INTRODUCING NAMES, PREDICATES, AND QUANTIFIERS

Paal Antonsen
antonsp@tcd.ie
https://sites.google.com/site/paalantonsen/teaching/logic

Formal Logic
When propositional logic comes short

God necessarily exists.
________________________._
God exists.

\( p \not\equiv q \)

All these arguments are valid, but not by virtue of their propositional argument form. We need a more sophisticated analysis if we are to correctly evaluate them.

Whitney will always love you.
________________________._
Whitney will love you tomorrow.

\( p \not\equiv q \)

All Trojans fear Achilles.
Hector is a Trojan.
________________________._
Hector fears Achilles.

\( p, q \not\equiv r \)
When propositional logic comes short

God necessarily exists.

—from—

God exists.

Necessarily $p \models p$

The argument is valid, because of the meaning of Necessarily. We can use modal logic to handle expressions like Necessarily, Possibly, Must, and Might.

Whitney will always love you.

—from—

Whitney will love you tomorrow.

Always $p \models$ Tomorrow $q$

The argument is valid, because of the meaning of Always and Tomorrow. We can use temporal logic to handle such time-sensitive expressions.

All Trojans fear Achilles.

Hector is a Trojan.

—from—

Hector fears Achilles.

$p, q \not\models r$

The argument is valid, because of the meaning of All, and some relation between individuals. We will learn how to handle such cases with the system of predicate logic.
Example 1
“Bianca is brave” is true iff the individual picked out by “Bianca” has the property expressed by “is brave”.
Back to truth conditions

**Example 1**
“Bianca is brave” is true iff the individual picked out by “Bianca” has the property expressed by “is brave”.

- “Bianca is brave” can be analyzed as built up by a name and a one-place predicate.
Example 1
“Bianca is brave” is true iff the individual picked out by “Bianca” has the property expressed by “is brave”.

▶ “Bianca is brave” can be analyzed as built up by a name and a one-place predicate.

▶ To capture the logical form of this sentence we start with the vocabulary:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Predicate logic</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a, b, c, \ldots$</td>
<td>Name</td>
<td>singular terms</td>
</tr>
<tr>
<td>$P, Q, R, \ldots$</td>
<td>Predicate</td>
<td>adjectives, verbs, some nouns</td>
</tr>
</tbody>
</table>
Example 1
“Bianca is brave” is true iff the individual picked out by “Bianca” has the property expressed by “is brave”.

▶ “Bianca is brave” can be analyzed as built up by a name and a one-place predicate.

▶ To capture the logical form of this sentence we start with the vocabulary:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Predicate logic</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a, b, c, \ldots$</td>
<td>Name</td>
<td>singular terms</td>
</tr>
<tr>
<td>$P, Q, R, \ldots$</td>
<td>Predicate</td>
<td>adjectives, verbs, some nouns</td>
</tr>
</tbody>
</table>

\[
\begin{aligned}
    a &= \text{Bianca} \\
    Px &= x \text{ is brave}
\end{aligned}
\]
Let’s consider some more examples

We handle connectives in the same way as we did in propositional logic: the same rules for complex formulas. The difference is that we now also analyze the structure of the atomic formulas.
Let’s consider some more examples

We handle connectives in the same way as we did in propositional logic: the same rules for complex formulas. The difference is that we now also analyze the structure of the atomic formulas.

If the Hatter is mad then the March Hare is mad
Let’s consider some more examples

We handle connectives in the same way as we did in propositional logic: the same rules for complex formulas. The difference is that we now also analyze the structure of the atomic formulas.

If the Hatter is mad then the March Hare is mad

<table>
<thead>
<tr>
<th>a</th>
<th>=</th>
<th>the Hatter</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>=</td>
<td>the March Hare</td>
</tr>
<tr>
<td>Px</td>
<td>=</td>
<td>x is mad</td>
</tr>
</tbody>
</table>
Let's consider some more examples

We handle connectives in the same way as we did in propositional logic: the same rules for complex formulas. The difference is that we now also analyze the structure of the atomic formulas.

If the Hatter is mad then the March Hare is mad

| a    | = the Hatter |
| b    | = the March Hare |
| Px   | = x is mad |

$Pa \supset Pb$
Let’s consider some more examples

We handle connectives in the same way as we did in propositional logic: the same rules for complex formulas. The difference is that we now also analyze the structure of the atomic formulas.

