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The Semantics of Tense and Aspect

A finite-state perspective

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1 Introduction: Prior and beyond

The present chapter describes a range of formal semantic accounts of tense and aspect, constituting a modest portion of the vast literature on tense and aspect (e.g., Binnick, 2012; Mani *et al.*, 2005). The focus is on the nature of the ingredients assumed, including the pairs $\langle w, t \rangle$ of possible worlds w and moments t of time in Montague (1973), expansions of the moments t to intervals (Bennett & Partee, 1972; Dowty, 1979) which generalize to formal occurrences (Galton, 1987), reductions of worlds w to situations (Barwise & Perry, 1983) events/eventualities (Kamp, 1979; Bach, 1981; van Lambalgen & Hamm, 2005), incomplete events (Parsons, 1990), branching (Landman, 1992), event nuclei (Moens & Steedman, 1988), and related complexes (Pustejovsky, 1991; Kamp & Reyle, 1993; Pulman, 1997). The chapter formulates these notions in finite-state terms, building strings that approximate timelines, a logical starting point for which is Priorean tense logic (Prior, 1967).

At the heart of Priorean tense logic, commonly called temporal logic (e.g., Emerson, 1992), is a satisfaction relation $\models_{\mathfrak{A}}$ defined relative to a model $\mathfrak{A}$. A simple example of $\models_{\mathfrak{A}}$ at work is the analysis (1b) below of (1a) as $\text{Past}(\mathsf{adam\text{-}leave\text{-}the\text{-}garden})$, with a time parameter changing from t to t'.

(1) a. Adam left the garden.

b. $t \models_{\mathfrak{A}} \text{Past}(\mathsf{adam\text{-}leave\text{-}the\text{-}garden}) \iff$
$(\exists t' \prec t)\ t' \models_{\mathfrak{A}} \mathsf{adam\text{-}leave\text{-}the\text{-}garden}$

The model $\mathfrak{A}$ is assumed to specify

(i) an *earlier-than* relation $\prec$ on a set $T_{\mathfrak{A}}$ of $\mathfrak{A}$-*times*, and

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(ii) a set $\mathfrak{A}[\mathsf{adam\text{-}leave\text{-}the\text{-}garden}]$ of $\mathfrak{A}$-times satisfying adam-leave-the-garden

$$t' \models_{\mathfrak{A}} \mathsf{adam\text{-}leave\text{-}the\text{-}garden} \iff t' \in \mathfrak{A}[\mathsf{adam\text{-}leave\text{-}the\text{-}garden}].$$

Taking t in (1b) to be the speech time S, and t' to be the event time E, the right hand side of (1b) says $E \prec S$, in accordance with the simple past (1a), as well as the present perfect (2a) and the past perfect (2b) below.

(2) a. Adam has left the garden.
 b. Adam had left the garden.

1.1 Reichenbach

(1a), (2a) and (2b) are differentiated in Reichenbach (1947) through a third parameter, the *reference time* R, which is related to

(i) event time E to determine aspect, as in (3), and
(ii) speech time S to determine tense, as in (4).

(3) a. simple: $E = R$
 b. perfect: $E \prec R$

(4) a. present: $R = S$
 b. past: $R \prec S$

(3) and (4) yield $E \prec S$ for each of (1a), (2a) and (2b), but with R at distinct positions relative to E and S. Reichenbach claims that R (not E or S) is "the carrier of the time position" to which a temporal adverb such as *yesterday* pertains, explaining the contrast in (5).

(5) a. Adam left the garden yesterday.
 b. *Adam has left the garden yesterday.

(5b), the argument goes, is odd because R is in the present whereas *yesterday* is in the past. A second past occurs in (6), distinguishing (2b) from (1a) and (2a), neither of which can replace (2b) in (6).

(6) Eve was in bits. Adam had left the garden. She had followed.
Now, paradise was lost and hard labour lay ahead.

1.2 The imperfective, intervals and aspectual classes

Another variant of (1a) in the past is the past progressive (7).

(7) Adam was leaving the garden (when it started to rain).

Unlike (1a), (2a) or (2b), however, (7) stops short of asserting that an adam-leave-the-garden event was completed, saying only that it was in progress. (7) is an imperfective, which contrasts with perfectives roughly according to (8) (e.g., Comrie, 1976; Smith, 1991; Klein & Li, 2009) .

(8) a. imperfective: ongoing, viewed from the inside, open-ended

b. perfective: completed, viewed from the outside, closed/bounded

We can flesh out the intuitions in (8) against a linear order $\prec$ on the set $T_{\mathfrak{A}}$ of time points as follows. An *interval* is a non-empty subset I of $T_{\mathfrak{A}}$ such that for all t and t' in I and $x \in T_{\mathfrak{A}}$, if x falls between t and t' (i.e., $t \prec x \prec t'$), then $x \in I$. An interval I is said to be *inside* an interval J, written $I \sqsubset J$, if J contains points to the left and to the right of all of I

$$I \sqsubset J \iff (\exists l, r \in J)(\forall t \in I)\ l \prec t \prec r.$$

Next, we introduce an interval V from which the event is viewed, and take the event time E also to be an interval. V is inside E for imperfectives with event time E (8a,9a), while E is inside V for perfectives with event time E (8b,9b).

(9) a. imperfective: V $\sqsubset$ E

b. perfective: E $\sqsubset$ V

Replacing V by R in (9a) yields R $\sqsubset$ E, a common Reichenbachian account of the progressive.[1] Just how the perfective in (9b) fits alongside either the simple or perfect in (3) is not clear.

V and/or R aside, something akin to the perfective/imperfective distinction is refined by the aspectual classes *States*, *Activities*, *Achievements* and *Accomplishments*, going back to Aristotle, Ryle, Kenny and Vendler (Vendler, 1957; Dowty, 1979). The progressive can be applied to distinguish an activity (such as *walking*) from an accomplishment (such as *walking a mile*); the former carries an entailment, (10a),[2] that the latter does not, (10b).

(10) a. Adam was walking $\vdash$ Adam walked

b. Adam was walking a mile $\nvdash$ Adam walked a mile

The progressives of states and achievements are more delicate matters; states cannot, in general, be put in the progressive (**Adam is loving Eve*), while the trouble with progressives of achievements (such as *arriving*) is that achievements are conceptualized as punctual, with temporal extents smaller than that of an event in the progressive (which, under (9a) above, is large enough to contain V). Assuming times to the left of the satisfaction relation

[1] See Moens & Steedman (1988), pages 22 and 28 (footnote 3).

[2] In fact, (10a) is questionable inasmuch as the possibility that Adam is still walking conflicts with the conclusion "Adam walked." If so, add the assumption "Adam is not walking" to (10a), and "Adam is not walking a mile" to (10b).

$\models_{\mathfrak{A}}$ are intervals (Bennett & Partee, 1972) but otherwise leaving progressives out, we can check how the truth, $I \models_{\mathfrak{A}} \varphi$, of φ at an interval I changes with subintervals of I[3] according to the aspectual class of φ. (11) is essentially item (13) in page 42 of Dowty (1986).

(11) Given $I \models_{\mathfrak{A}} \varphi$ and a subinterval I' of I, what more do we need to conclude $I' \models_{\mathfrak{A}} \varphi$?

a. For stative φ, nothing further required.

b. For an activity φ, I' is not too small.

c. For an achievement or accomplishment φ, $I' = I$.

Missing from (11) for the sake of simplicity is a world parameter varied in Dowty (1979) to account for events in progress that (as anticipated by (10b)) do not run to completion.

(12) Adam was leaving the garden when he was slain.

Aspectual classes are represented in Dowty (1979) by formulas in an aspect calculus, interpreted relative to interval-world pairs $\langle I, w \rangle$. Rather than building aspectual classes from pairs $\langle I, w \rangle$, *event nuclei* are described in Moens & Steedman (1988) consisting of culminations bracketed by preparatory processes (activities) to the left, and consequent states to the right.

The consequent state of an event is linked to the Reichenbachian analysis (3b) of the perfect in cases such as (2a) where the event (Adam's departure from the garden) has a clearly associated consequent state (Adam not in the garden).

(3) b. perfect: E $\prec$ R

(2) a. Adam has left the garden.

In such cases, E $\prec$ R follows from identifying E as the temporal projection of an event e that has a consequent state with temporal projection R. The equation R = S from the present (4a) entails the consequent state holds at speech time. This puts Adam outside the garden at S, unless the consequent state is understood as some condition other than Adam not being in the garden. An extreme choice of a consequent state of e, called the *resultant* state of e in Parsons (1990), is that e has occurred. Resultant or not, the consequent state is, we are assuming, derived from an event e. What if e is already a state as in (13a) or in the progressive as in (13b)?

(13) a. Adam has been outside the garden.

b. Adam has been sitting in the garden all afternoon.

As explained below, consequent-state accounts of the perfect appeal to type coercion (Moens & Steedman, 1988; Kamp & Reyle, 1993; Pulman, 1997), but in recent years, the "extended now" approach to the perfect (going back to

[3] A *subinterval of* an interval I is a subset of I that is an interval.

McCoard (1978); Dowty (1979)) has become a popular alternative, adding a *Perfect Time Span* (Iatridou *et al.*, 2001) on top of V in (9).

1.3 Prior extended three ways

Prior's use of evaluation time in (1b) for event time t' (Reichenbach's E) and speech time t (Reichenbach's S) are extended by the works mentioned in Sections 1.1 and 1.2 along at least three directions, listed in (14).

(1) b. $t \models_{\mathfrak{A}} \textsc{Past}(\textsf{adam-leave-the-garden}) \iff$
$(\exists t' \prec t)\ t' \models_{\mathfrak{A}} \textsf{adam-leave-the-garden}$

(14) a. add temporal parameters (e.g., R, V, Perfect Time Span)

b. expand times from points to intervals

c. bring out the events and states timed by E, R, S, etc.

If we generalize (1b) from $\prec$ to an arbitrary binary relation r on $T_{\mathfrak{A}}$, and λ-abstract for a categorial compositional analysis, we obtain the recipe (15a), which together with (15b), yields (15c).

(15) a. $\mathrm{ap}_{\mathrm{r}} = (\lambda P)(\lambda x)(\exists x'\ \mathrm{r}\ x)\ P(x')$
I.e., $\mathrm{ap}_{\mathrm{r}}(P)(x)$ says: $P(x')$ for some x' such that $x'\ \mathrm{r}\ x$

b. $\mathfrak{A}[\varphi](t') \iff t' \models_{\mathfrak{A}} \varphi$

c. $\mathrm{ap}_{\mathrm{r}}(\mathfrak{A}[\varphi])(t) \iff (\exists t'\ \mathrm{r}\ t)\ t' \models_{\mathfrak{A}} \varphi$ (given 15a,15b)

For φ equal to adam-leave-the-garden, we can approximate the Reichenbachian analysis E=R≺S of (1b) as $\mathrm{ap}_{\prec}(\mathrm{ap}_{=}(\mathfrak{A}[\varphi]))(\mathrm{S})$, which reduces to

$$(\exists \mathrm{R} \prec \mathrm{S})(\exists \mathrm{E} = \mathrm{R})\ \mathrm{E} \models_{\mathfrak{A}} \textsf{adam-leave-the-garden}.$$

The Reichenbachian present perfect E≺R=S has an equivalent approximation

$$\mathrm{ap}_{=}(\mathrm{ap}_{\prec}(P))(t) \iff \mathrm{ap}_{\prec}(\mathrm{ap}_{=}(P))(t)$$

as $\mathrm{ap}_{=}$ can be dropped without effect. The existential quantifier in (1b)/(15a) buries the reference time R (never mind the event time E, which $\mathfrak{A}[\varphi]$ picks out). In a sentence such as (16) from Partee (1973), it is useful to bring R out as a contextual parameter, specifying an interval (before S) over which the speaker fails to turn off the stove.

(16) I didn't turn off the stove.

Revising (1b) slightly, (17) puts R explicitly alongside S.

(17) $\textsc{Past}(\varphi)$ is $\mathfrak{A}$-true at R,S $\iff$ $\mathrm{R} \prec \mathrm{S}$ and $\mathrm{R} \models_{\mathfrak{A}} \varphi$

As a contextual parameter in (17), R becomes available for update, and can move time forward in narratives such as (18a), if not (18b).

(18) a. Adam left the garden. Eve wept.

b. The sky was dark. Eve was asleep.

A multi-sentence discourse typically describes a number of events and states, the temporal relations between which can be a problem to specify. This problem is investigated at length in dynamic approaches to discourse such as *Discourse Representation Theory* (DRT, Kamp & Reyle (1993)), which have arisen in no small part from the limitations of existential quantification. The fitness of R for various anaphoric purposes has been challenged (Kamp & Reyle, 1993; Nelken & Francez, 1997), and a slew of temporal parameters beyond S and R have been proposed to link sentences in a discourse. These links go beyond temporal intervals to events and states, employed in Asher & Lascarides (2003) as semantic indices for an account of discourse coherence based on rhetorical relations.

Stepping back to (17) and proceeding more conservatively from a timeline, let us refine (17) two ways. First, sentences such as (19) from Kamp (1971) suggest doubling the temporal parameter to the left of $\models_{\mathfrak{A}}$ to include the speech time S so that *will become* is placed after not just the child's birth but also S.[4]

(19) A child was born who will become ruler of the world.

And second, we can attach R as a subscript on PAST in (17), giving as many different $\mathrm{PAST}_{\mathrm{R}}$'s as there are choices of R, with the choice of R analogous to pronoun resolution (Kratzer, 1998). These two refinements can be implemented by treating R and S as variables assigned values by a function g (from context), which we adjoin to a model $\mathfrak{A}$ for the expanded model $(\mathfrak{A}, g)$. Generalizing (again) from $\prec$ to a binary relation on $T_{\mathfrak{A}}$, we can sharpen (17) to (20a) and (15b) to (20b).

