From program to data

Program: `tran/3, final/1`

```
accept(String) :- steps(q0,String,Q), final(Q).

steps(Q,[],Q).
steps(Q,[H|T],N) :- tran([Q,H,Qn), steps(Qn,T,N).
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From program to data (predicates $\rightsquigarrow$ lists)

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Data: `Tran, Final, q0`

```
acc(String) :- setof([Q,X,N], tran(Q,X,N), Tran),
               setof(Q, final(Q), Final),
               accept(String, Tran,Final,q0).
                              finite automaton
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From program to data (predicates ⇝ lists)

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                              finite automaton

accept([],_,Final,Q) :- member(Q,Final).
accept([H|T],Tran,Fi,Q) :- member([Q,H,N],Tran),
                           accept(T,Tran,Fi,N).
```

From accept/4 to search/1

$$\texttt{accept([],\_,Final,Q) :- }\underbrace{\texttt{member(Q,Final)}}_{\texttt{goal(Q)}}\texttt{.}$$

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search(Q) :- goal(Q).
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N.B. Finite automaton specifies initial state, goals & state space
+ `String` constrains `moves`.

Finite automaton as a finite model

Tran,Final,q0 $\rightsquigarrow$ U, move_a/2, goal/1, q0

universe U is set of states given by q0 and Tran

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setof(Q,(Q=q0; state(Tran,Q)), U).
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predicates: `goal/1`, `move_`a`/2` $(a \in \Sigma)$

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Finite automaton as a finite model

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Focus on models $\langle U, R_1, \ldots, R_n, c_1, \ldots, c_m \rangle$ where U is a finite set, R_i is an n_i-ary relation on U,

$$R_i \subseteq \underbrace{U \times \cdots \times U}_{n_i \text{ copies of } U} \quad (\text{for } 1 \leq i \leq n)$$

and c_j is a member of U (for $1 \leq j \leq m$).

Models from Datalog Knowledge Bases

Datalog KB $\approx$ declarative Prolog program with constants and predicates but NO functions of non-zero arity

$\texttt{U}$ = set of constants mentioned in KB

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Example from SWISH-Prolog (click here)

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U = [vincent, mia, marcellus, pumpkin, honey_bunny]
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U = [vincent, mia, marcellus, pumpkin, honey_bunny]
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Bound search: instantiate variables in U

```
?- loves(X,Y).
```

$\leadsto$

```
?- member(X,[vincent, mia, ..., honey_bunny]),
   member(Y,[vincent, mia, ..., honey_bunny]),
   loves(X,Y).
```

Complications from loops – e.g.

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loves(X,Y) :- loves(Y,X).
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[CSU33061: Constraint satisfaction, Herbrand models]

Strings as finite models

abbc $\rightsquigarrow$ model

$$\langle D_4, [\![S]\!], [\![P_a]\!], [\![P_b]\!], [\![P_c]\!] \rangle$$

Strings as finite models

$abbc \quad \rightsquigarrow \quad$ model

$$\langle D_4, [\![S]\!], [\![P_a]\!], [\![P_b]\!], [\![P_c]\!] \rangle$$

where

$$D_4 := \{1, 2, 3, 4\}$$
$$[\![S]\!] := \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 4\rangle\}$$

a binary relation symbol (successor) S

$$(\exists x_1)(\exists x_2)(\exists x_3)(\exists x_4) \quad S(x_1, x_2) \wedge S(x_2, x_3) \wedge S(x_3, x_4) \wedge \neg(\exists x)(S(x, x_1) \vee S(x_4, x))$$

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a binary relation symbol (successor) S and
a unary relation symbol P_σ for each symbol σ

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`accept/4` as a relation between models?

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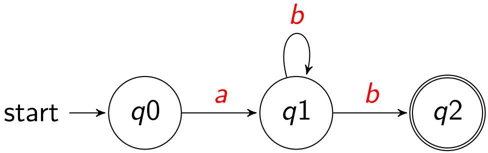
reconstrue finite automaton as predicate logic sentence



ab^*b

$$(\exists x)(\exists y)(P_a(x) \wedge P_b(y) \wedge \\ (\forall z)(\neg S(z,x) \wedge (z = x \vee P_b(z))))$$

Procedural/declarative divide

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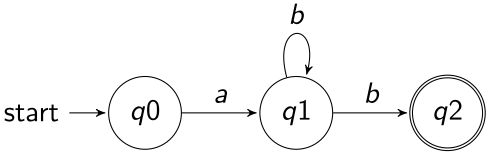

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`accept/4` as notion of satisfaction in predicate logic

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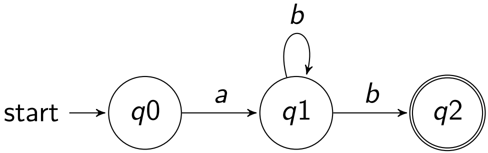

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Logic programming is neither logic nor programming.

\- Anonymous