If the Hatter is mad then the March Hare is mad

\[
\begin{align*}
a &= \text{the Hatter} \\
b &= \text{the March Hare} \\
Px &= x \text{ is mad}
\end{align*}
\]

\[Pa \supset Pb\]

Batman is a psychopath or a hero, but not both
Let's consider some more examples

We handle connectives in the same way as we did in propositional logic: the same rules for complex formulas. The difference is that we now also analyze the structure of the atomic formulas.

If the Hatter is mad then the March Hare is mad

\[
\begin{align*}
  a & = \text{ the Hatter} \\
  b & = \text{ the March Hare} \\
  Px & = x \text{ is mad}
\end{align*}
\]

\( Pa \supset Pb \)

Batman is a psychopath or a hero, but not both

\[
\begin{align*}
  a & = \text{ Batman} \\
  Px & = x \text{ is a psychopath} \\
  Qx & = x \text{ is a hero}
\end{align*}
\]
Let’s consider some more examples

We handle connectives in the same way as we did in propositional logic: the same rules for complex formulas. The difference is that we now also analyze the structure of the atomic formulas.

If the Hatter is mad then the March Hare is mad

\[
\begin{align*}
a & = \text{the Hatter} \\
b & = \text{the March Hare} \\
P_x & = x \text{ is mad}
\end{align*}
\]

\[Pa \supset Pb\]

Batman is a psychopath or a hero, but not both

\[
\begin{align*}
a & = \text{Batman} \\
P_x & = x \text{ is a psychopath} \\
Q_x & = x \text{ is a hero}
\end{align*}
\]

\[(Pa \lor Qa) \land \lnot(Pa \land Qa)\]
Example 2
“Bernhard loves Bianca" is true iff the individual picked out by "Bernhard" is in a "loves" relation to the individual picked out by "Bianca".
Example 2

“Bernhard loves Bianca” is true iff the individual picked out by “Bernhard” is in a “loves” relation to the individual picked out by “Bianca”.

“Bernhard loves Bianca” can be analyzed as built up by two names and a two-place predicate.
Example 2

“Bernhard loves Bianca” is true iff the individual picked out by “Bernhard" is in a “loves" relation to the individual picked out by “Bianca”.

- “Bernhard loves Bianca” can be analyzed as built up by two names and a two-place predicate.

- To capture the logical form of this sentence we start with the vocabulary:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Predicate logic</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a, b, c, \ldots$</td>
<td>Name</td>
<td>singular terms</td>
</tr>
<tr>
<td>$P, Q, R, \ldots$</td>
<td>Predicate</td>
<td>adjectives, verbs, some nouns</td>
</tr>
</tbody>
</table>
Example 2

“Bernhard loves Bianca" is true iff the individual picked out by “Bernhard" is in a "loves" relation to the individual picked out by “Bianca".

▶ “Bernhard loves Bianca" can be analyzed as built up by two names and a two-place predicate.

▶ To capture the logical form of this sentence we start with the vocabulary:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Predicate logic</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a, b, c, \ldots$</td>
<td>Name</td>
<td>singular terms</td>
</tr>
<tr>
<td>$P, Q, R, \ldots$</td>
<td>Predicate</td>
<td>adjectives, verbs, some nouns</td>
</tr>
</tbody>
</table>

$a = $ Bernhard

$b = $ Bianca

$R_{xy} = x$ loves $y$

$Rab$
Let’s consider some more examples

All predicates come with a fixed **arity** (one-place, two-place, . . .). When we give the dictionary we must specify how many places a predicate has.
Let's consider some more examples

All predicates come with a fixed arity (one-place, two-place, ...). When we give the dictionary we must specify how many places a predicate has.

Prince Charming kisses Princess Bella if and only if she is rich.
Let’s consider some more examples

All predicates come with a fixed **arity** (one-place, two-place, ...). When we give the dictionary we must specify how many places a predicate has.

Prince Charming kisses Princess Bella if and only if she is rich

| $a$   | = Prince Charming |
| $b$   | = Princess Bella  |
| $Px$  | = $x$ is rich     |
| $Rxy$ | = $x$ kisses $y$  |
Let’s consider some more examples

All predicates come with a fixed arity (one-place, two-place, …). When we give the dictionary we must specify how many places a predicate has.