(20) a. $\mathrm{TENSE}^{\mathrm{r}}_{\mathrm{R}}(\varphi)$ is $(\mathfrak{A}, g)$-true $\iff$ $g(\mathrm{R})\ \mathrm{r}\ g(\mathrm{S})$ and $g(\mathrm{R}) \models_{\mathfrak{A},g} \varphi$

b. $(\mathfrak{A}, g)[\varphi](t) \iff t \models_{\mathfrak{A},g} \varphi$

Whereas the satisfaction relation $\models_{\mathfrak{A}}$ occurs on both sides of (1b), $\models_{\mathfrak{A},g}$ occurs in (20a) only on the right, the idea being to distinguish $\mathrm{TENSE}^{\mathrm{r}}_{\mathrm{R}}$ from the modal operator ap_{r} linked, as in (15c), to $\models$ through (20b).[5]

What choices can we make for r in (20a) apart from $\prec$ and $=$ from (4)? There is a tradition going back to Chomsky (1957) that Past and Present are the only two English tense morphemes. This leaves the Future to be expressed through a modal auxiliary WOLL (Abusch, 1985), interpreted as essentially $\mathrm{ap}_{\succ}$ (stripped of worlds and types on variables, which we can safely put aside for the present discussion).

[4] This change to (17) gives essentially *true*$_2$ in Dowty (1982).

[5] The occurrence of R (but not S) on the left-hand side of (20a) makes R (but not S) essentially a meta-variable (insofar as different choices of R are possible). Generalizations of S to perspective time (Kamp & Reyle, 1993) suggest including S (alongside R) as a subscript on $\mathrm{TENSE}^{\mathrm{r}}_{\mathrm{R}}$.

(21) $t \models_{\mathfrak{A},g} \mathrm{WOLL}(\varphi) \iff (\exists t' \succ t)\ t' \models_{\mathfrak{A},g} \varphi$
$\iff \mathrm{ap}_{\succ}((\mathfrak{A},g)[\varphi])(t)$

As a modal auxiliary alongside *can* and *must*, WOLL sits below tense, and is pronounced *would* under the scope of $\textsc{Tense}^{\prec}_{\mathrm{R}}$ (i.e., past) and *will* under the scope of $\textsc{Tense}^{=}_{\mathrm{R}}$ (i.e., present).

(22) $\textsc{Tense}^{\mathrm{r}}_{\mathrm{R}}(\mathrm{WOLL}(\varphi))$ is $(\mathfrak{A},g)$-true $\iff g(\mathrm{R})\ \mathrm{r}\ g(\mathrm{S})$ and
$(\exists t \succ g(\mathrm{R}))\ t \models_{\mathfrak{A},g} \varphi$

But does the argument against treating the past as a modal operator $\mathrm{ap}_{\prec}$ not carry over to *will* and $\mathrm{ap}_{\succ}$? Consider the temporal anaphora in (23).

(23) a. Adam left. Eve starved.

b. Adam will leave. Eve will starve.

It is not clear that the pressure to temporally relate Adam's departure to Eve's starvation diminishes from (23a) to (23b).

Discourse considerations aside, there is a strong compositional pull to align semantic and syntactic accounts of phrases within a single sentence, using crosslinguistic morphosyntactic evidence. A challenge that has attracted wide attention is posed by the different types of perfect, including the resultative (2a), the existential (13a), and the universal (13b).

(2) a. Adam has left the garden.

(13) a. Adam has been outside the garden.

b. Adam has been sitting in the garden all afternoon.

Event structure from the verbal predicate has been implicated in the different readings (e.g., Kiparsky, 2002; Iatridou *et al.*, 2001); the universal requires a stative (as well as an adverbial), while the resultative requires a change in state. An attempt to derive the different readings of the perfect as different mappings of the event structure to the parameters E and R is made in Kiparsky (2002), assuming the Reichenbachian configuration $\mathrm{E} \prec \mathrm{R}$. An alternative considered in Iatridou *et al.* (2001) trades $\prec$ away for the Extended Now relation *xn* in (24a), applied in (24b) to the parameter V in (9).[6]

(24) a. $I\ xn\ J \iff J$ is a final subinterval of I
(i.e., I is J extended back/to the left)

b. perfect (XN): V *xn* R

(9) a. imperfective: $\mathrm{E} \sqsupset \mathrm{V}$ (V inside E)

b. perfective: $\mathrm{E} \sqsubset \mathrm{V}$ (E inside V)

[6] Writing R for the Perfect Time Span in (24b) preserves Reichenbach's conception of tense as a relation between R and S.

(24b) combines with (9a) so that E $\sqsupset$ R, as desired for (13b). Together with (9b), (24b) puts E sometime before or during R, for (13a). (9) and (24) nicely illustrate (14a, 14b).

(14) a. add temporal parameters (e.g., R, V)

b. expand times from points to intervals

c. bring out the events and states timed by E and R

An instance of (14c) is the assumption (25) that the set $\mathfrak{A}[\varphi]$ of times t such that $t \models_{\mathfrak{A}} \varphi$ are the temporal traces $time(e)$ of events e from some set $\varphi^{\mathfrak{A}}$. $\mathfrak{A}$ is henceforth understood to include any required contextual function g within it, allowing us to simplify $(\mathfrak{A}, g)$ to $\mathfrak{A}$.

(25) $\mathfrak{A}[\varphi] \;=\; \{time(e) \mid e \in \varphi^{\mathfrak{A}}\}$

Treating the function *time* in (25) as a binary relation, observe that by (15a),

$$\mathrm{ap}_{time}(\varphi^{\mathfrak{A}})(t) \iff t \models_{\mathfrak{A}} \varphi$$

and we can link a reference time R to some event in $\varphi^{\mathfrak{A}}$ through a sequence (26) of modal operators, at the cost of quantifying away V, E and e.

(26) $\mathrm{ap}_{xn}(\mathrm{ap}_{\mathrm{r}}(\mathrm{ap}_{time}(\varphi^{\mathfrak{A}})))(\mathrm{R}) \iff$
$(\exists \mathrm{V}\ xn\ \mathrm{R})(\exists \mathrm{E}\ \mathrm{r}\ \mathrm{V})(\exists e \in \varphi^{\mathfrak{A}})\ time(e) = \mathrm{E}$

The resultative reading (e.g. for (2a)) does not quite fit the scheme (26), requiring that φ and $\mathfrak{A}$ supply a set $\mathrm{Res}_{\varphi}^{\mathfrak{A}}$ of pairs $\langle e, s\rangle$ of events e and (consequent) states s that induce a set $\mathrm{Res}(\varphi)^{\mathfrak{A}}$ of times according to (27a), fed to the modification (27b) of (26).

(27) a. $\mathrm{Res}(\varphi)^{\mathfrak{A}}(t) \iff (\exists \langle e, s\rangle \in \mathrm{Res}_{\varphi}^{\mathfrak{A}})\ time(s) = t$
I.e., $\mathrm{Res}(\varphi)^{\mathfrak{A}}(t)$ says: $t = time(s)$ for some (e, s) in $\mathrm{Res}_{\varphi}^{\mathfrak{A}}$

b. $\mathrm{ap}_{xn}(\mathrm{ap}_{\sqsupset}(\mathrm{Res}(\varphi)^{\mathfrak{A}}))(\mathrm{R}) \iff$
$(\exists \mathrm{V}\ xn\ \mathrm{R})(\exists \langle e, s\rangle \in \mathrm{Res}_{\varphi}^{\mathfrak{A}})\ time(s) \sqsupset \mathrm{V}$

A wrinkle on the augmented extended-now account of the perfect (Iatridou *et al.*, 2001; Pancheva, 2003), the appeal to pairs $\langle e, s\rangle$ in $\mathrm{Res}_{\varphi}^{\mathfrak{A}}$ is the decisive feature of the perfect under a consequent-state approach (Moens & Steedman, 1988; Kamp & Reyle, 1993; Pulman, 1997). The consequent-state approach explains deviations from the resultative perfect pragmatically through type coercion based on aspectual classes, in contrast to the grammatical (viewpoint) orientation of (24), (9), (27). Under either approach, the extensions (14a–14c) take us far beyond the simple past of Prior. That said, we can implement (14a–14c) using little more than the ingredients of Priorean tense logic, as we will see below.

1.4 Fluents, segmentations, strings and automata

A basic ingredient of Priorean tense logic is a temporal proposition, or *fluent* (McCarthy & Hayes, 1969; van Lambalgen & Hamm, 2005) for short. A fluent can be used (as in Blackburn (1994)) to represent the temporal parameters mentioned in (14a). But rather than restricting the times $t \in T_{\mathfrak{A}}$ over which fluents are interpreted to points, we can take them to be intervals, in accordance with (14b). In particular, we can identify the name I of an interval $\mathrm{I}_{\mathfrak{A}}$ in $\mathfrak{A}$ with the fluent picking that interval out in $\mathfrak{A}$,[7] and weaken the fluent I to a fluent $\mathrm{I}_\circ$, pronounced I *segment*, true of subintervals of $\mathrm{I}_{\mathfrak{A}}$.

(28) a. $I \models_{\mathfrak{A}} \mathrm{I} \iff I = \mathrm{I}_{\mathfrak{A}}$

b. $I \models_{\mathfrak{A}} \mathrm{I}_\circ \iff I \subseteq \mathrm{I}_{\mathfrak{A}}$

We can then picture, for instance, the assertion $\mathrm{V} \sqsubset \mathrm{E}$ that V is inside E as a string

$$\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{V}}\boxed{\mathrm{E}_\circ}$$

segmenting E into 3 subintervals, the second of which is V (the first, the part of E before V; the third, the part of E after V). The idea, formally spelled out in section 2, is that a segmentation of an interval I is a finite sequence $I_1 I_2 \cdots I_n$ of intervals I_i partitioning I, and that the segmentation satisfies a string $\alpha_1\alpha_2 \cdots \alpha_n$ of sets α_i of fluents precisely if each fluent in α_i holds at I_i, for $1 \leq i \leq n$. With these strings, we can represent not just intervals but also the events and their kin mentioned in (14c), referred to as *situations* in Comrie (1976) and *eventualities* in Bach (1981). Event radicals in Galton (1987) and event nuclei in Moens & Steedman (1988) have natural formulations in terms of strings (Section 2.2, below). Further refinements are effected by introducing more and more fluents into the boxes. It will prove useful to analyze the refinements in reverse, de-segmenting by abstracting fluents away; for example, if we abstract V away, then the string

$$\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{V}}\boxed{\mathrm{E}_\circ}$$

(of length 3) projects to the string

$$\boxed{\mathrm{E}}$$

(of length 1), in which E is whole and unbroken, much like a perfective. These projections are systematized in section 3, yielding worlds via an inverse limit. Short of that limit, we consider various relations between strings in section 3, including mereological relations generalizing Carnap-Montague intensions, and accessibility relations (in the sense of Kripke semantics) between alternative possibilities. Inasmuch as these relations are computable by finite-state transducers, a string in these relations may be conceived as

[7] Recall from Section 1.3 (just before (25)) that we are assuming a model $\mathfrak{A}$ includes any necessary contextual information g. The interval $\mathrm{I}_{\mathfrak{A}}$ here is just $g(\mathrm{I})$.

a run of a program. Section 4 takes up ontological questions about such a conception, providing a curious twist on what Zucchi (1999) calls the *problem of indirect access*. A conceptual shift is suggested from a declarative semantics around truth to a procedural one around change.

As the technical details that follow may tax the most patient reader, some words of motivation are perhaps in order. The agenda behind this chapter is to present a finite-state approach to tense and aspect, the attraction of finite-state methods being that *less is more* (the simpler the better). Three inter-related hypotheses are put forward (hinting that the question of a finite-state implementation might be of interest also to theoreticians).

Ha Timelines can be segmented into strings representing situations.

Hb The relations between strings required by tense and aspect are computable by finite-state transducers.

Hc Change arises, up to bounded granularity, from finite automata.

These hypotheses are intended to be falsifiable. Indeed, finite automata are demonstrably inadequate for quantificational adverbials such as "as often as" (Kelleher & Vogel, 2013). The viability of finite-state methods for tense and aspect is, however, a different (if not altogether separate) question. In Klein & Li (2009), Wolfgang Klein more than once makes the point that many languages "have no categories as tense and aspect in their grammatical system" (page 1) and "in those languages which do have it, it is largely redundant" (page 43). Klein argues that "any real understanding of how the expression of time works requires a somewhat broader perspective" including "adverbials, inherent temporal features of the verb and discourse principles" (page 1), not unlike (one might add) DRT. Do finite-state methods carve out a subsystem of natural language temporality covering the tense and aspect of a language? This is vacuously the case for a language with*out* tense and aspect. But a language such as English poses a genuine challenge. The remainder of this chapter is organized around the notion of a timeline (as string) to make (Ha), (Hb) and (Hc), in turn, plausible and worthy of falsification (for any language with tense and/or aspect). Insights into tense and aspect from the literature seldom (if ever) come in finite-state terms; it would surely be impertinent and unnecessarily restrictive to insist that they should — which makes it all the more remarkable when they are shown to have finite-state formulations.

2 Within a timeline

Throughout this section, we fix in the background some set Φ of fluents and a model $\mathfrak{A}$ that specifies, amongst possibly other things, a linearly ordered set $(T_{\mathfrak{A}}, \prec)$ of time points, and a satisfaction relation $\models_{\mathfrak{A}}$ between intervals and fluents from Φ. Worlds are left out of this section, but will appear in the next. For the sake of brevity, we will often leave $\mathfrak{A}$ implicit when speaking of satisfaction or times, although we will try to keep the subscript $\mathfrak{A}$ on $\models_{\mathfrak{A}}$ and $T_{\mathfrak{A}}$ (but, somewhat inconsistently, not $\prec$). A commonly held view (shared by the avowedly Davidsonian Taylor (1977) and Montagovian Dowty (1979)) is that a fluent φ representing a stative is satisfied by an interval precisely if it is satisfied by every point in that interval — i.e., φ is pointwise according to the definition (29).

(29) φ is $\mathfrak{A}$-*pointwise* if for all intervals I,

$$I \models_{\mathfrak{A}} \varphi \iff (\forall t \in I)\ \{t\} \models_{\mathfrak{A}} \varphi.$$

Under the classical notion of negation $\neg$ given by

$$I \models_{\mathfrak{A}} \neg\varphi \iff \text{not } I \models_{\mathfrak{A}} \varphi,$$

the negation $\neg\varphi$ of a pointwise fluent φ may fail to be pointwise; that is, an interval I may satisfy $\neg\varphi$ even though for some point $t \in I$, $\{t\}$ satisfies φ. This complicates the task of tracking changes in a stative φ, on which we base our analysis of non-statives. We show how to overcome these complications in Section 2.1, before representing non-statives in Section 2.2 by strings $\alpha_1 \cdots \alpha_n$ of finite sets α_i of fluents. We look more closely at fluents in Section 2.3, be they pointwise or not. Along the way, we examine widely known parallels with the count/mass distinction (e.g., Mourelatos, 1978; Bach, 1986a), and the aspect hypothesis that

> "the different aspectual properties of the various kinds of verbs can be explained by postulating a single homogeneous class of predicates — *stative predicates* — plus three or four sentential operators and connectives" (Dowty, 1979, page 71).

