CSU34011 Symbolic Programming
First of Two Assessed Labs[1]

We know from Chapter 3 of LPN that addition over numerals `0, s(0), s(s(0))`, ... is captured by the predicate `add` below

```
add(0,X,X).
add(s(X),Y,s(Z)) :- add(X,Y,Z).
```

The aim of this assignment is to get you to think recursively. Accordingly, your answers should *not* use `is/2` or any of Prolog's built-in arithmetic predicates (described in Chapter 5).

Exercise 1 (20/100 points)
Suppose `X+Y` were numerals whenever `X` and `Y` are numerals. Define a predicate `add2(X,Y,Z)` such that for instance,

```
?- add2(s(0)+s(s(0)), s(s(0)), Z).
Z = s(s(s(s(s(0)))))

?- add2(0, s(0)+s(s(0)), Z).
Z = s(s(s(0)))

?- add2(s(s(0)), s(0)+s(s(0)), Z).
Z = s(s(s(s(s(0)))))

?- add2(s(0)+s(0), s(0+s(s(0))), Z).
Z = s(s(s(s(s(0)))))
```

etc. so that `Z` takes the form $\mathtt{s}^n\mathtt{(0)}$ for some $n \geq 0$.

Exercise 2 (20/100 points)
Next we introduce negative numbers via the function symbol `p` (for predecessor or -1, just as `s` stands for successor or $+1$). Suppose `p(X)` were a numeral whenever `X` is a numeral. Modify the predicate `add2` to `add3` such that for instance,

```
?- add3(p(s(0)), s(s(0)), Z).
Z = s(s(0))

?- add3(0, s(p(0)), Z).
Z = 0

?- add3(p(0)+s(s(0)),s(s(0)),Z).
Z = s(s(s(0)))

?- add3(p(0), p(0)+s(p(0)), Z).
Z = p(p(0))
```

so that `Z` takes the form $\mathtt{s}^n\mathtt{(0)}$ or $\mathtt{p}^n\mathtt{(0)}$ for some $n \geq 0$.

[1] Due in Blackboard by Oct 12 (23:59). Submit one text file suitable for cut-and-pasting onto SWISH-Prolog. Comment out code that causes queries to fail but keep it as a comment for possible partial credit.

Exercise 3 (20/100 points)
Define a predicate `minus(X,Y)` such that for instance,

```
?- minus(0, Z).
Z = 0

?- minus(s(s(0)), Z).
Z = p(p(0))

?- minus(s(p(0)), Z).
Z = 0

?- minus(p(s(p(0))), Z).
Z = s(0)
```

so that `Z` takes the form `s`^{n}`(0)` or `p`^{n}`(0)` for some $n \geq 0$.

Exercise 4 (20/100 points)
Suppose further that `-X` were a numeral whenever `X` is a numeral. Revise the predicate `add3` to `add4` such that for instance,

```
?- add4(-p(s(0)), s(s(0)), Z).
Z = s(s(0))

?- add4(p(0)+s(s(0)), -s(s(0)), Z).
Z = p(0)
```

so that `Z` takes the form `s`^{n}`(0)` or `p`^{n}`(0)` for some $n \geq 0$.

Exercise 5 (10/100 points)
Define the predicate `subtract(X,Y,Z)` for subtracting `Y` from `X` to get `Z` such that for instance,

```
?- subtract(p(s(0)), s(s(0)), Z).
Z = p(p(0))

?- subtract(p(0), -s(s(0)), Z).
Z = s(0)
```

so that `Z` takes the form `s`^{n}`(0)` or `p`^{n}`(0)` for some $n \geq 0$.

Exercise 6 (10/100 points)
Revise the predicates `add4` to `add5` and `subtract` to `subtract2` in order to handle numerals `X-Y`, for numerals `X` and `Y`. For instance,

```
?- add5(-(s(0)-p(0)),s(0),X).
X = p(0)

?- subtract2(p(0), p(s(0))-s(s(0)), Z).
Z = s(0)
```

so that `Z` takes the form `s`^{n}`(0)` or `p`^{n}`(0)` for some $n \geq 0$.