Prince Charming kisses Princess Bella if and only if she is rich

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Prince Charming</td>
</tr>
<tr>
<td>$b$</td>
<td>Princess Bella</td>
</tr>
<tr>
<td>$Px$</td>
<td>$x$ is rich</td>
</tr>
<tr>
<td>$Rxy$</td>
<td>$x$ kisses $y$</td>
</tr>
</tbody>
</table>

$Rab \equiv Pb$
Let’s consider some more examples

All predicates come with a fixed arity (one-place, two-place, \ldots). When we give the dictionary we must specify how many places a predicate has.

Prince Charming kisses Princess Bella if and only if she is rich

\[
\begin{array}{lcl}
a & = & \text{Prince Charming} \\
b & = & \text{Princess Bella} \\
Px & = & x \text{ is rich} \\
Rxy & = & x \text{ kisses } y
\end{array}
\]

\[R_{ab} \equiv P_b\]

Romeo doesn’t love himself unless Godzilla and Granny love him
Let’s consider some more examples

All predicates come with a fixed arity (one-place, two-place, . . .). When we give the dictionary we must specify how many places a predicate has.

Prince Charming kisses Princess Bella if and only if she is rich

\[
\begin{array}{ll}
a & = \text{Prince Charming} \\
b & = \text{Princess Bella} \\
Px & = x \text{ is rich} \\
Rxy & = x \text{ kisses } y
\end{array}
\]

\[Rab \equiv Pb\]

Romeo doesn’t love himself unless Godzilla and Granny loves him

\[
\begin{array}{ll}
a & = \text{Romeo} \\
b & = \text{Godzilla} \\
c & = \text{Granny} \\
Rxy & = x \text{ loves } y
\end{array}
\]
Let’s consider some more examples

All predicates come with a fixed arity (one-place, two-place, . . .). When we give the dictionary we must specify how many places a predicate has.

Prince Charming kisses Princess Bella if and only if she is rich

<table>
<thead>
<tr>
<th>a</th>
<th>=</th>
<th>Prince Charming</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>=</td>
<td>Princess Bella</td>
</tr>
<tr>
<td>Px</td>
<td>=</td>
<td>x is rich</td>
</tr>
<tr>
<td>Rxy</td>
<td>=</td>
<td>x kisses y</td>
</tr>
</tbody>
</table>

$Rab \equiv Pb$

Romeo doesn’t love himself unless Godzilla and Granny loves him

<table>
<thead>
<tr>
<th>a</th>
<th>=</th>
<th>Romeo</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>=</td>
<td>Godzilla</td>
</tr>
<tr>
<td>c</td>
<td>=</td>
<td>Granny</td>
</tr>
<tr>
<td>Rxy</td>
<td>=</td>
<td>x loves y</td>
</tr>
</tbody>
</table>

$\sim Raa \lor (Rba \land Rca)$
Example 3
“There is a jedi” is true iff there is some individual $x$, such that $x$ has the property expressed by “is a jedi".
Example 3

“There is a jedi” is true iff there is some individual $x$, such that $x$ has the property expressed by “is a jedi”.

▶ “There is a jedi” can be analyzed as built up by a quantifier, a variable and a one-place predicate.
Example 3

“There is a jedi” is true iff there is some individual $x$, such that $x$ has the property expressed by “is a jedi”.

- “There is a jedi” can be analyzed as built up by a quantifier, a variable and a one-place predicate.

- To capture the logical form of this sentence we start with the vocabulary:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Predicate logic</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x, y, z, \ldots$</td>
<td>Individual variables</td>
<td>(pronouns)</td>
</tr>
<tr>
<td>$\exists \ldots$</td>
<td>Existential quantifier</td>
<td>there is some ...</td>
</tr>
</tbody>
</table>
Back to truth conditions

Example 3

“There is a jedi" is true iff there is some individual $x$, such that $x$ has the property expressed by “is a jedi".

▶ “There is a jedi" can be analyzed as built up by a quantifier, a variable and a one-place predicate.