At the heart of our account is a satisfaction relation between a segmentation of an interval and a string $\alpha_1 \cdots \alpha_n$ of sets of fluents, plagued by issues of homogeneity (Fernando, 2013a).

2.1 Homogeneity, segmentations and strings

Pointwise fluents (29) are often described as homogeneous (e.g., Dowty, 1979). Applying the description to an interval I rather than a fluent φ, we say I is φ-*homogeneous* if φ is satisfied by either all or none of the subintervals of I — i.e., some subinterval of I satisfies φ iff every subinterval of I does

$$(\exists J \sqsubseteq I) J \models_{\mathfrak{A}} \varphi \iff (\forall J \sqsubseteq I) J \models_{\mathfrak{A}} \varphi$$

where the subinterval relation $\sqsubseteq$ is the subset relation $\subseteq$ restricted to intervals. The intuition is that *no* surprises about φ are buried within a φ-homogeneous interval.[8] If an interval I fails to be φ-homogeneous, we can bring out all of φ's changes within I by segmenting I into φ-homogeneous subintervals. More precisely, let us lift $\prec$ to intervals I and J by universal quantification (30a) for *full* precedence, and define a sequence $I_1 I_2 \cdots I_n$ of intervals I_i to be a *segmentation of* an interval I, written $I_1 \cdots I_n \nearrow I$, if I is the union of all intervals I_i, each of which is related to the next by $\prec$, (30b).

(30) a. $I \prec J \iff (\forall t \in I)(\forall t' \in J)\ t \prec t'$

b. $I_1 \cdots I_n \nearrow I \iff I = \bigcup_{i=1}^{n} I_i$ and for $1 \leq i < n$, $I_i \prec I_{i+1}$

Next, we say a segmentation $I_1 \cdots I_n$ of I is *φ-homogeneous* if for every subinterval I' of I, I' satisfies φ precisely if I' is covered by components I_i that satisfy φ

$$I' \models_{\mathfrak{A}} \varphi \iff I' \subseteq \bigcup\{I_i \mid 1 \leq i \leq n \text{ and } I_i \models \varphi\}.$$

Observe that an interval is φ-homogeneous as a segmentation (with $n = 1$) iff it is φ-homogeneous as an interval. What's more, it is not difficult to see

Fact 1 *For any pointwise fluent φ, a segmentation $I_1 \cdots I_n$ of an interval I is φ-homogeneous iff each I_i is φ-homogeneous for $1 \leq i \leq n$.*

Fact 1 explains why φ-homogeneous intervals are interesting — because segmentations of I built from φ-homogeneous subintervals specify exactly which subintervals of I satisfy φ. But when can we segment an interval I into φ-homogeneous subintervals? An obvious necessary condition is that φ not alternate between true and false in I infinitely often. To be more precise, for any positive integer n, we define a *(φ, n)-alternation in I* to be a string $t_1 \cdots t_n \in I^n$ such that for $1 \leq i < n$, $t_i \prec t_{i+1}$ and

$$\{t_i\} \models_{\mathfrak{A}} \varphi \iff \{t_{i+1}\} \models_{\mathfrak{A}} \neg\varphi$$

(e.g. $\{t_1\} \models_{\mathfrak{A}} \varphi$, $\{t_2\} \not\models_{\mathfrak{A}} \varphi$, $\{t_3\} \models_{\mathfrak{A}} \varphi$, $\{t_4\} \not\models_{\mathfrak{A}} \varphi$, etc). An interval I is *φ-stable* if there is a positive integer n such that *no* (φ, n)-alternation in I exists. The obvious necessary condition is, in fact, sufficient.

Fact 2 *For any pointwise fluent φ, there is a φ-homogeneous segmentation of an interval I iff I is φ-stable.*

As we will be interested in tracking more than one stative at a time, we generalize the notion of a φ-homogeneous segmentation from a single fluent φ to a set X of fluents (pointwise or otherwise). A segmentation is *X-homogeneous* if it is φ-homogeneous for every $\varphi \in X$. Fact 1 readily extends to any set X of pointwise fluents:

[8] An interval satisfying a pointwise fluent φ is φ-homogeneous; the problem is an interval may not satisfy φ even though some subinterval of it does.

a segmentation $I_1 \cdots I_n$ of an interval I is X-homogeneous iff for all i from 1 to n and all $\varphi \in X$, I_i is φ-homogeneous.

Extending Fact 2 to a set X of pointwise fluents requires a bit more work and the assumption that X is finite.

Fact 3 *For any finite set X of pointwise fluents, there is a X-homogeneous segmentation of an interval I iff I is φ-stable for every $\varphi \in X$.*

Fact 3 demonstrably fails for infinite X. But we will make do with finite sets X of fluents, extending satisfaction $\models_{\mathfrak{A}}$ from intervals to segmentations $I_1 \cdots I_n$ to model-theoretically interpret strings $\alpha_1 \cdots \alpha_m$ of finite sets α_i of fluents according to (31).

(31) $$I_1 \cdots I_n \models_{\mathfrak{A}} \alpha_1 \cdots \alpha_m \iff n = m \text{ and for } 1 \leq i \leq n, \ (\forall \varphi \in \alpha_i)\ I_i \models_{\mathfrak{A}} \varphi$$

(31) says a segmentation $I_1 \cdots I_n$ satisfies a string $\alpha_1 \cdots \alpha_m$ precisely if they have the same length, and each set α_i consists only of fluents that I_i satisfies. We enclose the sets α_i in boxes, as we did with the string

$$\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{V}}\boxed{\mathrm{E}_\circ}$$

from Section 1.4, above, for which

$$I_1 \cdots I_n \models_{\mathfrak{A}} \boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{V}}\boxed{\mathrm{E}_\circ} \iff n = 3 \text{ and } I_2 = \mathrm{V}_{\mathfrak{A}} \text{ and } I_1 \cup I_2 \cup I_3 \subseteq \mathrm{E}_{\mathfrak{A}}$$

for any segmentation $I_1 \cdots I_n$, assuming (28) for I equal to E or V.

(28) a. $I \models_{\mathfrak{A}} \mathrm{I} \iff I = \mathrm{I}_{\mathfrak{A}}$

b. $I \models_{\mathfrak{A}} \mathrm{I}_\circ \iff I \subseteq \mathrm{I}_{\mathfrak{A}}$

Under (31), a string $\alpha_1 \cdots \alpha_m$ can be construed as a film/comic strip, model-theoretically interpreted against segmentations. (31) applies whether or not for each $\varphi \in \alpha_i$, the segmentation $I_1 \cdots I_n$ is φ-homogeneous, and whether or not φ is pointwise. The notions of a pointwise fluent φ and a φ-homogeneous interval depend on the underlying model $\mathfrak{A}$. In the case of

$$\boxed{\mathrm{E}_\circ}\boxed{\mathrm{E}_\circ,\mathrm{V}}\boxed{\mathrm{E}_\circ}$$

it follows from (28) that $\mathrm{E}_\circ$ is pointwise. V is another matter, although we can arrange it to be pointwise by assuming the interval $\mathrm{V}_{\mathfrak{A}}$ consists of a single point. Indeed, we can construe a string $\alpha_1\alpha_2 \cdots \alpha_n$ as a model $\mathfrak{A}$ over the set $\bigcup_{i=1}^{n} \alpha_i$ of fluents, with $T_{\mathfrak{A}} := \{1, 2, \ldots, n\}$ under the usual ordering $<$ (restricted to $\{1, 2, \ldots, n\}$), and for intervals I and $\varphi \in \bigcup_{i=1}^{n} \alpha_i$,

$$I \models_{\mathfrak{A}} \varphi \iff \varphi \in \bigcap_{i \in I} \alpha_i$$

provided this does not clash with conditions we impose on $\models$ — there is no clash in

$$\boxed{E_\circ}\boxed{E_\circ, V}\boxed{E_\circ}$$

with (28). But even then, there should be no confusing strings with models, especially as the real line $\mathbb{R}$ is a popular choice for $T_{\mathfrak{A}}$.

2.2 Durative and telic strings

A segmentation $I_1 \cdots I_n$ of the full set $T_{\mathfrak{A}}$ of time points is, for $n \in \{2,3\}$, called a *formal occurrence* in Galton (1987), where non-statives are called *event radicals*. An event radical ψ is interpreted there as a set $[\![\psi]\!]$ of formal occurrences $I_1 \cdots I_n$ such that I_1 is *before* an occurrence of ψ, and I_n *after* that occurrence. Given an event radical ψ, we can form stative propositions Prog(ψ), Perf(ψ) and Pros(ψ) such that for any interval I,

$$I \models \text{Prog}(\psi) \iff (\exists I_1 I_2 I_3 \in [\![\psi]\!])\ I \subseteq I_2$$

for the progressive of ψ,

$$I \models \text{Perf}(\psi) \iff (\exists I_1 \cdots I_n \in [\![\psi]\!])\ I \subseteq I_n$$

for the perfect of ψ, and

$$I \models \text{Pros}(\psi) \iff (\exists I_1 \cdots I_n \in [\![\psi]\!])\ I \subseteq I_1.$$

for the prospective of ψ. Under these definitions, a formal occurrence $I_1 I_2 I_3$ in $[\![\psi]\!]$ satisfies the string

$$\boxed{\text{Pros}(\psi)}\boxed{\text{Prog}(\psi)}\boxed{\text{Perf}(\psi)}$$

as does any segmentation $I I_2 I'$ with second component I_2. Similarly, for a formal occurrence $I_1 I_2$ in $[\![\psi]\!]$ and the string

$$\boxed{\text{Pros}(\psi)}\boxed{\text{Perf}(\psi)}.$$

Because a formal occurrence in $[\![\psi]\!]$ need not be unique, a fixed interval I may satisfy more than one of Pros(ψ), Prog(ψ) and Perf(ψ). In particular, (2a) comes out true even on Adam's return.

(2) a. Adam has left the garden.

This is problematic if (2a) is understood to mean Adam is still gone (with Adam-not-in-the garden as the consequent state of adam-leave-the-garden). We can sharpen our analysis by segmenting a smaller subinterval of the full set $T_{\mathfrak{A}}$ of times.

Apart from the interval we segment, there is also the matter of how finely we segment it (roughly, the number of component subintervals in the segmentation). Consider the notion that an event may be *punctual* — i.e., lacking in internal structure. This is captured in Galton (1987) by a formal occurrence $I_1 I_2$ with no intermediate interval between the before-set I_1 and after-set I_2 (developed further in Herweg (1991); Piñon (1997)). Comrie (1976) discusses the example of *cough*, noting that "the inherent punctuality of *cough* would restrict the range of interpretations that can be given to imperfective forms of this verb" to an iterative reading (of a series of coughs), as opposed to a single cough, which he refers to as *semelfactive*. Comrie concedes, however, that, in fact, one can imagine

> "a situation where someone is commenting on a slowed down film which incorporates someone's single cough, as for instance in an anatomy lecture: here, it would be quite appropriate for the lecturer to comment on the relevant part of the film *and now the subject is coughing*, even in referring to a single cough, since the single act of coughing has now been extended, and is clearly durative, in that the relevant film sequence lasts for a certain period of time" (Comrie, 1976, page 43).

The earlier contention that *coughing* can only be read iteratively suggests that the interval spanned by a single cough is too small for our "normal" segmentations to isolate. These segmentations consist of intervals too big to delineate "punctual" events. The special context provided above by an anatomy lecture produces a finer segmenting knife. The punctual-durative distinction evidently depends on context.

Part of that context is a set X of fluents available to describe the interior as well as immediate exterior of a situation. As Krifka notes, the telic-atelic distinction lies not "in the nature of the object described, but in the description applied to the object" as

> "one and the same event of running can be described by running (i.e. by an atelic predicate) or by running a mile (i.e. a telic, or delimited, predicate)" (Krifka, 1998, page 207).

Understood over a string $\alpha_1 \cdots \alpha_n$ of sets α_i of fluents, the terms durative and telic can be defined quite simply.

(32) a. $\alpha_1 \cdots \alpha_n$ is *durative* if its length n is at least 3

b. $\alpha_1 \cdots \alpha_n$ is *telic* if for some φ in α_n and all i such that $1 \leq i < n$, $\neg\varphi$ appears in α_i

Building on the analysis of durativity in Galton (1987), (32a) is based on the intuition that a string represents internal structure iff it has a box other than the first or last one (at the very least, a middle). (32b) says there is a fluent in the string's final box that distinguishes that box from the rest. The significance of (32a, 32b) rests on the classification (33) of situations from Moens & Steedman (1988); Smith (1991); Pulman (1997), among others.

(33) a. A semelfactive is non-durative and atelic (= non-telic)

b. An activity (= process) is durative but atelic

c. An achievement (= culmination) is non-durative but telic

d. An accomplishment (= culiminated process) is telic and durative

Left out of (33) are statives, which we have been representing not as strings but as pointwise fluents.

