Assumptions behind Datalog

- An agent’s knowledge can be usefully described in terms of *individuals* and *relations* among individuals.
- An agent’s knowledge base consists of *definite* and *positive* statements.
- The environment is *static*.
- There are only a finite number of individuals of interest in the domain.
  An individual can be named.
Datalog syntax

- **A variable** starts with upper-case letter.
- **A constant** starts with lower-case letter or is a sequence of digits (numeral).
- **A predicate symbol** starts with lower-case letter.
- **A term** is either a variable or a constant.
- **An atomic symbol** (atom) is of the form $p$ or $p(t_1, \ldots, t_n)$ where $p$ is a predicate symbol and $t_i$ are terms.
A definite clause is either an atomic symbol (a fact) or of the form:

\[ a \leftarrow b_1 \land \cdots \land b_m \]

where \( a \) and \( b_i \) are atomic symbols.

query is of the form \(?b_1 \land \cdots \land b_m\).

knowledge base is a set of definite clauses.
A **semantics** specifies the meaning of sentences in the language. An **interpretation** specifies:

- what objects (individuals) are in the world
- the correspondence between symbols in the computer and objects & relations in world
  - constants denote individuals
  - predicate symbols denote relations
An interpretation is a triple $I = \langle D, \phi, \pi \rangle$, where

- $D$, the **domain**, is a nonempty set. Elements of $D$ are **individuals**.

- $\phi$ is a mapping that assigns to each constant an element of $D$. Constant $c$ **denotes** individual $\phi(c)$.

- $\pi$ is a mapping that assigns to each $n$-ary predicate symbol a relation: a function from $D^n$ into $\{TRUE, FALSE\}$. 
Example Interpretation

Constants: *phone*, *pencil*, *telephone*.
Predicate Symbol: *noisy* (unary), *left_of* (binary).

- \( D = \{ \text{phone}, \text{pencil}, \text{telephone} \} \).
- \( \phi(\text{phone}) = \text{phone}, \phi(\text{pencil}) = \text{pencil}, \phi(\text{telephone}) = \text{telephone} \).
- \( \pi(\text{noisy}): \begin{array}{ccc} \langle \text{phone} \rangle & \text{FALSE} & \langle \text{pencil} \rangle & \text{TRUE} & \langle \text{telephone} \rangle & \text{FALSE} \end{array} \)
- \( \pi(\text{left_of}): \begin{array}{ccc} \langle \text{phone}, \text{phone} \rangle & \text{FALSE} & \langle \text{phone}, \text{pencil} \rangle & \text{TRUE} & \langle \text{phone}, \text{telephone} \rangle & \text{TRUE} \\ \langle \text{pencil}, \text{phone} \rangle & \text{FALSE} & \langle \text{pencil}, \text{pencil} \rangle & \text{FALSE} & \langle \text{pencil}, \text{telephone} \rangle & \text{TRUE} \\ \langle \text{telephone}, \text{phone} \rangle & \text{FALSE} & \langle \text{telephone}, \text{pencil} \rangle & \text{FALSE} & \langle \text{telephone}, \text{telephone} \rangle & \text{FALSE} \end{array} \)
Important points to note

- The domain $D$ can contain real objects. (e.g., a person, a room, a course). $D$ can’t necessarily be stored in a computer.
- $\pi(p)$ specifies whether the relation denoted by the $n$-ary predicate symbol $p$ is true or false for each $n$-tuple of individuals.
- If predicate symbol $p$ has no arguments, then $\pi(p)$ is either TRUE or FALSE.
Truth in an interpretation

A constant $c$ denotes in $I$ the individual $\phi(c)$. Ground (variable-free) atom $p(t_1, \ldots, t_n)$ is
- true in interpretation $I$ if $\pi(p)(\langle \phi(t_1), \ldots, \phi(t_n) \rangle) = \text{TRUE}$ in interpretation $I$ and
- false otherwise.

Ground clause $h \leftarrow b_1 \land \ldots \land b_m$ is false in interpretation $I$ if $h$ is false in $I$ and each $b_i$ is true in $I$, and is true in interpretation $I$ otherwise.
Example Truths

In the interpretation given before, which of following are true?

\[
\begin{align*}
&\text{noisy}(\text{phone}) \\
&\text{noisy}(\text{telephone}) \\
&\text{noisy}(\text{pencil}) \\
&\text{left}_{\bigcirc} (\text{phone}, \text{pencil}) \\
&\text{left}_{\bigcirc} (\text{phone}, \text{telephone}) \\
&\text{noisy}(\text{phone}) \leftarrow \text{left}_{\bigcirc} (\text{phone}, \text{telephone}) \\
&\text{noisy}(\text{pencil}) \leftarrow \text{left}_{\bigcirc} (\text{phone}, \text{telephone}) \\
&\text{noisy}(\text{pencil}) \leftarrow \text{left}_{\bigcirc} (\text{phone}, \text{pencil}) \\
&\text{noisy}(\text{phone}) \leftarrow \text{noisy}(\text{telephone}) \land \text{noisy}(\text{pencil})
\end{align*}
\]
Example Truths

In the interpretation given before, which of following are true?

\[
\begin{align*}
\text{noisy}(\text{phone}) & \quad \text{true} \\
\text{noisy}(\text{telephone}) & \quad \text{true} \\
\text{noisy}(\text{pencil}) & \quad \text{false} \\
\text{left}_\text{of}(\text{phone}, \text{pencil}) & \quad \text{true} \\
\text{left}_\text{of}(\text{phone}, \text{telephone}) & \quad \text{false} \\
\text{noisy}(\text{phone}) & \quad \text{left}_\text{of}(\text{phone}, \text{telephone}) \quad \text{true} \\
\text{noisy}(\text{pencil}) & \quad \text{left}_\text{of}(\text{phone}, \text{telephone}) \quad \text{true} \\
\text{noisy}(\text{pencil}) & \quad \text{left}_\text{of}(\text{phone}, \text{pencil}) \quad \text{false} \\
\text{noisy}(\text{phone}) & \quad \text{noisy}(\text{telephone}) \land \text{noisy}(\text{pencil}) \quad \text{true}
\end{align*}
\]
A knowledge base, $KB$, is true in interpretation $I$ if and only if every clause in $KB$ is true in $I$.

A model of a set of clauses is an interpretation in which all the clauses are true.

If $KB$ is a set of clauses and $g$ is a conjunction of atoms, $g$ is a logical consequence of $KB$, written $KB \models g$, if $g$ is true in every model of $KB$.

That is, $KB \models g$ if there is no interpretation in which $KB$ is true and $g$ is false.
User’s view of Semantics

1. Choose a task domain: intended interpretation.
2. Associate constants with individuals you want to name.
3. For each relation you want to represent, associate a predicate symbol in the language.
4. Tell the system clauses that are true in the intended interpretation: axiomatizing the domain.
5. Ask questions about the intended interpretation.
6. If $KB \models g$, then $g$ must be true in the intended interpretation.
Computer’s view of semantics

- The computer doesn’t have access to the intended interpretation.
- All it knows is the knowledge base.
- The computer can determine if a formula is a logical consequence of $KB$.
- If $KB \models g$ then $g$ must be true in the intended interpretation.
- If $KB \not\models g$ then there is a model of $KB$ in which $g$ is false. This could be the intended interpretation.
in(kim, r123).
part_of(r123, cs_building).
in(X, Y) ←
  part_of(Z, Y) ∧
in(X, Z).