▶ To capture the logical form of this sentence we start with the vocabulary:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Predicate logic</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x, y, z, \ldots$</td>
<td>Individual variables</td>
<td>(pronouns)</td>
</tr>
<tr>
<td>$\exists \ldots$</td>
<td>Existential quantifier</td>
<td>there is some \ldots</td>
</tr>
</tbody>
</table>

$Px = x$ is a jedi

$(\exists x)Px$
Let’s consider some more examples

We use parentheses to indicate the scope of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\exists x)$, then $x$ is bound by that quantifier.
Let’s consider some more examples

We use parentheses to indicate the scope of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\exists x)$, then $x$ is bound by that quantifier.

There is an evil jedi
Let’s consider some more examples

We use parentheses to indicate the scope of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\exists x)$, then $x$ is bound by that quantifier.

There is an evil jedi

<table>
<thead>
<tr>
<th>$Px$</th>
<th>$x$ is evil</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Qx$</td>
<td>$x$ is a jedi</td>
</tr>
</tbody>
</table>
Let's consider some more examples

We use parentheses to indicate the scope of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\exists x)$, then $x$ is bound by that quantifier.

There is an evil jedi

<table>
<thead>
<tr>
<th>$Px$</th>
<th>$x$ is evil</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Qx$</td>
<td>$x$ is a jedi</td>
</tr>
</tbody>
</table>

$(\exists x)(Px \land Qx)$
Let's consider some more examples

We use parentheses to indicate the scope of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\exists x)$, then $x$ is bound by that quantifier.

There is an evil jedi

\[
\begin{align*}
Px &= x \text{ is evil} \\
Qx &= x \text{ is a jedi}
\end{align*}
\]

$(\exists x)(Px \land Qx)$

Stevens is being followed by a moonshadow
Let’s consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\exists x)$, then $x$ is **bound** by that quantifier.

There is an evil jedi

\[
\begin{align*}
Px & = \text{ } x \text{ is evil} \\
Qx & = \text{ } x \text{ is a jedi}
\end{align*}
\]

$(\exists x)(Px \& Qx)$

Stevens is being followed by a moonshadow

\[
\begin{align*}
a & = \text{ } \text{Stevens} \\
Px & = \text{ } x \text{ is a moonshadow} \\
Rxy & = \text{ } x \text{ follows } y
\end{align*}
\]
Let's consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\exists x)$, then $x$ is **bound** by that quantifier.

There is an evil jedi

<table>
<thead>
<tr>
<th>$Px$</th>
<th>$x$ is evil</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Qx$</td>
<td>$x$ is a jedi</td>
</tr>
</tbody>
</table>

$(\exists x)(Px \& Qx)$

Stevens is being followed by a moonshadow

<table>
<thead>
<tr>
<th>$a$</th>
<th>Stevens</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Px$</td>
<td>$x$ is a moonshadow</td>
</tr>
<tr>
<td>$Rxy$</td>
<td>$x$ follows $y$</td>
</tr>
</tbody>
</table>

$(\exists x)(Px \& Rxa)$
Let’s consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\exists x)$, then $x$ is **bound** by that quantifier.

There is a jedi that doesn’t love Luke

$a = \text{Luke}$

$P_x = x \text{ is a jedi}$

$R_{xy} = x \text{ loves } y$

$(\exists x) (P_x \& \neg R_{xa})$

There is a clown that kicks Romeo or Juliet

$a = \text{Romeo}$

$b = \text{Juliet}$

$P_x = x \text{ is a clown}$

$R_{xy} = x \text{ kicks } y$

$(\exists x) (P_x \& (R_{xa} \lor R_{xb}))$
Let’s consider some more examples

We use parentheses to indicate the scope of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\exists x)$, then $x$ is bound by that quantifier.

There is a jedi that doesn’t love Luke

$(\exists x)(Px \& \neg Rxa)$

There is a clown that kicks Romeo or Juliet

$(\exists x)(Px \& (Rxa \lor Rxb))$
Let’s consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\exists x)$, then $x$ is **bound** by that quantifier.

There is a jedi that doesn’t love Luke

<table>
<thead>
<tr>
<th>$a$</th>
<th>$\equiv$</th>
<th>Luke</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Px$</td>
<td>$\equiv$</td>
<td>$x$ is a jedi</td>
</tr>
<tr>
<td>$Rxy$</td>
<td>$\equiv$</td>
<td>$x$ loves $y$</td>
</tr>
</tbody>
</table>
Let’s consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable \( x \) falls within the scope of an appropriate quantifier, e.g. \( (\exists x) \), then \( x \) is **bound** by that quantifier.