Let us be a bit more concrete about what the strings in (32) and (33) look like, starting with the set X of fluents that we can put into boxes. Recall that an event nucleus is made up of a culmination, with a preparatory process (activity) to the left, and a consequent state to the right (Moens & Steedman, 1988). Working from the string

$$\boxed{\text{Pros}(\psi)}\boxed{\text{Prog}(\psi)}\boxed{\text{Perf}(\psi)}$$

satisfied by a formal occurrence $I_1 I_2 I_3$ in the interpretation $[\![\psi]\!]$ of an event radical ψ (Galton, 1987), consider modifying the string to

$$\boxed{\text{pre}(\psi)}\boxed{\text{cul}(\psi)}\boxed{\text{csq}(\psi)}$$

for some preparatory process pre(ψ), culmination cul(ψ) and consequent state csq(ψ). This modification is too crude; while csq(ψ) is stative (as are Perf(ψ), Prog(ψ) and Pros(ψ)), neither a preparatory process nor a culmination is. To represent segmentations $I_1 I_2$ for punctual non-statives in Galton (1987), let us associate strings of length 2 with non-durative situations in (33a, 33c). Taking csq(ψ) to be φ in (32b), we associate a culmination (achievement) meeting (33c) and (32) with the string

$$\boxed{\neg\text{csq}(\psi)}\boxed{\text{csq}(\psi)}$$

rather than some fluent cul(ψ). For a non-durative semelfactive (33a), we adopt a Galton-like before-after representation

$$\boxed{\text{bef}_s(\psi)}\boxed{\text{aft}_s(\psi)}$$

for some pair of (before and after) fluents $\text{bef}_s(\psi)$ and $\text{aft}_s(\psi)$ (respectively) that differ from those of an achievement in that $\text{bef}_s(\psi)$ is not $\neg\text{aft}_s(\psi)$ (lest the semelfactive become telic). Indeed, an interval may satisfy both $\text{bef}_s(\psi)$ and $\text{aft}_s(\psi)$, allowing semelfactives to iterate for the set of strings

$$\boxed{\text{bef}_s(\psi)}\boxed{\text{bef}_s(\psi),\ \text{aft}_s(\psi)}^{+}\boxed{\text{aft}_s(\psi)}$$

representing an activity (e.g., Moens & Steedman, 1988; Rothstein, 2004). The idea is that $\text{bef}_s(\psi)$ expresses the exertion of a force, and $\text{aft}_s(s)$ the change resulting from that force. If ψ is mary-drink-water, for instance, $\text{bef}_s(\psi)$ might describe the drinking (as an action), and $\text{aft}_s(\psi)$ the consumption of some bit of water. We will have more to say about $\text{bef}_s(\psi)$ and $\text{aft}_s(\psi)$ when we take up forces and incremental change in section 4. For now, let us flesh (33) out with some sample strings.

(34) a. $\boxed{\text{bef}_s(\psi)}\boxed{\text{aft}_s(\psi)}$

b. $\boxed{\text{bef}_s(\psi)}\boxed{\text{bef}_s(\psi), \text{aft}_s(\psi)}^+\boxed{\text{aft}_s(\psi)}$

c. $\boxed{\neg\text{csq}(\psi)}\boxed{\text{csq}(\psi)}$

d. $\boxed{\text{bef}_s(\psi), \neg\text{csq}(\psi)}\boxed{\text{bef}_s(\psi), \text{aft}_s(\psi), \neg\text{csq}(\psi)}^+\boxed{\text{aft}_s(\psi), \text{csq}(\psi)}$

A semelfactive (34a) iterates to yield an activity (34b) that combines with an achievement (34c) for an accomplishment (34d).[9] All these strings can be refined further, as more fluents are brought into the picture. But before we do, we pause in the next section to consider two kinds of fluents (segmented and whole).

2.3 Segmented and whole fluents

The formal occurrences of Galton (1987) analyze non-statives ψ as perfectives, segmenting the full set $T_{\mathfrak{A}}$ of times into an interval before and an interval after the occurrence, but no further (leaving the middle, if it exists, whole) A segmentation $I_1 \cdots I_n$ of an interval I, as defined in (30b), may have any finite number n of subintervals, allowing us (for $n > 3$) to delve inside a non-stative and to break the perfective.

(30) b. $I_1 \cdots I_n \nearrow I \iff I = \bigcup_{i=1}^{n} I_i$ and for $1 \leq i < n$, $I_i \prec I_{i+1}$

In this subsection, we revisit the imperfective-perfective contrast (8) and develop the parallels

$$\frac{\text{imperfective}}{\text{perfective}} \approx \frac{\text{segmented}}{\text{whole}} \approx \frac{\text{mass}}{\text{count}}$$

(e.g., Mourelatos, 1978; Bach, 1986a). As a first step, we picture (8) as (9)′, with fluents E and V picking out the intervals for the event and view, respectively, and (28) holding for I equal to E or V.

(8) a. imperfective: ongoing, viewed from the inside, open-ended

b. perfective: completed, viewed from the outside, closed/bounded

(9) a. imperfective: $\boxed{\text{E}_\circ}\boxed{\text{E}_\circ,\text{V}}\boxed{\text{E}_\circ}$

[9] Notice that in (34d), $\neg\text{csq}(\psi)$ has been added to all non-final boxes of a string, not just the penultimate one. This is an instance of inertial flow, discussed in section 4.

b. perfective: | $V_\circ$ | $V_\circ$,E | $V_\circ$ |

(28) a. $I \models_{\mathfrak{A}} \mathrm{I} \iff I = \mathrm{I}_{\mathfrak{A}}$

b. $I \models_{\mathfrak{A}} \mathrm{I}_\circ \iff I \subseteq \mathrm{I}_{\mathfrak{A}}$

The contrast between the "segmented" fluents $E_\circ$ and $V_\circ$ and the "whole" fluents E and V is made precise by the definitions in (35).

(35) a. φ is $\mathfrak{A}$-*segmented* if for all intervals I and I' such that $I \cup I'$ is an interval,

$$I \models_A \varphi \text{ and } I' \models_A \varphi \iff I \cup I' \models_A \varphi$$

b. φ is $\mathfrak{A}$-*whole* if for all intervals I and I' such that $I \cup I'$ is an interval,

$$I \models_A \varphi \text{ and } I' \models_A \varphi \quad \text{implies} \quad I = I'$$

The direction $\Rightarrow$ in (35a) is illustrated in (36a), making $\mathfrak{A}$-segmented fluents additive (Bach, 1981); the converse, $\Leftarrow$, gives them the so-called subinterval property (Bennett & Partee, 1972) illustrated in (36b).

(36) a. Adam slept 3 to 5, Adam slept 4 to 6 $\vdash$ Adam slept 3 to 6

b. Adam slept from 3 to 6 $\vdash$ Adam slept from 3 to 5

$\mathfrak{A}$-pointwise fluents are $\mathfrak{A}$-segmented; $\mathfrak{A}$-segmented fluents need not be $\mathfrak{A}$-pointwise unless, for instance, $T_{\mathfrak{A}}$ is finite. Can we get $\mathfrak{A}$-segmented fluents by forming the *φ-segment*, $\varphi_\circ$, of an arbitrary fluent φ, with the understanding (37) that $\varphi_\circ$ holds exactly at subintervals of intervals where φ holds (generalizing (28b))?

(37) $I \models_{\mathfrak{A}} \varphi_\circ \iff (\exists I' \supseteq I)\ I' \models_{\mathfrak{A}} \varphi$

For any fluent φ, $\varphi_\circ$ satisfies the subinterval property, but not necessarily the other half of the equivalence in (35a) for $\mathfrak{A}$-segmented fluents. A sufficient condition for $\varphi_\circ$ to be $\mathfrak{A}$-segmented is that φ be $\mathfrak{A}$-whole. To relate the notion of an $\mathfrak{A}$-segmented fluent to a segmentation $\mathbb{I} = I_1 \cdots I_n$, it is useful to extend satisfaction $\models_{\mathfrak{A}}$ from strings $s = \alpha_1 \cdots \alpha_n$ to sets L of such strings (i.e., languages) disjunctively according to (38a), and then to define a fluent φ to be $\mathfrak{A}$-*segmeentable as* L when the satisfaction of φ at an interval I is equivalent to there being a segmentation of I that satisfies L, as well as every segmentation of L satisfying L, (38b).

(38) a. $\mathbb{I} \models_{\mathfrak{A}} L \iff (\exists s \in L)\ \mathbb{I} \models_{\mathfrak{A}} s$

b. φ is $\mathfrak{A}$-*segmentable as* L if for all intervals I,

$$\begin{aligned} I \models_{\mathfrak{A}} \varphi &\iff (\exists \mathbb{I} \nearrow I)\ \mathbb{I} \models_{\mathfrak{A}} L \\ &\iff (\forall \mathbb{I} \nearrow I)\ \mathbb{I} \models_{\mathfrak{A}} L \end{aligned}$$

Fact 4 *The following three conditions are equivalent.*

(i) φ *is* $\mathfrak{A}$*-segmented*

(ii) φ *is* $\mathfrak{A}$*-segmentable as* $\boxed{\varphi}^+$

(iii) φ *is* $\mathfrak{A}$*-segmentable as* $\boxed{\varphi_\circ}^+$

Fact 4 suggests that the map $\varphi \mapsto \varphi_\circ$ from $\mathfrak{A}$-whole to $\mathfrak{A}$-segmented fluents can be viewed as a grinder.

For a packager going the opposite direction, more definitions are in order. Given two intervals I and I', we say I *meets* I' and write $I \;\mathtt{m}\; I'$ if $I \prec I'$ and $I \cup I'$ is an interval (Allen & Ferguson, 1994).[10] Meet is implicit in the notion of a segmentation inasmuch as

$$I \;\mathtt{m}\; I' \iff II' \nearrow I \cup I'$$

and indeed for any $n \geq 2$,

$$I_1 \cdots I_n \nearrow \bigcup_{i=1}^{n} I_i \iff I_i \;\mathtt{m}\; I_{i+1} \text{ for } 1 \leq i < n.$$

Next, given a relation $\mathtt{r}$ between intervals, we form the fluent $\langle \mathtt{r} \rangle \varphi$ which an interval satisfies precisely if it is related by $\mathtt{r}$ to an interval satisfying φ, (39).

(39) $I \models_{\mathfrak{A}} \langle \mathtt{r} \rangle \varphi \iff (\exists I')\; I \;\mathtt{r}\; I' \text{ and } I' \models \varphi$

Note that $\varphi_\circ$ is just $\langle \subseteq \rangle \varphi$, and that $\langle \mathtt{m} \rangle \varphi$ is an existential interval form of the temporal formula Next(φ), and $\langle \mathtt{mi} \rangle \varphi$ is of Previous(φ) for $\mathtt{mi}$ the inverse of $\mathtt{m}$.

Fact 5 *The following three conditions are equivalent.*

(i) φ *is* $\mathfrak{A}$*-whole*

(ii) there is no segmentation $\mathbb{I}$ *such that* $\mathbb{I} \models_{\mathfrak{A}} \boxed{\varphi}\boxed{\varphi_\circ} + \boxed{\varphi_\circ}\boxed{\varphi}$

(iii) φ *is* $\mathfrak{A}$*-segmentable as*

$$\boxed{\varphi_\circ, \neg\langle \mathtt{m} \rangle \varphi_\circ, \neg\langle \mathtt{mi} \rangle \varphi_\circ} + \boxed{\varphi_\circ, \neg\langle \mathtt{mi} \rangle \varphi_\circ}\boxed{\varphi_\circ}^*\boxed{\varphi_\circ, \neg\langle \mathtt{m} \rangle \varphi_\circ}$$

Let us define fluents φ and φ' to be $\mathfrak{A}$-*equivalent*, $\varphi \equiv_{\mathfrak{A}} \varphi'$, if they satisfy exactly the same intervals,

$$\varphi \equiv_{\mathfrak{A}} \varphi' \iff (\forall \text{ interval } I)\; I \models_{\mathfrak{A}} \varphi \equiv \varphi'.$$

Combining the fluents in the first box in condition (iii) of Fact 5 by conjunction $\wedge$, we can add a fourth condition

[10] Meet is called *abutment on the left* in Hamblin (1971), and just *abutment* in Kamp & Reyle (1993).

(iv) $\varphi \ \equiv_{\mathfrak{A}} \ \varphi_\circ \wedge \neg\langle \mathtt{m} \rangle \varphi_\circ \wedge \neg\langle \mathtt{mi} \rangle \varphi_\circ$

to the list in Fact 5. The right hand side of (iv), $\varphi_\circ \wedge \neg\langle \mathtt{m} \rangle \varphi_\circ \wedge \neg\langle \mathtt{mi} \rangle \varphi_\circ$ is essentially the *pofective of* $\varphi_\circ$ (Galton, 1984, 1987), which we can reformulate as $\max(\varphi_\circ)$, where max is the operator defined in (40).

(40) $\max(\varphi) \ := \ \varphi \wedge \neg\langle \mathtt{m} \rangle \varphi \wedge \neg\langle \mathtt{mi} \rangle \varphi$

Given an $\mathfrak{A}$-segmented fluent φ, can we apply max and then $\cdot_\circ$ for (41)?

(41) $\varphi \equiv_{\mathfrak{A}} (\max(\varphi))_\circ$

If $T_{\mathfrak{A}}$ is finite, then we can. But if $T_{\mathfrak{A}}$ is say, the real line $\mathbb{R}$ and φ picks out bounded intervals

$$I \models_{\mathfrak{A}} \varphi \quad \iff \quad (\exists x, y \in \mathbb{R}) \ I \subseteq [x, y]$$

then $\max(\varphi)$ becomes $\mathfrak{A}$-unsatisfiable, and so does $(\max(\varphi))_\circ$. To rule out such pesky counter-examples to (41), we say φ is $\mathfrak{A}$-*chain-complete* if φ is $\mathfrak{A}$-satisfied by the union $\bigcup \mathcal{I}$ of every set $\mathcal{I}$ of intervals $\mathfrak{A}$-satisfying φ such that for all $I, I' \in \mathcal{I}$, $I \subseteq I'$ or $I' \subseteq I$. $\mathfrak{A}$-whole fluents are $\mathfrak{A}$-chain-complete (vacuously), as are all fluents, if $T_{\mathfrak{A}}$ is finite. For infinite $T_{\mathfrak{A}}$, the example of bounded intervals shows $\mathfrak{A}$-segmented fluents need not. Let us call an $\mathfrak{A}$-segmented fluent *chain-$\mathfrak{A}$-segmented* if it is also $\mathfrak{A}$-chain-complete. The equivalence (41) holds for chain-$\mathfrak{A}$-segmented fluents φ. For $\mathfrak{A}$-whole φ, $\varphi_\circ$ is chain-$\mathfrak{A}$-segmented. Moreover, the map $\varphi \mapsto \max(\varphi)$ from chain-$\mathfrak{A}$-segmented fluents to $\mathfrak{A}$-whole fluents is the lower (left) adjoint of the map $\varphi \mapsto \varphi_\circ$ from $\mathfrak{A}$-whole to chain-$\mathfrak{A}$-segmented fluents.[11]

Are the fluents $\mathrm{csq}(\psi)$, $\neg\mathrm{csq}(\psi)$, $\mathrm{bef}_s(\psi)$, $\mathrm{aft}_s(\psi)$ that appear in the strings in (34) $\mathfrak{A}$-segmented? Certainly, the stative fluent $\mathrm{csq}(\psi)$ is, assuming it is $\mathfrak{A}$-pointwise (being stative). But already $\neg\mathrm{csq}(\psi)$ is problematic, as $\mathfrak{A}$-segmented fluents are not closed under negation. To overcome this problem, it is useful to form the universal dual of the fluent $\langle \mathtt{r} \rangle \varphi$ in (39), where $\mathtt{r}$ is the inverse $\sqsupseteq$ of the subinterval relation $\sqsubseteq$.

