Computation as search

\[\text{search}(\text{Node}) :\text{-} \text{goal}(\text{Node}).\]

\[\text{search}(\text{Node}) :\text{-} \text{arc}(\text{Node},\text{Next}), \text{search}(\text{Next}).\]
Computation as search

```
search(Node) :- goal(Node).

search(Node) :- arc(Node,Next), search(Next).
```

More than one Next may satisfy `arc(Node,Next)`

\[ \iff \text{non-determinism} \]

Computation eliminates non-determinism (determinization)

Available choices depend on `arc`-actions specified by Turing machine (graph)

Bound number of calls to `arc` (iterations of `search`)
Computation as search

\[
\text{search}(\text{Node}) \leftarrow \text{goal}(\text{Node}).
\]

\[
\text{search}(\text{Node}) \leftarrow \text{arc}(\text{Node}, \text{Next}), \text{search}(\text{Next}).
\]

More than one \text{Next} may satisfy \text{arc}(\text{Node}, \text{Next})
\[\rightsquigarrow\] non-determinism

Choose \text{Next} closest to goal (heuristic: best-first),
keeping track of costs (min cost, A*)
Computation as search

\[
\text{search}(\text{Node}) :\text{ }\text{a} \Rightarrow \text{goal}(\text{Node}).
\]

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Feasibility and non-determinism: P vs NP

Cobham’s Thesis

A problem is feasibly solvable iff some deterministic Turing machine (dTm) solves it in polynomial time.

\[ P = \{ \text{problems a dTm solves in polynomial time} \} \]
Feasibility and non-determinism: P vs NP

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\( P = NP \) says non-determinism makes no difference to feasibility.
A closer look

Given a set $L$ of strings, and a Tm $M$. 

\[
\text{TIME}(n^k) := \{ L | \text{some dTm solves } L \text{ in time } n^k \}
\]

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A closer look

Given a set $L$ of strings, and a Tm $M$.

$M$ solves $L$ in time $n^k$ if there is a fixed integer $c > 0$ such that for every string $s$ of size $n$,

$$s \in L \iff M \text{ accepts } s \text{ within } c \cdot n^k \text{ steps.}$$
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Boolean satisfiability (SAT)

**SAT.** Given a Boolean expression \( \varphi \) with variables \( x_1, \ldots, x_n \), can we make \( \varphi \) true by assigning true/false to \( x_1, \ldots, x_n \)?

e.g., \( (x_1 \lor \overline{x_2} \lor x_3) \land (\overline{x_1} \lor \overline{x_3}) \)
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**CSAT:** \( \varphi \) is a conjunction of clauses, where a *clause* is an OR of literals, and a *literal* is a variable \( x_i \) or negated variable \( \overline{x_i} \).
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But is SAT in \( P \)? There are \( 2^n \) assignments to try.

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Horn-SAT: every clause has at most one positive literal — linear