There is a jedi that doesn't love Luke

\[
\begin{align*}
a &= \text{Luke} \\
P_x &= x \text{ is a jedi} \\
R_{xy} &= x \text{ loves } y
\end{align*}
\]

\((\exists x)(P_x \& \sim R_xa)\)
Let’s consider some more examples

We use parentheses to indicate the scope of a quantifier. If a variable \( x \) falls within the scope of an appropriate quantifier, e.g. \( (\exists x) \), then \( x \) is bound by that quantifier.

There is a jedi that doesn’t love Luke

\[
\begin{array}{ll}
    a &= \text{Luke} \\
    Px &= x \text{ is a jedi} \\
    Rxy &= x \text{ loves } y \\
\end{array}
\]

\((\exists x)(Px \& \sim Rxa)\)

There is a clown that kicks Romeo or Juliet
Let’s consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\exists x)$, then $x$ is **bound** by that quantifier.

There is a Jedi that doesn’t love Luke

$\begin{align*} a &= \text{Luke} \\ Px &= x \text{ is a Jedi} \\ Rxy &= x \text{ loves } y \end{align*}$

$(\exists x)(Px & \land \sim Rxa)$

There is a clown that kicks Romeo or Juliet

$\begin{align*} a &= \text{Romeo} \\ b &= \text{Juliet} \\ Px &= x \text{ is a clown} \\ Rxy &= x \text{ kicks } y \end{align*}$
Let’s consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable \(x\) falls within the scope of an appropriate quantifier, e.g. \((\exists x)\), then \(x\) is **bound** by that quantifier.

There is a jedi that doesn’t love Luke

\[
\begin{align*}
a &= \text{Luke} \\
P_x &= \text{x is a jedi} \\
R_{xy} &= \text{x loves y}
\end{align*}
\]

\((\exists x)(P_x \land \sim R_{xa})\)

There is a clown that kicks Romeo or Juliet

\[
\begin{align*}
a &= \text{Romeo} \\
b &= \text{Juliet} \\
P_x &= \text{x is a clown} \\
R_{xy} &= \text{x kicks y}
\end{align*}
\]

\((\exists x)(P_x \land (R_{xa} \lor R_{xb}))\)
Example 4

“Everything is an illusion" is true iff for all individuals $x$, $x$ has the property expressed by “is an illusion”.
Back to truth conditions

Example 4

“Everything is an illusion" is true iff for all individuals $x$, $x$ has the property expressed by “is an illusion".

“Everything is an illusion" can be analyzed as built up by a quantifier, a variable and a one-place predicate.

![Diagram](Everything is an illusion
For all $x$ is an illusion)
Example 4

“Everything is an illusion" is true iff for all individuals \( x \), \( x \) has the property expressed by “is an illusion".

- “Everything is an illusion" can be analyzed as built up by a quantifier, a variable and a one-place predicate.

- To capture the logical form of this sentence we start with the vocabulary:
Example 4

“Everything is an illusion" is true iff for all individuals $x$, $x$ has the property expressed by “is an illusion”.

▶ “Everything is an illusion" can be analyzed as built up by a quantifier, a variable and a one-place predicate.

▶ To capture the logical form of this sentence we start with the vocabulary:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Predicate logic</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x, y, z, \ldots$</td>
<td>Individual variables</td>
<td>(pronouns)</td>
</tr>
<tr>
<td>$\forall \ldots$</td>
<td>Universal quantifier</td>
<td>for all . . .</td>
</tr>
</tbody>
</table>
Example 4

“Everything is an illusion" is true iff for all individuals $x$, $x$ has the property expressed by “is an illusion”.

“Everything is an illusion" can be analyzed as built up by a quantifier, a variable and a one-place predicate.

To capture the logical form of this sentence we start with the vocabulary:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Predicate logic</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x, y, z, \ldots$</td>
<td>Individual variables</td>
<td>(pronouns)</td>
</tr>
<tr>
<td>$\forall \ldots$</td>
<td>Universal quantifier</td>
<td>for all \ldots</td>
</tr>
</tbody>
</table>

$Px = x$ is an illusion

$(\forall x)Px$
Let's consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable \( x \) falls within the scope of an appropriate quantifier, e.g. \((\forall x)\), then \( x \) is **bound** by that quantifier.
Let’s consider some more examples

We use parentheses to indicate the scope of a quantifier. If a variable \( x \) falls within the scope of an appropriate quantifier, e.g. \((\forall x)\), then \( x \) is bound by that quantifier.

