ARGUMENT FORM IN PROPOSITIONAL LOGIC

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Formal Logic
We’ve described what counts as a formula of propositional logic:

**Vocabulary** The language of propositional logic consists of:

| ATOMIC PROPOSITIONS | p, q, r, ... |
| LOGICAL CONNECTIVES | ¬, &, ∨, ⊃, ≡ |
| PARENTHESES          | (, )         |

**Grammar**

| ATOMIC CONNECTIVES | If A is an atomic proposition then A is a formula. |
| CONNECTIVES        | If A is a formula then ¬A is a formula. |
|                    | If A and B are formulas then (A & B), (A ∨ B), (A ⊃ B) and (A ≡ B) are formulas. |
| CLOSURE            | Nothing else counts as a formula. |
We’ve defined **truth on an evaluation** for those formulas:

**Truth on an evaluation**

For every atomic formula $p$, $v(p) = 1$ or $v(p) = 0$.

\[
v(\neg A) = \begin{cases} 
1 & \text{if } v(A) = 0 \\
0 & \text{otherwise}
\end{cases}
\]

\[
v(A \& B) = \begin{cases} 
1 & \text{if } v(A) = 1 \text{ and } v(B) = 1 \\
0 & \text{otherwise}
\end{cases}
\]

\[
v(A \lor B) = \begin{cases} 
0 & \text{if } v(A) = 0 \text{ and } v(B) = 0 \\
1 & \text{otherwise}
\end{cases}
\]

\[
v(A \supset B) = \begin{cases} 
0 & \text{if } v(A) = 1 \text{ and } v(B) = 0 \\
1 & \text{otherwise}
\end{cases}
\]

\[
v(A \equiv B) = \begin{cases} 
1 & \text{if } v(A) = 1 \text{ and } v(B) = 1 \\
1 & \text{if } v(A) = 0 \text{ and } v(B) = 0 \\
0 & \text{otherwise}
\end{cases}
\]
Argument form in propositional logic

HOW TO: REPRESENT THE GENERAL ARGUMENT FORM

(a) Identify the premises and the conclusion.

(b) Identify the atomic propositions and provide a dictionary that assigns a (lower case) letter to each atomic proposition.

(c) Replace the atomic propositions with letters, in accordance with the provided dictionary.

(d) Replace the natural language connectives with logical connectives and enclose subformulas with parantheses.

If Kaidan doesn't kill Udina, Udina survives and escapes. Udina doesn't escape. Therefore, Kaidan kills Udina.
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q = \text{Udina survives} \\
r = \text{Udina escapes}
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<table>
<thead>
<tr>
<th>$p$</th>
<th>Kaidan kills Udina</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>Udina survives</td>
</tr>
<tr>
<td>$r$</td>
<td>Udina escapes</td>
</tr>
</tbody>
</table>

If it isn’t the case that $p$, $q$ and $r$  
It isn’t the case that $r$  
$\neg r$  
$p$
Argument form in propositional logic

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(a) Identify the premises and the conclusion.
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\[ p = \text{Kaidan kills Udina} \]
\[ q = \text{Udina survives} \]
\[ r = \text{Udina escapes} \]

If it isn’t the case that \( p, q \) and \( r \)
It isn’t the case that \( r \)
\[ p \]
HOW TO: REPRESENT THE GENERAL ARGUMENT FORM

(a) Identify the premises and the conclusion.
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If Kaidan doesn’t kill Udina, Udina survives and escapes. Udina doesn’t escape. Therefore, Kaidan kills Udina.

\[ \sim p \supset (q \& r) \]
\[ \sim r \]
\[ \sim p \subset (q \& r) \]
\[ \sim r \]
\[ \sim p \subset (q \& r) \]
\[ \sim r \]
\[ p \]

\[
\begin{array}{|c|}
\hline
p & \text{Kaidan kills Udina} \\
q & \text{Udina survives} \\
r & \text{Udina escapes} \\
\hline
\end{array}
\]
Argument form: example 1

If Todd’s vegan, he can’t be defeated. Todd isn’t vegan if he eats gelato. Therefore, Todd doesn’t eat gelato or he can be defeated.
If Todd’s vegan, he can’t be defeated. Todd isn’t vegan if he eats gelato. Therefore, Todd doesn’t eat gelato or he can be defeated.

\[ p \rightarrow q \]

\[ \neg r \rightarrow p \]

\[ \neg r \lor q \]

**Question**: test your intuition, is the argument valid?

**Answer**: No, argument is invalid.
If Todd’s vegan, he can’t be defeated. Todd isn’t vegan if he eats gelato. Therefore, Todd doesn’t eat gelato or he can be defeated.

\[
p \quad \rightarrow \quad \neg q \\

\neg r \quad \rightarrow \quad p
\]

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Question: test your intuition, is the argument valid?