(42) $[\sqsupseteq]\varphi \ := \ \neg\langle \sqsupseteq \rangle \neg\varphi$

Under (42) and (39), we have for any interval I and fluent φ,

$$I \models_{\mathfrak{A}} [\sqsupseteq]\varphi \quad \iff \quad \text{for every subinterval } I' \text{ of } I, \ I' \models_{\mathfrak{A}} \varphi.$$

Applying $[\sqsupseteq]$ to $\neg\varphi$ yields a negation

$$\mathrm{neg}(\varphi) \quad := \quad [\sqsupseteq]\neg\varphi$$

called *predicate negation* in Hamblin (1971) and *strong negation* in Allen & Ferguson (1994). It is easy to see that if φ is $\mathfrak{A}$-segmented, so is $\mathrm{neg}(\varphi)$. We

[11] The assumption of $\mathfrak{A}$-chain-completeness was mistakenly left out of the discussion in Fernando (2013b) of the adjunction between max and $\cdot_\circ$ (Section 2.1).

can apply the prefix $[\sqsupseteq]$ not only to $\neg\mathrm{csq}(\psi)$, but as we will see in section 4, also to $\mathrm{bef}_s(\psi)$ and $\mathrm{aft}_s(\psi)$ for $\mathfrak{A}$-segmented fluents. Henceforth, we assume that in the descriptions (32b) and (34c, 34d) of telicity, $\neg\varphi$ is $\mathrm{neg}(\varphi)$.

Next, we step from the fluents inside strings in (34) to the strings themselves. Given a set L of strings of sets of fluents, let us collect all intervals that have segmentations $\mathfrak{A}$-satisfying L in the set

$$L_{\mathfrak{A}} \; := \; \{ I \mid (\exists \mathbb{I} \nearrow I)\ \mathbb{I} \models_{\mathfrak{A}} L \}.$$

We can then ask if

(Q1) $L_{\mathfrak{A}}$ is segmented in the sense that for all intervals I and I' such that $I \cup I'$ is an interval,

$$I \in L_{\mathfrak{A}} \text{ and } I' \in L_{\mathfrak{A}} \quad \Longleftrightarrow \quad I \cup I' \in L_{\mathfrak{A}}$$

or if

(Q2) $L_{\mathfrak{A}}$ is whole in the sense that for all intervals I and I' such that $I \cup I'$ is an interval,

$$I \in L_{\mathfrak{A}} \text{ and } I' \in L_{\mathfrak{A}} \;\text{ implies }\; I \cup I' \in L_{\mathfrak{A}}.$$

Given what little we have said so far about $\mathrm{bef}_s(\psi)$ and $\mathrm{aft}_s(\psi)$, we are only in a position to answer these questions for the strings in (34c, 34d) involving $\mathrm{csq}(\psi)$ and $\neg\mathrm{csq}(\psi)$.

(34) c. $\boxed{\neg\mathrm{csq}(\psi)}\boxed{\mathrm{csq}(\psi)}$

d. $\boxed{\mathrm{bef}_s(\psi), \neg\mathrm{csq}(\psi)}\boxed{\mathrm{bef}_s(\psi),\, \mathrm{aft}_s(\psi), \neg\mathrm{csq}(\psi)}^{+}\boxed{\mathrm{aft}_s(\psi), \mathrm{csq}(\psi)}$

As telicity is incompatible with the subinterval property, it should not be surprising that the answer to (Q1) for L given by (34c) or (34d) is no. It turns out the answer to (Q2) is no different. In fact, we can say more. Let us call L $\mathfrak{A}$-*quantized* if it is *not* the case that there are distinct intervals I and $I' \in L_{\mathfrak{A}}$ such that $I \subset I'$. (This is the notion of quantized in Krifka (1998), with parts as subintervals.) Note that if $L_{\mathfrak{A}}$ is whole in the sense of (Q2), then L is $\mathfrak{A}$-quantized. Neither (34c) nor (34d) is $\mathfrak{A}$-quantized. Consider, for instance, a run to the post-office; the second half of any run to the post-office is also a run to the post-office. The trouble is that the notion of quantized is not "sensitive to the arrow of time" (Landman & Rothstein, 2012, page 97); the part relation $\subset$ carries no sense of temporal direction. The strings in (34) do. The main concern of Landman & Rothstein (2012) is a notion of incremental homogeneity partially related to the question (Q1) for (34a, 34b).

(34) a. $\boxed{\mathrm{bef}_s(\psi)}\boxed{\mathrm{aft}_s(\psi)}$

b. $\boxed{\mathrm{bef}_s(\psi)}\boxed{\mathrm{bef}_s(\psi),\, \mathrm{aft}_s(\psi)}^{+}\boxed{\mathrm{aft}_s(\psi)}$

Anticipating the discussion in section 4 of (34a, 34b), suffice it to say the languages L in (34) describe sets $L_{\mathfrak{A}}$ of intervals that are neither whole nor segmented. Rather, the languages pick out parts of intervals that can be segmented to track the changes described. The existential quantifier $\exists$ on segmentations defining $L_{\mathfrak{A}}$ above contrasts strikingly with $\forall$ and $\exists$ behind $\mathfrak{A}$-segmentability in Facts 4 and 5 (characterizing $\mathfrak{A}$-segmented and $\mathfrak{A}$-whole fluents). The map $\varphi \mapsto \varphi_\circ$ from whole to segmented fluents is comparable to the "in progress" predicate modifier IP of Szabo (2008), but reveals in $\varphi_\circ$ very little about internal structure, describing an undifferentiated (homogeneous?) mass that says nothing about progress (incremental or otherwise). Suggestive as the parallel

$$\frac{\text{imperfective}}{\text{perfective}} \approx \frac{\text{mass}}{\text{count}}$$

might be of applications to aspectual composition (e.g., Verkuyl, 2005), it is clear from examples such as runs to the post-office, and the interest in paths and degrees (e.g., Jackendoff, 1996; Krifka, 1998; van Lambalgen & Hamm, 2005; Kennedy & McNally, 2005) that we need more information than can be expected from $\langle\subseteq\rangle\varphi$, known above as $\varphi_\circ$.

3 Between timelines

If the previous section revolves around strings $\alpha_1 \cdots \alpha_n$ of finite sets α_i of fluents model-theoretically interpreted relative to segmentations of intervals, the present section centers around relations between these strings (computed by finite-state transducers). The importance of such relations is hinted in the following paragraph.

> "The expression of time in natural languages relates a *clause-internal temporal structure* to a *clause-external temporal structure*. The latter may shrink to a single interval, for example, the time at which the sentence is uttered; but this is just a special case. The clause-internal temporal structure may also be very simple – it may be reduced to a single interval without any further differentiation, the 'time of the situation'; but if this ever happens, it is only a borderline case. As a rule, the clause-internal structure is much more complex" (Klein & Li, 2009, page 75).

The simplest case described by the passage is illustrated by the picture

$$\boxed{\text{E}}\boxed{\text{S}} + \boxed{\text{E}}\boxed{}\boxed{\text{S}}$$

of the clause-internal event (or situation) time E preceding the clause-external speech (utterance) time S for the simple past. Elaborating on the event timed by E, we can replace

$$\boxed{\mathrm{E}}$$

by any of the strings in the language (34d) for an accomplishment ψ (Section 2.2).

(34) d. $\boxed{\mathrm{bef}_s(\psi), \neg\mathrm{csq}(\psi)}\boxed{\mathrm{bef}_s(\psi), \mathrm{aft}_s(\psi), \neg\mathrm{csq}(\psi)}^{+}\boxed{\mathrm{aft}_s(\psi), \mathrm{csq}(\psi)}$

From the model-theoretic interpretation of strings, there is a sense in which we can reduce (34d) to the single string

$$\boxed{\mathrm{bef}_s(\psi), \neg\mathrm{csq}(\psi)}\boxed{\mathrm{bef}_s(\psi), \mathrm{aft}_s(\psi), \neg\mathrm{csq}(\psi)}\boxed{\mathrm{aft}_s(\psi), \mathrm{csq}(\psi)}$$

of length 3, which we systematize in Section 3.1. An important contextual parameter that we shall vary is a finite set X of fluents (under consideration) fixing a level of granularity; strings get longer as X is enlarged, and shorter as X is reduced. For example, the Reichenbachian account of tense can be based on $X := \{\mathrm{R,S}\}$, and the Reichenbachian account of aspect on $X := \{\mathrm{R,E}\}$. For any set Φ of fluents (infinite or otherwise), we can let X vary over the finite subsets of Φ to construct worlds via an inverse limit, outlined in Section 3.2, with branching time. Carnap-Montague intensions generalize to relations between strings representing indices and denotations alike, and notions of containment between strings designed in Sections 3.3, 3.4 to express constraints.

3.1 Desegmenting by block compression

A 12-month calendar from January to December can be represented as a string

$$s_{mo} \ := \ \boxed{\mathsf{Jan}}\boxed{\mathsf{Feb}}\boxed{\mathsf{Mar}}\cdots\boxed{\mathsf{Dec}}$$

of length 12, or were we interested also in days, a string

$$s_{mo,dy} \ := \ \boxed{\mathsf{Jan,d1}}\boxed{\mathsf{Jan,d2}}\cdots\boxed{\mathsf{Jan,d31}}\boxed{\mathsf{Feb,d1}}\cdots\boxed{\mathsf{Dec,d31}}$$

of length 365 (for a non-leap year). In contrast to the points in the real line $\mathbb{R}$, a box can split, as $\boxed{\mathsf{Jan}}$ in s_{mo} does (30 times) to

$$\boxed{\mathsf{Jan,d1}}\boxed{\mathsf{Jan,d2}}\cdots\boxed{\mathsf{Jan,d31}}$$

in $s_{mo,dy}$, on introducing days $\mathsf{d1}$, $\mathsf{d2}, \ldots,$ $\mathsf{d31}$. Reversing direction and generalizing from

$$mo \ := \ \{\mathsf{Jan,Feb,\ldots Dec}\}$$

to any set X, we define the function ρ_X on strings (of sets) to componentwise intersect with X

$$\rho_X(\alpha_1 \cdots \alpha_n) \;:=\; (\alpha_1 \cap X) \cdots (\alpha_n \cap X)$$

(throwing out non-X's from each box) so that

$$\rho_{mo}(s_{mo,dy}) \;=\; \boxed{\text{Jan}}^{31}\boxed{\text{Feb}}^{28} \cdots \boxed{\text{Dec}}^{31}.$$

Next, the *block compression* $bc(s)$ *of* a string s compresses all repeating blocks α^n (for $n \geq 1$) of a box α in a string s to α for

$$bc(s) \;:=\; \begin{cases} bc(\alpha s') & \text{if } s = \alpha\alpha s' \\ \alpha \; bc(\beta s') & \text{if } s = \alpha\beta s' \text{ with } \alpha \neq \beta \\ s & \text{otherwise} \end{cases}$$

so that if $bc(s) = \alpha_1 \cdots \alpha_n$ then $\alpha_i \neq \alpha_{i+1}$ for i from 1 to $n-1$. In particular,

$$bc(\boxed{\text{Jan}}^{31}\boxed{\text{Feb}}^{28} \cdots \boxed{\text{Dec}}^{31}) \;=\; s_{mo}.$$

Let bc_X be the function mapping s to $bc(\rho_X(s))$. For example,

$$bc_{mo}(s_{mo,dy}) \;=\; s_{mo}.$$

The motto behind the maps bc_X is

as simple as possible and as complicated as necessary.

While bc simplifies a string by compressing it, enlarging X can lead to a longer, more complicated string.

The functions bc_X provide a handle on the X-homogeneous segmentations defined in section 2 which track changes in X. Let the *X-diagram* $\Delta_X(I)$ *of* an interval I be the set of fluents in X that I satisfies

$$\Delta_X(I) \;\;:=\;\; \{\varphi \in X \mid I \models_{\mathfrak{A}} \varphi\}$$

and the *X-diagram* $\Delta_X(\mathbb{I})$ *of* a segmentation $\mathbb{I} = I_1 \cdots I_n$ be the string

$$\Delta_X(I_1 \cdots I_n) \;\;:=\;\; \Delta_X(I_1) \cdots \Delta_X(I_n)$$

of X-diagrams of I_i for i from 1 to n. An X-diagram $\Delta_X(\mathbb{I})$ is more correctly an $(X, \mathfrak{A})$-diagram $\Delta_{X,\mathfrak{A}}(\mathbb{I})$; we suppress $\mathfrak{A}$ for simplicity.

Fact 6 *Let X be a finite set of $\mathfrak{A}$-segmented fluents φ and I be an interval such that for every $\varphi \in X$, there is a φ-homogeneous segmentation of I. Then there is a unique segmentation $\mathbb{I}_{X,I}$ of I that is X-homogeneous such that for every X-homogeneous segmentation $\mathbb{I}$ of I,*

$$\Delta_X(\mathbb{I}_{X,I}) \;=\; bc(\Delta_X(\mathbb{I})).$$

Moreover, for all $X' \subseteq X$,

$$\Delta_{X'}(\mathbb{I}_{X',I}) \;=\; \mathit{bc}_{X'}(\Delta_X(\mathbb{I}_{X,I})).$$

Let us henceforth refer to the segmentation $\mathbb{I}_{X,I}$ as the *X-segmentation of I*. Observe that for a chain-complete $\mathfrak{A}$-segmented fluent φ, there is a φ-homogeneous segmentation of I exactly if the set

$$\{I \cap I' \mid I' \models_{\mathfrak{A}} \max(\varphi)\}$$

of intersections of I with intervals satisfying $\max(\varphi)$ is finite, where $\max(\varphi)$ is the $\mathfrak{A}$-whole fluent (40) from Section 2.3.