Only logicians can dance
Let’s consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\forall x)$, then $x$ is **bound** by that quantifier.

Only logicians can dance

<table>
<thead>
<tr>
<th>$P_x$</th>
<th>$x$ is logician</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_x$</td>
<td>$x$ can dance</td>
</tr>
</tbody>
</table>
Let’s consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $\forall x$, then $x$ is **bound** by that quantifier.

Only logicians can dance

\[
\begin{align*}
Px &= x \text{ is logician} \\
Qx &= x \text{ can dance}
\end{align*}
\]

\[(\forall x)(Qx \supset Px)\]
Let’s consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\forall x)$, then $x$ is **bound** by that quantifier.

Only logicians can dance

<table>
<thead>
<tr>
<th>$Px$</th>
<th>$x$ is logician</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Qx$</td>
<td>$x$ can dance</td>
</tr>
</tbody>
</table>

$(\forall x)(Qx \supset Px)$

Boys don’t cry
Let’s consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable \( x \) falls within the scope of an appropriate quantifier, e.g. \((\forall x)\), then \( x \) is **bound** by that quantifier.

Only logicians can dance

\[
\begin{align*}
Px &= x \text{ is logician} \\
Qx &= x \text{ can dance}
\end{align*}
\]

\((\forall x)(Qx \supset Px)\)

Boys don’t cry

\[
\begin{align*}
Px &= x \text{ is a boy} \\
Qx &= x \text{ cries (does cry)}
\end{align*}
\]
Let's consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\forall x)$, then $x$ is **bound** by that quantifier.

Only logicians can dance

<table>
<thead>
<tr>
<th>$Px$</th>
<th>$x$ is logician</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Qx$</td>
<td>$x$ can dance</td>
</tr>
</tbody>
</table>

$(\forall x)(Qx \supset Px)$

Boys don’t cry

<table>
<thead>
<tr>
<th>$Px$</th>
<th>$x$ is a boy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Qx$</td>
<td>$x$ cries (does cry)</td>
</tr>
</tbody>
</table>

$(\forall x)(Px \supset \neg Qx)$
Let’s consider some more examples

We use parentheses to indicate the scope of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\forall x)$, then $x$ is bound by that quantifier.
Let’s consider some more examples

We use parentheses to indicate the scope of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\forall x)$, then $x$ is **bound** by that quantifier.

Every Jedi that isn’t disciplined is evil
Let’s consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\forall x)$, then $x$ is **bound** by that quantifier.

Every Jedi that isn’t disciplined is evil

\[
\begin{align*}
Px &= \text{ $x$ is a Jedi} \\
Qx &= \text{ $x$ is disciplined} \\
Rx &= \text{ $x$ is evil}
\end{align*}
\]
Let's consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable \( x \) falls within the scope of an appropriate quantifier, e.g. \((\forall x)\), then \( x \) is **bound** by that quantifier.

Every Jedi that isn’t disciplined is evil

| \( Px \) | = | \( x \) is a Jedi |
| \( Qx \) | = | \( x \) is disciplined |
| \( Rx \) | = | \( x \) is evil |

\((\forall x)((Px \& \sim Qx) \supset Rx)\)
Let’s consider some more examples

We use parentheses to indicate the scope of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\forall x)$, then $x$ is bound by that quantifier.

Every Jedi that isn’t disciplined is evil

$Px = x$ is a Jedi
$Qx = x$ is disciplined
$Rx = x$ is evil

$(\forall x)((Px \& \sim Qx) \supset Rx)$

Everybody who loves Jabba loves themselves
Let's consider some more examples

We use parentheses to indicate the **scope** of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. $(\forall x)$, then $x$ is **bound** by that quantifier.

