $Raa, Pb, (\forall x)(Px \supset x = a)$; therefore $Rab$
Trees with identity

Example: $Raa, Pb, (\forall x)(Px \supset x = a)$; therefore $Rab$

\[
\begin{align*}
&Raa \\
&Pb \\
&(\forall x)(Px \supset x = a) \\
&\sim Rab
\end{align*}
\]
Trees with identity

Example: $Raa$, $Pb$, $(\forall x)(Px \supset x = a)$; therefore $Rab$

$$\begin{align*}
Raa \\
Pb \\
(\forall x)(Px \supset x = a) \\
\sim Rab
\end{align*}$$
Trees with identity

Example: $Ra, Pb, (\forall x)(Px \supset x = a)$; therefore $Rab$

$$
\begin{array}{l}
Ra \\
Pb \\
(\forall x)(Px \supset x = a) \vdash b \\
\sim Rab \\
| \\
Pb \supset b = a
\end{array}
$$
Trees with identity

Example: \( Raa, Pb, (\forall x)(Px \ni x = a); \) therefore \( Rab \)

\[
\begin{array}{c}
Raa \\
Pb \\
(\forall x)(Px \ni x = a) / b \\
\sim Rab \\
| \\
Pb \ni b = a
\end{array}
\]
Trees with identity

Example: $Raa, Pb, (\forall x)(Px \supset x = a)$; therefore $Rab$

```
  Raa
   Pb
(\forall x)(Px \supset x = a) /b
  \sim Rab
       \\|\
\checkmark Pb \supset b = a
\sim Pb \quad b = a
```
Trees with identity

Example: $Raa$, $Pb$, $(\forall x)(Px \supset x = a)$; therefore $Rab$

```
Raa
 Pb
(\forall x)(Px \supset x = a) /b
\sim Rab
  \checkmark Pb \supset b = a
    \sim Pb b = a
      \times
```
Trees with identity

Example: \( Raa, Pb, (\forall x)(Px \supset x = a) \); therefore \( Rab \)

\[
\begin{array}{c}
Raa \\
Pb \\
(\forall x)(Px \supset x = a) / b \\
\sim Rab \\
\checkmark Pb \supset b = a \\
\sim Pb \\
\otimes
\end{array}
\]
Trees with identity

Example:  \( Raa, Pb, (\forall x)(Px \supset x = a) \); therefore  \( Rab \)

\[
\begin{array}{c}
Raa \\
Pb \\
(\forall x)(Px \supset x = a) /b \\
\sim Rab \\

\checkmark Pb \supset b = a \\
\sim Pb \quad b = a \\
\otimes \quad \sim Raa
\end{array}
\]
Example: $Raa$, $Pb$, $(\forall x)(Px \supset x = a)$; therefore $Rab$

$$
\begin{array}{c}
\text{Raa} \\
\text{Pb} \\
(\forall x)(Px \supset x = a) / b \\
\sim Rab \\
\vdash Pb \supset b = a \\
\sim Pb \quad \quad b = a \\
\otimes \quad \quad \sim Raa \\
\otimes
\end{array}
$$
Trees with identity

Example: $(\forall x)(\forall y)(Rxy \supset x \neq y)$, therefore $\sim(\exists x)Rxx$
Trees with identity

Example: \((\forall x)(\forall y)(Rxy \supset x \neq y)\), therefore \(\sim(\exists x)Rxx\)

\[(\forall x)(\forall y)(Rxy \supset x \neq y)\]
\[\sim\sim(\exists x)Rxx\]
Trees with identity

Example: $(\forall x)(\forall y)(Rxy \supset x \neq y)$, therefore $\sim(\exists x)Rxx$

$(\forall x)(\forall y)(Rxy \supset x \neq y)$

$\sim\sim(\exists x)Rxx$
Trees with identity

Example: $(\forall x)(\forall y)(Rxy \supset x \neq y)$, therefore $\sim(\exists x)Rxx$

$(\forall x)(\forall y)(Rxy \supset x \neq y)$

$\checkmark \sim \sim(\exists x)Rxx$

$\mid (\exists x)Rxx$
Trees with identity

Example: \((\forall x)(\forall y)(Rxy \supset x \neq y)\), therefore \(\sim(\exists x)Rxx\)

\[
(\forall x)(\forall y)(Rxy \supset x \neq y) \\
\checkmark \sim \sim (\exists x)Rxx \\
\mid \quad (\exists x)Rxx
\]
Trees with identity

Example: \((\forall x)(\forall y)(Rxy \supset x \neq y)\), therefore \(\sim(\exists x)Rxx\)

\[
(\forall x)(\forall y)(Rxy \supset x \neq y) \\
\check{\sim} (\exists x)Rxx
\]

\[
\check{(\exists x)Rxx} / a
\]

\[
Raa
\]
Example: $(\forall x)(\forall y)(Rxy \supset x \neq y)$, therefore $\sim(\exists x)Rxx$