(40) $\max(\varphi) \;:=\; \varphi \wedge \neg\langle \mathtt{m}\rangle\varphi \wedge \neg\langle \mathtt{mi}\rangle\varphi$

A concrete example of $\max(\varphi)$ is the fluent in (28a), for φ equal to the $\mathfrak{A}$-segmented fluent $\mathrm{I}_\circ$ in (28b).

(28) a. $I \models_{\mathfrak{A}} \mathrm{I} \iff I = \mathrm{I}_{\mathfrak{A}}$

b. $I \models_{\mathfrak{A}} \mathrm{I}_\circ \iff I \subseteq \mathrm{I}_{\mathfrak{A}}$

It is instructive to analyze $\mathrm{I}_\circ$ in terms of bc_X and a function *unpad* on strings that strips off any initial or final empty boxes

$$\mathit{unpad}(s) \;=\; \begin{cases} \mathit{unpad}(s') & \text{if } s = \square s' \text{ or else } s = s'\square \\ s & \text{otherwise} \end{cases}$$

so that $\mathit{unpad}(s)$ neither begins nor ends with $\square$. For example,

$$\mathit{unpad}(\mathit{bc}_X(s_{mo,dy})) \;=\; \begin{cases} \boxed{\mathsf{Feb}} & \text{if } X \text{ is } \{\mathsf{Feb}\} \\ \boxed{\mathsf{d3}}(\boxed{}\boxed{\mathsf{d3}})^{11} & \text{if } X \text{ is } \{\mathsf{d3}\}. \end{cases}$$

Given a string s, we define a fluent φ to be an *s-interval* if

$$\mathit{unpad}(\mathit{bc}_{\{\varphi\}}(s)) \;=\; \boxed{\varphi}.$$

Thus, Feb is an $s_{mo,dy}$-interval but d3 is not. Next, given a finite set X of fluents, let us collect strings s in which every $\varphi \in X$ is an s-interval, and apply bc_X and *unpad* to s for

$$\begin{aligned} \mathrm{Ivl}(X) \;:=\; \{&\mathit{unpad}(\mathit{bc}_X(s)) \mid s \in \mathit{Pow}(X)^+ \text{ and} \\ &(\forall\varphi \in X)\; \mathit{unpad}(\mathit{bc}_{\{\varphi\}}(s)) = \boxed{\varphi}\} \end{aligned}$$

(where the power set $\mathit{Pow}(X)$ of X is the set of all subsets of X). For two distinct fluents e and e', there are 13 strings in $\mathrm{Ivl}(\{e,e'\})$, one per Allen interval relation (e.g., Allen & Ferguson, 1994), refining the relations $\prec$ of full precedence and $\bigcirc$ of *overlap* used in the Russell-Wiener construction of time from events (e.g., Kamp & Reyle, 1993); see Table 1.

We have

Table 1. From Russell-Wiener to Allen

RW	Allen	$\mathtt{Ivl}(\{e,e'\})$	Allen	$\mathtt{Ivl}(\{e,e'\})$	Allen	$\mathtt{Ivl}(\{e,e'\})$
$e \bigcirc e'$	e = e'	$\fbox{$e,e'$}$	e fi e'	$\fbox{$e$}\fbox{$e,e'$}$	e f e'	$\fbox{$e'$}\fbox{$e,e'$}$
	e si e'	$\fbox{$e,e'$}\fbox{$e$}$	e di e'	$\fbox{$e$}\fbox{$e,e'$}\fbox{$e$}$	e oi e'	$\fbox{$e'$}\fbox{$e,e'$}\fbox{$e$}$
	e s e'	$\fbox{$e,e'$}\fbox{$e'$}$	e o e'	$\fbox{$e$}\fbox{$e,e'$}\fbox{$e'$}$	e d e'	$\fbox{$e'$}\fbox{$e,e'$}\fbox{$e'$}$
$e \prec e'$	e m e'	$\fbox{$e$}\fbox{$e'$}$	$e < e'$	$\fbox{$e$}\fbox{$$}\fbox{e'}$		
$e' \prec e$	e mi e'	$\fbox{$e'$}\fbox{$e$}$	$e > e'$	$\fbox{$e'$}\fbox{$$}\fbox{e}$		

$$\mathrm{Ivl}(\{e,e'\}) \;=\; \mathrm{Allen}(e \bigcirc e') + \mathrm{Allen}(e \prec e') + \mathrm{Allen}(e' \prec e)$$

where Allen$(e \bigcirc e')$ consists of the 9 strings in which e overlaps e'

$$\mathrm{Allen}(e \bigcirc e') \;:=\; (\fbox{e} + \fbox{e'} + \epsilon)\fbox{e,e'}(\fbox{e} + \fbox{e'} + \epsilon)$$

(with empty string ϵ), and Allen$(e \prec e')$ consists of the 2 strings in which e precedes e'

$$\mathrm{Allen}(e \prec e') \;:=\; \fbox{e}\fbox{e'} + \fbox{e}\fbox{}\fbox{e'}$$

and similarly for Allen$(e' \prec e)$. For an exact match between Ivl$(\{e,e'\})$ and Russell-Wiener, we need to add to $\{e,e'\}$ the fluents Prosp(x) and Perf(x) for $x \in \{e,e'\}$ so that, for instance,

$$\fbox{e}\fbox{}\fbox{e'}$$

becomes

$$\fbox{$e, \mathrm{Pros}(e')$}\fbox{$\mathrm{Perf}(e), \mathrm{Pros}(e')$}\fbox{$\mathrm{Perf}(e), e'$}$$

no two boxes in which are related by $\subset$ (as required by Russell-Wiener). With this adjustment, the Russell-Wiener notion of time based on events X coincides with Ivl(X), for any finite set X (not just pairs). For infinitely many events, an inverse limit construction is described next.

3.2 IL inverted and strung out

Given some large set Φ of fluents, let $Fin(\Phi)$ be the set of finite subsets of Φ. A function f with domain $Fin(\Phi)$ mapping $X \in Fin(\Phi)$ to a string $f(X)$ over the alphabet $Pow(X)$ of subsets of X is a (bc, Φ)-*system* if

$$f(X) \;=\; bc_X(f(X')) \;\text{ whenever } X \subseteq X' \in Fin(\Phi).$$

If I is an interval that has a φ-segmentation for all $\varphi \in \Phi$, then by Fact 6, the map $X \mapsto \Delta_X(\mathbb{I}_{X,I})$ with domain $Fin(\Phi)$ is a (bc, Φ)-system.

Let us write $\mathfrak{I}\mathfrak{L}_{bc}(\Phi)$ for the set of all (bc, Φ)-systems. "IL" here stands not for intensional logic (e.g. Montague, 1973) but for inverse limit — to be precise, the *inverse limit of* the restrictions of bc_X to $Pow(X')^*$ for $X \subseteq X' \in Fin(\Phi)$, all computable by finite-state transducers. That said, there is intensional variation in $\mathfrak{I}\mathfrak{L}_{bc}(\Phi)$ with a branching notion of time based on the prefix relation on strings s, s'

$$s \text{ prefix } s' \iff s' = s\hat{s} \text{ for some string } \hat{s}.$$

Let $\prec_\Phi$ be the binary relation on $\mathfrak{I}\mathfrak{L}_{bc}(\Phi)$ holding between distinct $f, f' \in \mathfrak{I}\mathfrak{L}_{bc}(\Phi)$ such that $f(X)$ is a prefix of $f'(X)$ for every $X \in Fin(\Phi)$

$$f \prec_\Phi f' \iff f \neq f' \text{ and } (\forall X \in Fin(\Phi))\ f(X) \text{ prefix } f'(X).$$

The intuition is that a temporal moment comes with its past, and that an $f \in \mathfrak{I}\mathfrak{L}_{bc}(\Phi)$ encodes the moment that is X-approximated, for each $X \in Fin(\Phi)$, by the last box in $f(X)$, with past given by the remainder of $f(X)$ (leading to that box). The relation $\prec_\Phi$ makes $\mathfrak{I}\mathfrak{L}_\pi(\Phi)$ tree-like in the sense of (e.g. Dowty, 1979, page 152).

Fact 7 *$\prec_\Phi$ is transitive and left linear: for every $f \in \mathfrak{I}\mathfrak{L}(\Phi)$, and all $f_1 \prec_\Phi f$ and $f_2 \prec_\Phi f$,*

$$f_1 \prec_\Phi f_2 \ \text{ or } \ f_2 \prec_\Phi f_1 \ \text{ or } \ f_1 = f_2.$$

Moreover, no element of $\mathfrak{I}\mathfrak{L}_\pi(\Phi)$ is $\prec_\Phi$-maximal: for any $f \in \mathfrak{I}\mathfrak{L}_\pi(\Phi)$, there is an $f' \in \mathfrak{I}\mathfrak{L}_\pi(\Phi)$ such that $f \prec_\Phi f'$.

Maximal chains, called *histories* in Dowty (1979), figure prominently in possible worlds semantics. While we can pick one out in $\mathfrak{I}\mathfrak{L}_{bc}(\Phi)$ to represent an actual history, it is far from obvious what significance maximal $\prec_\Phi$-chains have in the present framework, which is closer in spirit to situation semantics in the sense of Barwise & Perry (1983), updated in Cooper & Ginzburg (2015)[12].

The asymmetry in the notion of a prefix accounts for $\prec_\Phi$ branching forward as in historical necessity (e.g., Thomason, 1984), rather than backwards. We have been careful not to incorporate *unpad* into the projections shaping $\mathfrak{I}\mathfrak{L}_{bc}(\Phi)$, lest we forget the past. For a fixed temporal span, there is also the question of how much of Φ to consider. Given strings s and s' of sets, we say *s subsumes s'* and write $s \trianglerighteq s'$ if they have the same length and are related componentwise by inclusion.

(43) $\alpha_1 \cdots \alpha_n \trianglerighteq \alpha'_1 \cdots \alpha'_m \iff n = m \text{ and } \alpha_i \supseteq \alpha'_i \text{ for } 1 \leq i \leq n$

Subsumption $\trianglerighteq$ generalizes ρ_X (i.e., $\bigcup_{X \subseteq \Phi} \rho_X$ is a subset of $\trianglerighteq$) and holds, for instance, between the durative strings (34b) and (34d) of the same length

[12] Chapter 12 of this volume.

describing activities and accomplishments

$$\boxed{\mathrm{bef}_s(\psi), \neg\mathrm{csq}(\psi)}\boxed{\mathrm{bef}_s(\psi),\, \mathrm{aft}_s(\psi), \neg\mathrm{csq}(\psi)}\boxed{\mathrm{aft}_s(\psi), \mathrm{csq}(\psi)}$$
$$\trianglerighteq \boxed{\mathrm{bef}_s(\psi)}\boxed{\mathrm{bef}_s(\psi),\, \mathrm{aft}_s(\psi)}\boxed{\mathrm{aft}_s(\psi)}$$

We extend subsumption $\trianglerighteq$ to languages L (to the right) existentially

$$s \trianglerighteq L \iff (\exists s' \in L)\ s \trianglerighteq s'$$

just as we did with $\models_{\mathfrak{A}}$.

(38) a. $\mathbb{I} \models_{\mathfrak{A}} L \iff (\exists s \in L)\ \mathbb{I} \models_{\mathfrak{A}} s$

Some useful consequences are recorded in (44), where α is any subset of Φ, and L is any language over the alphabet $Pow(X)$.

(44) a. s is durative iff $s \trianglerighteq \boxed{\ }\boxed{\ }\boxed{\ }^{+}$

b. $s\alpha$ is telic iff $s \trianglerighteq \sum_{\varphi\in\alpha} \boxed{\neg\varphi}^{*}$

c. $\mathbb{I} \models_{\mathfrak{A}} L$ iff $\Delta_{X,\mathfrak{A}}(\mathbb{I}) \trianglerighteq L$

In (44c), we have attached $\mathfrak{A}$ as a subscript on the $(X, \mathfrak{A})$-diagram $\Delta_{X,\mathfrak{A}}(\mathbb{I})$ of $\mathbb{I}$, which we will presently vary. We can treat the model $\mathfrak{A}$ behind the notion $\models_{\mathfrak{A}}$ of satisfaction as a component of the index in a Carnap-Montague intension[13] CM_L of L mapping a pair $\mathfrak{A}, \mathbb{I}$ to one of two truth values, 0 or 1, with 1 just in case $\mathbb{I} \models_{\mathfrak{A}} L$

$$\mathrm{CM}_L(\mathfrak{A}, \mathbb{I}) \;=\; \begin{cases} 1 & \text{if } \mathbb{I} \models_{\mathfrak{A}} L \\ 0 & \text{otherwise.} \end{cases}$$

By (38a) and (44c),

$$\mathbb{I} \models_{\mathfrak{A}} L \iff (\exists d \in L)\ \Delta_{X,\mathfrak{A}}(\mathbb{I}) \;\trianglerighteq\; d$$

suggesting we can sharpen CM_L using the binary relation

$$\trianglerighteq_L \;:=\; \{(i, d) \mid i \trianglerighteq d \text{ and } d \in L\}$$

on strings, returning truth-witnesses or proofs d insofar as

$$\mathrm{CM}_L(\mathfrak{A}, \mathbb{I}) = 1 \iff (\exists d)\ \Delta_{X,\mathfrak{A}}(I) \trianglerighteq_L d.$$

Although $\trianglerighteq_L$ need not be a function (as it may return no output or may return several), we can encode it in a revised Carnap-Montague intension CM'_L with

[13] A *Carnap-Montague intension* of an expression γ is understood here to be a function CM_γ mapping an *index* $\mathbf{i}$ for evaluating γ to a *denotation* (or *extension* or *value*) $\mathrm{CM}_\gamma(\mathbf{i})$.

indices expanded to include d (following the tradition of many-dimensional modal logic)

$$\mathrm{CM}'_L(\mathfrak{A}, \mathbb{I}, d) \;=\; \begin{cases} 1 & \text{if } \Delta_{X,\mathfrak{A}}(\mathbb{I}) \trianglerighteq_L d \\ 0 & \text{otherwise.} \end{cases}$$

From a computational perspective, however, the output d of $\trianglerighteq_L$ is arguably more interesting (as Barwise & Perry (1983)'s *described situation*) than the truth value returned by CM'_L (or CM_L), and the pair $\mathfrak{A}, \mathbb{I}$ is only relevant up to the string $\Delta_{X,\mathfrak{A}}(\mathbb{I})$ it induces. Moreover, we can ask of $\trianglerighteq_L$, being a relation between strings, whether it is computable by a finite-state transducer (i.e. regular). As long as L is a regular language and the alphabet $Pow(X)$ of the input strings is finite, the answer is yes. Reflecting on the move made in section 2 from an interval I satisfying a fluent φ, $I \models_{\mathfrak{A}} \varphi$, to a segmentation $\mathbb{I}$ satisfying a set L of strings, $\mathbb{I} \models_{\mathfrak{A}} L$, we can say that (44c) takes a further step to a relation $\trianglerighteq$ between strings, conceived as indices (such as $\Delta_{X,\mathfrak{A}}(\mathbb{I})$) to the left of $\trianglerighteq$ and denotations to the right — such as the strings in

$$\boxed{\,}\boxed{\,}\boxed{\,}^{+}$$

from (44a). That said, it will become clear below (if it is not already) that there are problems with viewing subsumption $\trianglerighteq$ as *the* definitive relation between strings-as-indices and strings-as-denotations.