Every Jedi that isn’t disciplined is evil

<table>
<thead>
<tr>
<th>$P_!x$</th>
<th>$x$ is a Jedi</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_!x$</td>
<td>$x$ is disciplined</td>
</tr>
<tr>
<td>$R_!x$</td>
<td>$x$ is evil</td>
</tr>
</tbody>
</table>

$(\forall x)((P_\!x \& \sim Q_\!x) \supset R_\!x)$

Everybody who loves Jabba loves themselves

<table>
<thead>
<tr>
<th>$a$</th>
<th>Jabba</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Rxy$</td>
<td>$x$ loves $y$</td>
</tr>
</tbody>
</table>
Let’s consider some more examples

We use parentheses to indicate the scope of a quantifier. If a variable $x$ falls within the scope of an appropriate quantifier, e.g. ($\forall x$), then $x$ is bound by that quantifier.

Every Jedi that isn’t disciplined is evil

<table>
<thead>
<tr>
<th>$Px$</th>
<th>$x$ is a Jedi</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Qx$</td>
<td>$x$ is disciplined</td>
</tr>
<tr>
<td>$Rx$</td>
<td>$x$ is evil</td>
</tr>
</tbody>
</table>

$$(\forall x)(((Px \& \neg Qx) \supset Rx)$$

Everybody who loves Jabba loves themselves

<table>
<thead>
<tr>
<th>$a$</th>
<th>Jabba</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Rxy$</td>
<td>$x$ loves $y$</td>
</tr>
</tbody>
</table>

$$(\forall x)(Rxa \supset Rxx)$$
When there is more than one quantifier, we have to be careful about the order in which we place them.

Everybody loves somebody
Somebody loves everybody

\[ R_{xy} = x \text{ loves } y \]
When there is more than one quantifier, we have to be careful about the order in which we place them.

Everybody loves somebody  \((\forall x)(\exists y)R_{xy}\)
Somebody loves everybody

\[
R_{xy} = x \text{ loves } y
\]
When there is more than one quantifier, we have to be careful about the order in which we place them.

- Everybody loves somebody: \((\forall x)(\exists y)R_{xy}\)
- Somebody loves everybody: \((\exists x)(\forall y)R_{xy}\)

\[R_{xy} = x \text{ loves } y\]
When there is more than one quantifier, we have to be careful about the order in which we place them.

Everybody loves somebody: \((\forall x)(\exists y)Rxy\)
Somebody loves everybody: \((\exists x)(\forall y)Rxy\)

\[ Rxy = x \text{ loves } y \]

The only boy who can teach Dusty is a son of a preacher man.
When there is more than one quantifier, we have to be careful about the order in which we place them.

- Everybody loves somebody: $\forall x (\exists y) R_{xy}$
- Somebody loves everybody: $(\exists x) (\forall y) R_{xy}$

$R_{xy} = x$ loves $y$

The only boy who can teach Dusty is a son of a preacher man

- $a = $ Dusty
- $Px = x$ is a boy
- $Qx = x$ is a preacher man
- $R_{xy} = x$ can teach $y$
- $S_{xy} = x$ is the son of $y$
When there is more than one quantifier, we have to be careful about the order in which we place them.

Everybody loves somebody: \((\forall x)(\exists y)Rxy\)

Somebody loves everybody: \((\exists x)(\forall y)Rxy\)

\[ Rxy = x\text{ loves } y \]

The only boy who can teach Dusty is a son of a preacher man:

\[ (\forall x)((Px \& Rxa) \supset (\exists y)(Qy \& Sxy)) \]
Language of predicate logic

**Vocabulary**  The language of predicate logic consists of:

- **NAMES** (INDIVIDUAL CONSTANTS)  $a, b, c, \ldots$
- **INDIVIDUAL VARIABLES**  $x, y, z, \ldots$
- **PREDICATES**  $P, Q, R, \ldots$
- **CONNECTIVES**  $\neg, \&,$  $\lor,$  $\supset,$  $\equiv$
- **QUANTIFIERS**  $\exists,$  $\forall$
- **PARENTHESES**  $(,)$

**Grammar**

- **ATOMIC**  If $P$ is a predicate with $n$-places and $a_1, \ldots a_n$ are names then $Pa_1\ldots a_n$ is a formula.
- **QUANTIFIERS**  If $A$ is a formula then $(\exists x)A(a := x)$ and $(\exists x)A(a := x)$ formulas, $A(a := x)$ is the result of replacing all occurences of $a$ with $x$.
- **CONNECTIVES**  If $A$ is a formula then $\neg A$ is a formula.
- **CLOSEURE**  Nothing else counts as a formula.