$(\forall x)(\forall y)(Rxy \supset x \neq y)$

$\checkmark \sim \sim(\exists x)Rxx$

$\checkmark (\exists x)Rxx /a$

$Ra a$
Trees with identity

Example: $(\forall x)(\forall y)(Rxy \supset x \neq y)$, therefore $\sim(\exists x)Rxx$

$(\forall x)(\forall y)(Rxy \supset x \neq y) / a$

$\checkmark \sim \sim (\exists x)Rxx$

$\checkmark (\exists x)Rxx / a$

$Ra a$

$(\forall y)(Ray \supset a \neq y)$
Trees with identity

Example: $(\forall x)(\forall y)(Rxy \supset x \neq y)$, therefore $\sim(\exists x)Rxx$

$(\forall x)(\forall y)(Rxy \supset x \neq y) /a$
$\checkmark \sim(\exists x)Rxx$

$\checkmark (\exists x)Rxx /a$

$Raa$

$(\forall y)(Ray \supset a \neq y)$
Trees with identity

Example: $(\forall x)(\forall y)(Rxy \supset x \neq y)$, therefore $\neg(\exists x)Rxx$

$(\forall x)(\forall y)(Rxy \supset x \neq y) / a$

$\checkmark \neg(\exists x)Rxx$

$\checkmark (\exists x)Rxx / a$

$Raa$

$(\forall y)(Ray \supset a \neq y) / a$

$Raa \supset a \neq a$
Trees with identity

Example: $(\forall x)(\forall y)(Rxy \supset x \neq y)$, therefore $\sim(\exists x)Rxx$

\[
(\forall x)(\forall y)(Rxy \supset x \neq y) / a \\
\check{\sim}(\exists x)Rxx \\
\check{(\exists x)Rxx} / a \\
Raa \\
(\forall y)(Ray \supset a \neq y) / a \\
Raa \supset a \neq a
\]
Trees with identity

Example: $\forall x \forall y (Rxy \supset x \neq y)$, therefore $\neg (\exists x) Rxx$

$$(\forall x)(\forall y)(Rxy \supset x \neq y) / a$$

$\checkmark \sim \sim (\exists x) Rxx$

$\checkmark (\exists x) Rxx / a$

$Raa$

$$(\forall y)(Ray \supset a \neq y) / a$$

$Raa \supset a \neq a$

$\sim Raa \quad a \neq a$
Trees with identity

Example: \((\forall x)(\forall y)(Rxy \supset x \neq y)\), therefore \(\sim(\exists x)Rxx\)

\[
(\forall x)(\forall y)(Rxy \supset x \neq y) \vdash a
\]

\[
\check\sim\sim(\exists x)Rxx
\]

\[
\check (\exists x)Rxx \vdash a
\]

\[
Raa
\]

\[
(\forall y)(Ray \supset a \neq y) \vdash a
\]

\[
Raa \supset a \neq a
\]

\[
\sim Raa \quad a \neq a
\]

\[
\otimes \quad \otimes
\]
Gottlob Frege
A puzzle about psychological attitudes
A puzzle about psychological attitudes

In this course we’ve talked about evaluating propositions and formulas. Propositions are also things that we evaluate by adopting psychological attitudes towards them: we can believe that $p$, desire that $p$, hope that $p$, imagine that $p$, wonder whether $p$.

We use psychological attitudes to account for people’s behaviour.

**Behaviour**
If someone desires that $p$ and believes that doing $Q$ will bring about $p$, then all things equal they do $Q$.

We use psychological attitude to account for rationality constraints.

**Belief**
One should believe $p$ only if $p$ is true.

**Reasoning**
If one believes $X$ and $p$ is entailed by $X$ then one should believe $p$. 
A puzzle about psychological attitudes

I kissed a hero and I liked it

Batman has been out beating up oddly dressed villains for the entire night, saving the Mayor’s son at the same time. Rachel is impressed, and thinks that he’s the noblest guy in Gotham (not anything like that degenerate partyboy Bruce, she says to herself). In a moment of passion, she walks up to Batman and kisses him.
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\[
\begin{align*}
\text{Batman} &= \text{Bruce} \\
\hline
\text{Rachel kisses Bruce}
\end{align*}
\]

(2) Rachel believes Batman is noble

\[
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\text{Batman} &= \text{Bruce} \\
\hline
\text{Rachel believes Bruce is noble}
\end{align*}
\]

▶ While (1) is a valid argument, (2) doesn’t seem to be valid. But why not?
A puzzle about psychological attitudes

The propositional solution
When occurring under a psychological attitude, names and predicates refer to their contents instead of their normal extension.
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\[(3) \text{ Rachel believes Bruce is noble} \]

is associated with two different readings:
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(3) Rachel believes Bruce is noble
is associated with two different readings:

(4) Rachel believes about Bruce that Bruce is noble.
    (Bruce : x)(Rachel believes x is noble)

(5) Rachel believes that Bruce is noble