3.3 From subsumption to superposition

A binary operation on strings of the same length complementing subsumption $\trianglerighteq$ is *superposition* & obtained by componentwise union

$$\alpha_1 \cdots \alpha_n \;\&\; \alpha'_1 \cdots \alpha'_n \;\;:=\;\; (\alpha_1 \cup \alpha'_1) \cdots (\alpha_n \cup \alpha'_n).$$

For instance,

$$\boxed{\varphi}\boxed{\varphi}\boxed{\varphi} \;\&\; \boxed{\neg\psi}\boxed{\neg\psi}\boxed{\psi} = \boxed{\varphi, \neg\psi}\boxed{\varphi, \neg\psi}\boxed{\varphi, \psi}$$

and for strings s and s' of the same length,

$$s \trianglerighteq s' \iff s = s \,\&\, s'$$
$$s \,\&\, s' \;=\; \text{least } \trianglerighteq\text{-upper bound of } s \text{ and } s'.$$

It will be convenient to extend & to sets L and L' of strings (of possibly different lengths) by collecting superpositions of strings from L and L' of the same length

$$L \,\&\, L' \;=\; \{s \,\&\, s' \mid s \in L,\; s' \in L' \text{ and length}(s)\text{=length}(s')\}$$

(a regular language provided L and L' are (Fernando, 2004)). Notice that

$$\{s\} \mathbin{\&} \{s'\} \;=\; \{s \mathbin{\&} s'\} \quad \text{if length}(s)\text{= length}(s')$$

and the language $\text{dur}(L)$ defined in (45a) returns the set of strings in L that are durative.

(45) a. $\text{dur}(L) \;=\; L\& \fbox{}\fbox{}^{+}$

b. $\text{cul}(L,\varphi) \;=\; L\& \fbox{$\neg\varphi$}^{+}\fbox{φ}$

From (45b), we get a telic language $\text{cul}(L,\psi)$, including achievements (34c)

$$\text{cul}(\fbox{}, \text{csq}(\psi)) \;=\; \fbox{$\neg\text{csq}(\psi)$}\fbox{$\text{csq}(\psi)$}$$

and accomplishments (34d)

$$\text{cul}(\fbox{$\text{bef}_s(\psi)$}\fbox{$\text{bef}_s(\psi), \text{aft}_s(\psi)$}^{+}\fbox{$\text{aft}_s(\psi)$}, \text{csq}(\psi)) \;=$$
$$\fbox{$\text{bef}_s(\psi), \neg\text{csq}(\psi)$}\fbox{$\text{bef}_s(\psi), \text{aft}_s(\psi), \neg\text{csq}(\psi)$}^{+}\fbox{$\text{aft}_s(\psi), \text{csq}(\psi)$}$$

from (34b).

(34) b. $\fbox{$\text{bef}_s(\psi)$}\fbox{$\text{bef}_s(\psi), \text{aft}_s(\psi)$}^{+}\fbox{$\text{aft}_s(\psi)$}$

c. $\fbox{$\neg\text{csq}(\psi)$}\fbox{$\text{csq}(\psi)$}$

d. $\fbox{$\text{bef}_s(\psi), \neg\text{csq}(\psi)$}\fbox{$\text{bef}_s(\psi), \text{aft}_s(\psi), \neg\text{csq}(\psi)$}^{+}\fbox{$\text{aft}_s(\psi), \text{csq}(\psi)$}$

Next, we apply superposition & to temporal *for* and *in*-modification, (46), related to (non-)entailments of the progressive, (10).

(46) a. Adam walked for an hour.

b. Adam walked a mile in an hour.

(10) a. Adam was walking $\vdash$ Adam walked

b. Adam was walking a mile $\not\vdash$ Adam walked a mile

To interpret a duration D such as *one hour*, we construe D as a fluent true of intervals in a set $D_{\mathfrak{A}}$ with that duration

$$I \models_{\mathfrak{A}} D \iff I \in D_{\mathfrak{A}}.$$

We build a language $\mathcal{L}_x(D)$ for an interval named by x of duration D, treating the name x as a fluent picking out an interval $x_{\mathfrak{A}}$

$$I \models_{\mathfrak{A}} x \iff I = x_{\mathfrak{A}}$$

and building modal fluents (39)

(39) $I \models_{\mathfrak{A}} \langle \text{r} \rangle \varphi \iff (\exists I')\ I \text{ r } I' \text{ and } I' \models_{\mathfrak{A}} \varphi$

from the interval relations *il* and *fn* given by

$$I \; il \; I' \iff I \text{ is an initial subinterval of } I'$$
$$I \; fn \; I' \iff I \text{ is a final subinterval of } I'$$

(i.e., *fn* is the inverse of the extended now relation, (24)).[14] We mark an initial subinterval of $x_{\mathfrak{A}}$ by the fluent $x_i := \langle il \rangle x$ and a final subinterval of $x_{\mathfrak{A}}$, taken to be in $D_{\mathfrak{A}}$ by $D_x := \langle fn \rangle (x \wedge D)$. We can then segment the fluent $D \wedge x$ as the language

$$\mathcal{L}_x(D) \; := \; \boxed{x_i, D_x} + \boxed{x_i}\boxed{}^* \boxed{D_x}.$$

Next, to modify a language L (representing, for example, Adam's walk) by an interval x of duration D, we superpose $\mathcal{L}_x(D)$ with L, building in durativity and either iterativity or telicity as follows. We collect the fluents appearing in the last box of every string of L in

$$\omega(L) \; = \; \{\varphi \mid (\forall s \in L) \; s \trianglerighteq \boxed{}^* \boxed{\varphi}\}$$

(with $\omega(L) = \{\text{aft}_s(\psi)\}$ for ψ-activities in (34b), and $\{\text{aft}_s(\psi), \text{csq}(\psi)\}$ for ψ-accomplishments in (34d)) and adopt (47), with strings containing contradictory pairs $\varphi, \neg\varphi$ in the same box to be discarded (as unsatisfiable).

(47) a. $\mathsf{for}_x(L, D) \; = \; \text{dur}(L) \; \& \; \mathcal{L}_x(D) \; \& \; \boxed{}\omega(L)^+$

b. $\mathsf{in}_x(L, D) \; = \; \text{dur}(L) \; \& \; \mathcal{L}_x(D) \; \& \sum_{\varphi \in \omega(L)} \boxed{\neg\varphi}^+ \boxed{}$

3.4 Containment and constraints

A string s may have a subpart s' even if s does not $\trianglerighteq$-subsume s'. For instance, s' might be obtained from s by truncating either end of s — which is to say, s may have s' as a factor

$$s \; \mathsf{has\text{-}factor} \; s' \iff s = s_1 s' s_2 \text{ for some (possibly null) strings } s_1 \text{ and } s_2.$$

Combining has-factor with subsumption $\trianglerighteq$ leads to a more general subpart relation, which we shall refer to as *containment* $\sqsupseteq$

$$s \sqsupseteq s' \iff (\exists s'') \; s \text{ has-factor } s'' \text{ and } s'' \trianglerighteq s'.$$

By factoring in variations in temporal extent, containment $\sqsupseteq$ brings us closer than subsumption $\trianglerighteq$ to "the nicest theory" in Bach (1986b), featuring "possible histories" (indices) and "temporal manifestations" (denotations) that "pick out subparts of histories" (page 591). It is notable Bach should declare that

[14] In terms of Table 1 from Section 3.1 above, *il* is = or s, while *fn* is = or f.

> "it seems downright wrong to insist that everything that happens in a possible history, let alone separate possible histories, be mappable onto a single time line" (Bach, 1986b, page 587).

Certainly, "sequences of causally or otherwise contingently related sequences of events" (Moens & Steedman, 1988, page 26) are more clearly understood separate from (rather than indiscriminately lumped in with) independent sequences of such. If the strings above are to be traced to (runs of) finite automata, it makes sense to decompose an automaton into distinct components to the extent that it can. That is, we need not apologize that the inputs to our generalized Carnap-Montague intensions are strings that fall short of possible worlds. As for non-determinism, the analysis of action sentences as indefinite descriptions in Davidson (1967) is a well-tested classic (Parsons, 1990). And there is every reason computationally to process finite structures incrementally, feeding the outputs of one process as inputs to another process, thereby blurring the line between index (i.e. input) and denotation (i.e. output).

Part of that blurring is indeterminacy in temporal extent, which we will take up in the next section. With that in mind, we introduce a tool for expressing constraints on strings in $Pow(X)^*$, for any finite subset X of the full set Φ of fluents. Given languages $L, L' \subseteq Pow(X)^*$, let $L \Rightarrow L'$ be the set consisting of strings in $Pow(X)^*$ every factor of which subsumes L' if it subsumes L

$$L \Rightarrow L' \;:=\; \{s \in Pow(X)^* \mid (\text{for every factor } s' \text{ of } s) \text{ if } s' \unrhd L \text{ then } s' \unrhd L'\}.$$

For example, to say that once φ is true, it remains true, we form

$$\boxed{\varphi}\boxed{\;} \Rightarrow \boxed{\;}\boxed{\varphi} \;=\; \bigcup_{n \geq 0} \{\alpha_1 \cdots \alpha_n \in Pow(X)^n \mid \text{for } 1 \leq i < n, \text{ whenever } \varphi \in \alpha_i,\ \varphi \in \alpha_{i+1}\}.$$

To see that $L \Rightarrow L'$ is a regular language if L and L' are, note that for any relation R on strings computable by a finite-state transducer, the inverse image of L relative to R

$$\langle R \rangle L \;:=\; \{s \mid (\exists s' \in L)\ sRs'\}$$

is regular. As the counter-examples to $L \Rightarrow L'$ form the set

$$\langle \text{has-factor} \rangle (\langle \unrhd \rangle L \cap \overline{\langle \unrhd \rangle L'})$$

of strings with factors that subsume L but not L' (where the complement $\overline{L}$ is $Pow(X)^* - L$), complementing gives

$$L \Rightarrow L' \;=\; \overline{\langle \text{has-factor} \rangle (\langle \unrhd \rangle L \cap \overline{\langle \unrhd \rangle L'})}.$$

In the next section, we apply ⇒ to formulate inertial laws on statives (e.g., Comrie, 1976; Dowty, 1986; van Lambalgen & Hamm, 2005).

4 Behind timelines

Building on the dictum that "there could be no time if nothing changed" (traced in Prior, 1967, page 85, to J.M.E. McTaggart), we have assumed that change is manifested in a set Φ of fluents to reduce a timeline to a function f mapping a finite subset X of Φ to a string $f(X)$ that approximates the timeline up to granularity X (by recording changes in X). As X gets larger, more changes can be observed and the string $f(X)$ induced by X gets longer to record those changes. We draw this chapter to a close, showing how to enlarge X to (i) account for inertia associated with statives and (ii) record incremental change. The first point leads to notions of force behind timelines. The second takes us to degrees/grades and back to questions about homogeneity and indeterminacy of temporal extent. World-time pairs commonly taken for granted in the formal semantics of tense and aspect can, it is tempting to suggest, be put down to runs of many automata, only partially known, on different clocks, some cut short.

4.1 Inertial statives and force

Comrie (1976) observes that "unless something happens to change [a] state, then the state will continue" (page 49). Consider (48).

(48) Pat stopped the car before it hit the tree.

Unless something happens to change the state of the-car-at-rest after Pat stops it, we may assume the car continues to be at rest, preventing the car from hitting the tree (a precondition for which is the negation of the-car-at-rest). But what does it mean for "something happens to change the state of the-car-at-rest"? If all that means is the state of the-car-at-rest changes, then all we have said is: unless the state of the-car-at-rest changes, then the state of the-car-at-rest continues.

To avoid vacuity, let us recognize not only the-car-at-rest as a fluent, but also a fluent fφ saying "a force for φ occurs" so that the constraint (49a) saying "the-car-at-rest continues" can be modified to the constraint (49b) saying "the-car-at-rest continues or a force for the negation of the-car-at-rest has occurred."

(49) a. $\fbox{the-car-at-rest}\fbox{} \Rightarrow \fbox{}\fbox{the-car-at-rest}$

b. $\fbox{the-car-at-rest}\fbox{} \Rightarrow \fbox{}\fbox{the-car-at-rest} + \fbox{f$\neg$the-car-at-rest}\fbox{}$

(We assume + binds more tightly than ⇒.) In general, we can express Comrie's aforementioned observation about states φ as a constraint (50a) for φ persisting forward unless opposed, together with a constraint (50b) for φ persisting backward unless forced, and a "succeed unless opposed" constraint (50c) for fφ (Fernando, 2008).