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If Todd’s vegan, he can’t be defeated. Todd isn’t vegan if he eats gelato. Therefore, Todd doesn’t eat gelato or he can be defeated.

<table>
<thead>
<tr>
<th>$p$</th>
<th>Todd is vegan</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>Todd can be defeated</td>
</tr>
<tr>
<td>$r$</td>
<td>Todd eats gelato</td>
</tr>
</tbody>
</table>

$p \supset \sim q$
If Todd’s vegan, he can’t be defeated. Todd isn’t vegan if he eats gelato. Therefore, Todd doesn’t eat gelato or he can be defeated.

\[ p \implies \neg q \]

\[ \neg r \implies \neg p \]

\[ \neg r \lor q \]

**Question**: test your intuition, is the argument valid?

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If Todd’s vegan, he can’t be defeated. Todd isn’t vegan if he eats gelato. Therefore, Todd doesn’t eat gelato or he can be defeated.

$p = \text{Todd is vegan}
$q = \text{Todd can be defeated}
$r = \text{Todd eats gelato}$

$p \supset \sim q
\sim r \supset \sim p
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$\sim r \lor q$

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\[ p \implies \neg q \]
\[ r \implies \neg p \]
\[ \neg p \]

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\[ p = \text{Todd is vegan} \]
\[ q = \text{Todd can be defeated} \]
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If Todd’s vegan, he can’t be defeated. Todd isn’t vegan if he eats gelato. Therefore, Todd doesn’t eat gelato or he can be defeated.

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p \supset \sim q \\
r \supset \sim p \\
\hline
\sim r \lor q
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$p = \text{Todd is vegan}$
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$p \supset \neg q$
$r \supset \neg p$

\[
\therefore \neg r \lor q
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If Todd’s vegan, he can’t be defeated. Todd isn’t vegan if he eats gelato. Therefore, Todd doesn’t eat gelato or he can be defeated.

\[
p \supset \sim q \\
r \supset \sim p
\]

\[
\sim r \lor q
\]

**Question**: test your intuition, is the argument valid?

**Answer**: No, argument is invalid.
Nancy won’t survive if and only if Hartigan doesn’t kill himself. Roark takes revenge or Hartigan kills himself. Therefore, if Roark doesn’t take revenge, Nancy will survive.
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\[ p = \text{Nancy will survive} \]
\[ q = \text{Hartigan kills himself} \]
\[ r = \text{Roark takes revenge} \]
Argument form: example 2

Nancy won’t survive if and only if Hartigan doesn’t kill himself. Roark takes revenge or Hartigan kills himself. Therefore, if Roark doesn’t take revenge, Nancy will survive.

\[ p = \text{Nancy will survive} \]
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\[ r = \text{Roark takes revenge} \]

\[ \sim p \equiv \sim q \lor q \]

\[ \sim r \implies p \]

Question: test your intuition, is the argument valid?

Answer: No, argument is invalid.
Argument form: example 2

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\[ \sim p \equiv \sim q \]
Nancy won’t survive if and only if Hartigan doesn’t kill himself. Roark takes revenge or Hartigan kills himself. Therefore, if Roark doesn’t take revenge, Nancy will survive.

$p$ = Nancy will survive  
$q$ = Hartigan kills himself  
$r$ = Roark takes revenge

$\sim p \equiv \sim q$
Nancy won’t survive if and only if Hartigan doesn’t kill himself. Roark takes revenge or Hartigan kills himself. Therefore, if Roark doesn’t take revenge, Nancy will survive.

\[ \sim p \equiv \sim q \]
\[ r \lor q \]

\[ \sim r \implies p \]
Nancy won’t survive if and only if Hartigan doesn’t kill himself. Roark takes revenge or Hartigan kills himself. Therefore, if Roark doesn’t take revenge, Nancy will survive.

\[ \sim p \equiv \sim q \]
\[ r \lor q \]
\[ \therefore \sim r \rightarrow p \]

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\neg p & \equiv \neg q \\
\neg r \lor q & \\
\hline
\neg r \supset p
\end{align*}
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\[ r \lor q \]
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**Question**: test your intuition, is the argument valid?

**Answer**: Yes, argument is valid.
Dr. Manhattan destroys New York only if the world doesn’t blow up. Because, if the world doesn’t blow up then Adrian isn’t the world’s smartest man, but if Adrian is the world’s smartest man then Dr. Manhattan destroys New York.

\[(\sim q \supset \sim r) \& (r \supset r)\]

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\[
\begin{align*}
\text{Dr. Manhattan destroys New York} & \equiv p \\
\text{World blows up} & \equiv q \\
\text{Adrian is the world’s smartest man} & \equiv r \\
\end{align*}
\]

\[
(\sim q \supset \sim r) \land (r \supset p) \quad \text{———} \quad p \supset \sim q
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\((\sim q \supset \sim r) \& (r \supset p)\)

\[ p \supset \sim q \]

**Question**: test your intuition, is the argument valid?

**Answer**: No, argument is invalid.
If Cluracan sings, drinks and dances, then he is merry. If he doesn’t dance then he isn’t merry. He is merry unless he doesn’t sing. Therefore, Cluracan is merry, if he drinks.
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\[(p \& (q \& r)) \supset s\]
If Cluracan sings, drinks and dances, then he is merry. If he doesn’t dance then he isn’t merry. He is merry unless he doesn’t sing. Therefore, Cluracan is merry, if he drinks.

