Faculty of Engineering, Mathematics and Science
School of Computer Science & Statistics

B.A. (Mod.) Computer Science & Language Semester 2 2019
Year 4 Annual Examinations

Knowledge Representation and Automata

Fri, 26 April 2019 RDS-SIM COURT 9:30 – 11:30

Dr Tim Fernando

Instructions to Candidates:
Attempt two questions. All questions carry equal marks. Each question is scored out of a total of 50 marks.

You may not start this examination until you are instructed to do so by the Invigilator.

Exam paper is not to be removed from the venue.

Materials permitted for this examination:
Non-programmable calculators are permitted for this examination — please indicate the make and model of your calculator on each answer book used.
1. (a) What are the ingredients constituting a **Turing machine** (Tm), and what restrictions on these ingredients make the Tm a **finite state machine** (fsm)?

[10 marks]

(b) What does it mean for a Tm to be non-deterministic? Why does depth-first search suffice to process the non-determinism of an fsm, but is inadequate for a non-deterministic Tm in general? Describe an alternative to depth-first search that suffices for an arbitrary non-deterministic Tm.

[15 marks]

(c) Let $L_3$ be the set of strings

$$L_3 = (0 + 1)^*1(0 + 1)(0 + 1)$$

over the alphabet $\{0, 1\}$ with length at least 3 such that the third to the last symbol of the string is 1. For example, 100 and 0110 are both in $L_3$, but neither 11 nor 01011 is.

(i) One way to build an fsm accepting $L_3$ is by identifying a state with a set of $L_3$-inseparable histories. Describe the transitions for such states, specifying which state is initial and which are final/accepting.

[10 marks]

(ii) Another way to build an fsm accepting $L_3$ is by identifying a state with a set of $L_3$-acceptable futures. Describe the transitions for such states, specifying which state is initial and which are final/accepting.

[10 marks]

(iii) What is the minimum number of states that a deterministic fsm accepting $L_3$ must have?

[5 marks]
2. (a) Recall that an atom \( p(t_1, \ldots, t_n) \) consists of an \( n \)-ary predicate \( p \) and an \( n \)-tuple \( t_1, \ldots, t_n \) of terms \( t_i \). Given three atoms \( h, a \) and \( b \), we can formalize the conditional

\[ h \text{ if } a \text{ and } b \]

as either the definite clause (\( \dagger \)) or as the claim (\( \ddagger \)) that \( h \) is a logical consequence of the knowledge base \( a, b \).

\[ h :- a, b. \quad (\dagger) \]

\[ a, b \models h \quad (\ddagger) \]

(i) The meaning of (\( \dagger \)) and (\( \ddagger \)) can be made precise through the notion of an interpretation \( I = \langle D, \phi, \pi \rangle \). But just what are \( D, \phi \) and \( \pi \)?

[5 marks]

(ii) How is the notion of an interpretation used to specify what (\( \dagger \)) means?

[10 marks]

(iii) How is the notion of an interpretation used to specify what (\( \ddagger \)) means?

[10 marks]

(iv) How does the claim

\[ a \models h :- b \]

that \( h :- b \) is a logical consequence of \( a \) compare to (\( \dagger \)) and (\( \ddagger \))?

[10 marks]

(b) Description logics trade in concepts and roles, the meanings of which are also given relative to an interpretation \( I = \langle D, \phi, \pi \rangle \). A Description Logic due to Brachman and Levesque allows us to form the concept

\[ \text{[ALL } R C \text{]} \quad (*) \]

from a role \( R \) and concept \( C \), as well as the subsumption

\[ A \subseteq C \quad (***) \]

from an additional concept \( A \). Now, suppose \( I \) is an interpretation under
which the meaning of $R$ consists of pairs $(a, a)$ such that $a$ is an element of the meaning of $A$

$$\mathcal{M}[R] = \{(a, a) \mid a \in \mathcal{M}[A]\}.$$

What do $(\ast)$ and $(\ast\ast)$ mean under the interpretation $I$? Are their meanings related? Explain.

[15 marks]
3. (a) What does the Büchi-Elgot-Trakhtenbrot theorem say about **Monadic Second Order logic** (MSO) over strings? How can the string

\[
\begin{array}{c}
\text{a, b, a, c}
\end{array}
\]

be understood as a **model** of MSO over the vocabulary \( A = \{a, b, c\} \)?

[15 marks]

(b) For any finite alphabet \( \Sigma \), the set \( \text{Odd}(\Sigma) \) of strings over \( \Sigma \) that have odd length form a regular language.

(i) Give a regular expression specifying \( \text{Odd}(\Sigma) \).

[10 marks]

(ii) Give an MSO-sentence satisfied exactly by the models associated with the strings in \( \text{Odd}(\Sigma) \).

[25 marks]