(50) a. $\boxed{\varphi}\boxed{} \;\Rightarrow\; \boxed{}\boxed{\varphi} + \boxed{\mathsf{f}\neg\varphi}\boxed{}$

b. $\boxed{}\boxed{\varphi} \;\Rightarrow\; \boxed{\varphi}\boxed{} + \boxed{\mathsf{f}\varphi}\boxed{}$

c. $\boxed{\mathsf{f}\varphi}\boxed{} \;\Rightarrow\; \boxed{}\boxed{\varphi} + \boxed{\mathsf{f}\neg\varphi}\boxed{}$

An addendum to McTaggart's mantra 'no time without change' that can be extracted from (50) is: 'no change unless forced.' Lest we apply these constraints on all fluents, let us call fluents φ for which we impose (50) 'inertial.' These include fluents representing statives, but *not* fluents prefixed by f — henceforth called 'force fluents.' For inertial φ, the culimination $\mathrm{cul}(L,\varphi)$ in (45b) can be refined to $\mathrm{cul}_{\mathsf{f}}(L,\varphi)$ in (51), with the force fluent fφ inserted into the penultimate box.

(45) b. $\mathrm{cul}(L,\varphi) \;=\; L\& \;\boxed{\neg\varphi}^{+}\boxed{\varphi}$

(51) $\mathrm{cul}_{\mathsf{f}}(L,\varphi) \;=\; L\& \;\boxed{\sim\varphi}^{*}\boxed{\sim\varphi,\mathsf{f}\varphi}\boxed{\varphi}$

The adjustment (51) of (45b) illustrates a way to neutralize the constraints (50). Any change or non-change can be brought into compliance with (50) by positing some force responsible for it. In the case, for instance, of the string

$$\boxed{\text{the-car-at-rest}}\boxed{}$$

it suffices to introduce f¬the-car-at-rest and f(the-car-at-rest) to its first box for

$$\boxed{\text{the-car-at-rest, f}\neg\text{the-car-at-rest, f(the-car-at-rest)}}\boxed{}.$$

For (50) to have any bite, some restraint is required on admitting forces into a string. In particular, we cannot make the leap from

$$\boxed{\text{the-car-at-rest}}\boxed{} \text{ to } \boxed{\text{the-car-at-rest}}\boxed{\text{the-car-at-rest}}$$

on the basis of (50a) alone. For an inertial fluent to flow, we need a further principle banning the introduction of force fluents unless there is contextual support for them. This is how defeasibility arises from the otherwise strictly non-defeasible constraints (50).

To see how tricky inferences based on inertia can be, consider (52).

(52) a. Pat stopped the car. Chris restarted it.

b. In 1995, Amy was a toddler.

c. Adam has left the garden. He did so many years ago, before he reappeared in the garden this morning.

In (52a), we should be careful about inferring after the first sentence that the-car-at-rest holds at speech time. The second sentence (also in the past) describes a force that may overturn the consequent state of the first sentence. Under a Reichenbachian analysis of tense and aspect, the inertial constraints might be enforced during aspectual processing (before tense brings S in),[15] limiting the state the-car-at-rest to the reference time R of the first sentence. In effect, R introduces a force that acts as a barrier to inertial flow beyond it (Fernando, 2008). This same assumption accounts for blocking the inference in (52b) that at speech time, Amy is a toddler. The complication raised by (52c) is that the present tense of the first sentence (coupled with perfect aspect) suggests the consequent state ¬Adam-in-the-garden holds at speech time (= R for present tense). The second sentence in (52c) suggests that the perfect in the first sentence should be read existentially (as in Galton (1987)), much like

Adam has at some point in the past left the garden

in which case a force is added once the consequent state holds, blocking it from persisting forward to R=S. Herein, one might suggest, lies the force of the existential perfect.

The discussion above makes clear the importance of bounding the temporal span over which inertial calculations are made. Beyond a certain interval, worrying about what forces are or are not in play becomes more trouble than it is worth, and we may as well put (50) and force fluents aside. That said, Comrie (1976) has more to say, implicating forces.

4.2 Incremental change

Comrie writes

> "With a state, unless something happens to change that state, then the state will continue ... With a dynamic situation, on the other hand, the situation will only continue if it is continually subject to a new input of energy" (Comrie, 1976, page 49).

[15] Recall that Reichenbach's Reference time R breaks tense and aspect cleanly into two distinct processes: aspect positions an event with time E relative to R, while tense places the speech time S relative to R. Although the two processes need not be arranged in a pipeline, it has become common practice to proceed from the described event with time E (roughly the un-inflected verb phrase) to a larger situation, first adding R (via aspect) and then S (via tense), reversing the direction from a larger index to a smaller denotation in a Carnap-Montague intension. From the point of view of aspect, it is tempting to call the event with time E the described event or denotation (with R as part of the index); but from the point of view of tense, the denotation is arguably the situation marked by the reference time R (with S as part of the index). Proposals for additional temporal parameters such as V (for "higher aspect") introduce further processes intervening between indices and denotations (as conceptualized in a Carnap-Montague intension).

An example of a dynamic situation continuing for an hour is (53a) (Dowty, 1979).

(53) a. The soup cooled for an hour.

b. The soup cooled in an hour.

Before taking up (53a), let us consider (53b), a common intuition for which is that *in an hour* requires a culmination:

$$\boxed{\neg\mathrm{csq}(\psi)}\boxed{\mathrm{csq}(\psi)}$$

In this case, $\mathrm{csq}(\psi)$ is a fluent $sDg < d$ saying the soup temperature is below some threshold temperature d (supplied by context), interpreted homogeneously by a model $\mathfrak{A}$ so that

$$I \models_{\mathfrak{A}} sDg < d \iff (\forall t \in I)\ sdg_{\mathfrak{A}}(t) < d$$

for an interval $I \subseteq T_{\mathfrak{A}}$ with soup temperature $sdg_{\mathfrak{A}}(t)$ for $t \in I$. We let $d \leq sDg$ abbreviate $\neg\mathrm{csq}(\psi)$, interpreted as $[\sqsupseteq]\neg\mathrm{csq}(\psi)$ (as agreed in Section 2.3) so that

$$I \models_{\mathfrak{A}} d \leq sDg \iff (\forall t \in I)\ d \leq sdg_{\mathfrak{A}}(t)$$

assuming a soup temperature is defined at every $t \in I$. To describe an hour x that culminates with the soup temperature below d, we form the string (54).

(54) $\boxed{x_i, d \leq sDg}\boxed{d \leq sDg}\boxed{\mathrm{hour}_x, sDg < d}$

$= \boxed{d \leq sDg}\boxed{d \leq sDg}\boxed{sDg < d}\ \&\ \boxed{x_i}\boxed{\ }\boxed{\mathrm{hour}_x}$

While (54) is perhaps a passable string for (53b), the challenge of (53a) is that *for an hour* suggests a steady drop in temperature over that hour. We might track soup cooling by a descending sequence of degrees, $d_1 > d_2 > \cdots > d_n$, with d_1 at the beginning of the hour, and d_n at the end; but we can*not* assume a sample of finite size n is complete. Surely, continuous change here calls for the real line (van Lambalgen & Hamm, 2005)? But if we existentially quantify away the threshold temperature d above, we can use our “previous” modal operator $\langle\mathtt{mi}\rangle$ to express a drop in the soup temperature through the fluent

$$sDg_{\downarrow} \ := \ \exists x(sDg < x \wedge \langle\mathtt{mi}\rangle(x \leq sDg))$$

so that $I \models_{\mathfrak{A}} sDg_{\downarrow}$ iff for some d,

$$sdg_{\mathfrak{A}}(t) < d \text{ for all } t \in I \ \text{ and for some } I' \ \mathtt{m}\ I,\ d \leq sdg_{\mathfrak{A}}(t') \text{ for all } t' \in I'.$$

The condition that $sdg_{\mathfrak{A}}$ is decreasing over I

$$(\forall t, t' \in I)\ t \prec_{\mathfrak{A}} t' \text{ implies } sdg_{\mathfrak{A}}(t) > sdg_{\mathfrak{A}}(t')$$

follows if we prefix $sDg_\downarrow$ with $[\sqsupseteq]$. Superposing gives the string (55) for (53a).

(55) $\boxed{x_i}\boxed{[\sqsupseteq]sDg_\downarrow}\boxed{\text{hour}_x, [\sqsupseteq]sDg_\downarrow}$

$= \boxed{}\boxed{[\sqsupseteq]sDg_\downarrow}\boxed{[\sqsupseteq]sDg_\downarrow} \;\&\; \boxed{x_i}\boxed{}\boxed{\text{hour}_x}$

Next, let us compare (55) and (54) to our strings for semelfactives (34a), activities (34b), achievements (34c) and accomplishments (34d).

(34) a. $\boxed{\text{bef}_s(\psi)}\boxed{\text{aft}_s(\psi)}$

b. $\boxed{\text{bef}_s(\psi)}\boxed{\text{bef}_s(\psi), \text{aft}_s(\psi)}^+\boxed{\text{aft}_s(\psi)}$

c. $\boxed{\neg\text{csq}(\psi)}\boxed{\text{csq}(\psi)}$

d. $\boxed{\text{bef}_s(\psi), \neg\text{csq}(\psi)}\boxed{\text{bef}_s(\psi), \text{aft}_s(\psi), \neg\text{csq}(\psi)}^+\boxed{\text{aft}_s(\psi), \text{csq}(\psi)}$

We set $\text{aft}_s(\psi)$ to $sDg_\downarrow$ to express a fall in soup temperature, prefixing $sDg_\downarrow$ with $[\sqsupseteq]$ if we want the activity (34b) to be incrementally homogeneous. As for $\text{bef}_s(\psi)$, the passage from Comrie (1976) above suggests an "input of energy" or force (e.g. Talmy, 1988; Copley & Harley, 2012), leading to the "dynamic situation" $\text{aft}_s(\psi)$. To a first approximation, $\text{bef}_s(\psi)$ can be associated with the verb (e.g., *gulp*) describing manner, as opposed to the result (e.g., liquid consumed) encoded in $\text{aft}_s(\psi)$. It is noteworthy, however, that an intriguing "two-vector model of events including a force vector and a result vector" (Warglien *et al.*, 2012a) building on Gärdenfors (2000); Kiparsky (1997); Levin & Hovav (2013) has not gone unchallenged (Croft, 2012; Geuder, 2012; Kracht & Klein, 2012; Krifka, 2012; Wolff, 2012; Warglien *et al.*, 2012b). The syntax-semantics interface is a very delicate, thorny matter (Rappaport Hovav & Levin, 2015[16]).[17]

Be that as it may, let us generalize from soup cooling to some graded notion ψ that comes with degrees $\text{deg}(\psi)$. Let $\text{aft}_s(\psi)$ be the fluent

$$\psi_\uparrow \quad := \quad (\exists r)(\text{deg}(\psi) > r \wedge \langle\texttt{mi}\rangle(\text{deg}(\psi) \leq r))$$

spinning the drop into a rise, and $\text{bef}_s(\psi)$ be the force fluent $\mathsf{f}(\psi_\uparrow)$ for the set

$$\text{dur}_\uparrow(\psi) \quad = \quad \boxed{\mathsf{f}(\psi_\uparrow)}\boxed{\mathsf{f}(\psi_\uparrow), \psi_\uparrow}^+\boxed{\psi_\uparrow}$$

of strings expressing incremental (or, prefixing $\psi_\uparrow$ with $[\sqsupseteq]$, continuous) progress in ψ. This progress may culminate in $\text{csq}(\psi)$ once some threshold d is exceeded; i.e., $\text{csq}(\psi)$ is just $\text{deg}(\psi) > d$. Readers familar with van Lambalgen

[16] Chapter 19 of this volume.

[17] The finite-state hypotheses (Ha) – (Hc) outlined in Section 1.4 apply to semantics. Irregularity may well creep in from syntax.

& Hamm (2005) will notice a semblance of the Trajectory predicate deployed there to analyze continuous change. The essential difference is the restriction above to a finite set of fluents, subsets of which are strung out to approximate a timeline that need not be tied to the real line $\mathbb{R}$. The string approximations can, of course, be improved by adding more fluents, introducing names, for instance, of any finite number of degrees (among many other things). But the aim is to keep strings as simple as possible, whilst allowing for extensions to multi-sentence discourse with a network of states and events.

4.3 Temporal indeterminacy

The organization of the present chapter around timelines is implicit recognition of the importance of timelines to tense and aspect. How does this square with the proposal from Steedman (2005) that "the so-called temporal semantics of natural language is not primarily to do with time at all" (as given say, by the real line R), but rather that "the formal devices we need are those related to representation of causality and goal-directed action" (page ix)? Lurking not far from much of the discussions above are finite automata that are obvious candidates for such devices. If these automata have stayed largely in the dark, it is because the evidence for these comes largely from their runs in timelines. Zucchi describes a related problem in the truth-conditional semantics of tense and aspect:

> "in analyzing the meaning of temporal and aspectual features, we make assumptions about the truth conditions of uninflected clauses like 'Carnap fly to the moon', 'Terry build a house' and 'Terry be at home'. However, we have only indirect evidence of how these sentences are interpreted by native speakers, since they do not occur as independent clauses in English. I'll refer to the problem of determining the truth conditions of the base sentences that are the input to tense and aspect markers as the *problem of indirect access* in the semantics of tense and aspect." (Zucchi, 1999, page 180).

The problem of indirect access, as stated, presupposes base sentences have truth conditions. Even if some do, there is every chance that some do not, opening the problem up to the "Declarative Fallacy" (Belnap, 1990). Asking for an automaton's truth conditions does "have the feel of a category mistake" (to quote Carlson (1995) out of context). One asks not whether it is true or false, but what it does — or better, what it is designed to do. Conceptually prior to their runs, programs are commonly conceived and understood in splendid isolation, only to break down when executed alongside other programs running. If base sentences are programs, and fully inflected episodic sentences are runs, it is arguably premature to seek the truth conditions of base sentences.

Indirect access is an acute problem for programs that we can observe only through their runs, and only assuming we are right about which runs go with which programs. Nor can we pick out with the infinite precision of real numbers the temporal extent of statives and track their changes to delineate events completely. (Stepping back from models $\mathfrak{A}$ in which $T_{\mathfrak{A}}$ is the real line $\mathbb{R}$ to minimal strings is, it would seem, the feeblest acknowledgment of this limitation.) And even the atemporal is temporal; the causal structures at stake here are not the universal laws of physics, but everyday dispositions that may change over time. For all these reasons, strings of boxes, not transitions diagrams, have figured prominently above.[18]

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18 I regret that habituals and many other interesting topics in the semantics of tense and aspect have been left out of the present chapter. On a more positive note, I thank the editors of the handbook for their feedback and support.

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