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\end{align*}

\((p \land (q \land r)) \supset s\)
\(~r \supset \sim s\)

Question: test your intuition, is the argument valid?
Answer: No, argument is invalid.
If Cluracan sings, drinks and dances, then he is merry. If he doesn’t dance then he isn’t merry. **He is merry unless he doesn’t sing.** Therefore, Cluracan is merry, if he drinks.

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(p \land (q \land r)) \supset s \\
\sim r \supset \sim s
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| p   | Cluracan sings |
| q   | Cluracan drinks |
| r   | Cluracan dances |
| s   | Cluracan is merry |

\[(p \& (q \& r)) \supset s\]
\[\sim r \supset \sim s\]
\[s \lor \sim p\]

\[\underline{}\]
If Cluracan sings, drinks and dances, then he is merry. If he doesn’t dance then he isn’t merry. He is merry unless he doesn’t sing. Therefore, Cluracan is merry, if he drinks.

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$(p \land (q \land r)) \supset s$
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(p \land (q \land r)) \supset s \\
\sim r \supset \sim s \\
 s \lor \sim p \\
\hline
\sim p \\
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(p \land (q \land r)) \supset s \\
\sim r \supset \sim s \\
s \lor \sim p
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\[
\text{Therefore, } q \supset s
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\[
(p \land (q \land r)) \supset s \\
\sim r \supset \sim s \\
s \lor \sim p \\
\hline \\
q \supset s
\]

**Question**: test your intuition, is the argument valid?

**Answer**: No, argument is invalid.
If Kaidan doesn’t kill Udina, Udina survives and escapes. Udina doesn’t escape. Therefore Kaidan kills Udina.

\[ p = \text{Kaidan kills Udina} \]
\[ q = \text{Udina survives} \]
\[ r = \text{Udina escapes} \]
If Kaidan doesn’t kill Udina, Udina survives and escapes. Udina doesn’t escape. Therefore Kaidan kills Udina.

\[
\begin{align*}
\neg p & \implies (q \land r) \\
\neg r & \\
\therefore p
\end{align*}
\]

Winnie-the-pooh doesn’t keep his honey safe only if he falls asleep and heffalumps steal his honey. Heffalumps don’t steal his honey. Therefore, Winnie-the-pooh keeps his honey safe.

\[
\begin{align*}
p & = \text{Winnie-the-pooh keeps his honey safe} \\
q & = \text{Winnie-the-pooh falls asleep} \\
r & = \text{Heffalumps steal his honey}
\end{align*}
\]
If Kaidan doesn’t kill Udina, Udina survives and escapes. Udina doesn’t escape. Therefore Kaidan kills Udina.

\[ p = \text{Kaidan kills Udina} \]
\[ q = \text{Udina survives} \]
\[ r = \text{Udina escapes} \]

\[ \sim p \supset (q \& r) \]
\[ \sim r \]
\[ \hline \]
\[ p \]

Winnie-the-pooh doesn’t keep his honey safe only if he falls asleep and heffalumps steal his honey. Heffalumps don’t steal his honey. Therefore, Winnie-the-pooh keeps his honey safe.

\[ p = \text{Winnie-the-pooh keeps his honey safe} \]
\[ q = \text{Winnie-the-pooh falls asleep} \]
\[ r = \text{Heffalumps steal his honey} \]
# Some famous argument forms

<table>
<thead>
<tr>
<th>Modus Ponens</th>
<th>Modus Tollens</th>
<th>Modus Morons</th>
<th>Law of the Excluded Middle</th>
<th>Law of Non-Contradiction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \supset q$</td>
<td>$p \supset q$</td>
<td>$p \supset q$</td>
<td>$p \supset q$</td>
<td>$p \supset q$</td>
</tr>
<tr>
<td>$p$</td>
<td>$\sim q$</td>
<td>$\sim q$</td>
<td>$\sim q$</td>
<td>$\sim(q \land \sim q)$</td>
</tr>
<tr>
<td></td>
<td>$p$</td>
<td>$\sim p$</td>
<td>$p \lor \sim p$</td>
<td>$\sim(p \land \sim p)$</td>
</tr>
</tbody>
</table>