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S.I.: THE LOGIC AND PHILOSOPHY OF A.N. PRIOR

Prior and temporal sequences for natural language

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Abstract Logics of discrete time are, in Arthur Prior's words, "applicable in limited fields of discourse in which we are concerned with what happens in a sequence of discrete states," independent of "any serious metaphysical assumption that time is discrete." This insight is applied to natural language semantics, a widespread assumption in which is that time is, as is the real line, dense. "Limited fields of discourse" are construed as finite sets of temporal propositions, inducing bounded notions of temporal granularity that can be refined to expand the discourse. The construal is developed in line with Prior's view of what is "metaphysically fundamental".

Keywords Prior · Temporal sequence · Natural language

1 Introduction

In a prescient defense of logics of discrete time, Arthur Prior writes

> The usefulness of systems of this sort does not depend on any serious metaphysical assumption that time is discrete; they are applicable in limited fields of discourse in which we are concerned with what happens in a sequence of discrete states, e.g. in the workings of a digital computer.

(Prior 1967, p. 67). Forming sequences based on computational steps has proved remarkably fruitful in computer science (e.g., Emerson 1995). In linguistic semantics, however, there is no obvious analog to a computational step, and tense logics, with or without the assumption of discrete time, have arguably met less success (since their

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adoption in Montague 1973). Moving away from discrete time, formally inclined linguists have (from Bennett and Partee 1972 on) embraced intervals that (as in the real line) are arbitrarily divisible (e.g., Dowty 1979; Kamp and Reyle 1993; van Lambalgen and Hamm 2004; Pratt-Hartmann 2005; Klein 2009). And following Reichenbach (1947) and Davidson (1967), notions of reference and event have taken center stage—so much so that "tense logic has fallen into disuse in natural language semantics" (Blackburn 2006, p. 342). Focusing on the issue of discrete time, the present paper applies Prior's statement above to the widening distance between Priorean tense logic and temporal semantics in linguistics. Very briefly, "limited fields of discourse" are linked to finite sets of temporal propositions, and "a sequence of discrete states" associated with a finite automaton is induced from a choice X of such a finite set. While no single choice X can capture the open-endedness of ordinary language, certain choices suffice for certain purposes, yielding a serviceable notion of discrete time. Variations in these choices can lead to non-discrete time including the real line.

Metaphysical assumptions about discrete time aside, Prior has qualms about the very notion of an instant

> I find myself quite unable to take 'instants' seriously as individual entities; I cannot *understand* 'instants', and the earlier-later relation that is supposed to hold between them, except as logical constructions out of tensed facts. Tense logic is for me, if I may use the phrase, *metaphysically fundamental*

(Prior 1968, p. 200). The technicalities below can be developed from this stance. That said, a linear order on instants has become standard fare in temporal logic, and is taken for granted in Sect. 2 to formulate notions of past, present and future. We pause in Sect. 3 for ideas about discourse and models, and proceed in Sect. 4 to flesh out "limited fields of discourse" as finite sets X of fluents that can be tracked by strings. These strings serve not only as models but, as detailed in Sect. 5, also as formulae. What's more, in Sect. 6, they can be construed as instants (relative to X) that, in accordance with Prior (1967), branch into the future.

2 Past, present and future—and intervals

Fix a linear order $\prec$ on a set T of instants. An instant $t \in T$ divides T into 3 disjoint subsets, $\{t\}$, past(t) and future(t) where

$$\begin{aligned} \text{past}(t) &:= \{t' \in T \mid t' \prec t\} \\ \text{future}(t) &:= \{t' \in T \mid t \prec t'\} . \end{aligned}$$

Assuming t is neither $\prec$-least nor $\prec$-greatest, the sets past(t) and future(t) are non-empty, and we get the chain past$(t) \prec \{t\} \prec$ future(t), where $\prec$ is lifted to intervals $I, I' \subseteq T$ via universal quantification for *whole* precedence

$$I \prec I' \iff (\forall t \in I)(\forall t' \in I')\, t \prec t'.$$

Focusing on the sets past(t) and future(t), Galton 1987 defines a *formal occurrence* to be a pair (B, A) of intervals B and A such that

$$B \prec A, \quad B \prec \overline{B} \text{ and } \overline{A} \prec A$$

where the *complement* $\overline{C}$ *of* C is $\{t \in T \mid t \notin C\}$. The intuition is that the "*b*efore" set B is past(t), while the "*a*fter" set A is future(t), except that t (or better yet, $\overline{B \cup A}$) is allowed to stretch into an interval or vanish altogether into the empty set $\emptyset$. An *event radical* e is then interpreted as a set $[\![e]\!]$ of formal occurrences serving as an input/output relation between intervals

$$B[\![e]\!]A \iff e \text{ outputs } A \text{ on input } B, \text{ taking up time } \overline{B \cup A}$$

with the progessive Prog(e) of e holding at instants in $\overline{B \cup A}$ for $B[\![e]\!]A$.

(G) $$t \models \text{Prog}(e) \iff (\exists B \prec \{t\})(\exists A \succ \{t\})\ B[\![e]\!]A$$

(G) is similar to an earlier account (N) of the progressive from Nishimura (1980), under which some sentences are evaluated at instants (or moments) t and others (Galton's event radicals) at intervals (t, t') with $t \prec t'$.

(N) $$t \models \text{ING}(e) \iff (\exists x \prec t)(\exists y \succ t)\ (x, y) \models e$$

(N) is, in turn, a modification of a well-known proposal (S) by Dana Scott.

(S) $$t \models \text{PROG}(e) \iff (\exists x \prec t)(\exists y \succ t)(\forall t' \in (x, y))\ t' \models e$$

Over the real line, an open interval (x, y) around t includes instants in the past and future of t so that under (S), we have

(s) whenever $t \models \text{PROG}(e)$, $(\exists t' \prec t)\ t' \models e$ and $(\exists t'' \succ t)\ t'' \models e$.

The spillover (s) reflects the "ongoing" character of imperfectives (including progressives), but is lost in (G), defeating the point of distinguishing instants from formal occurrences to capture the contrasts (1) between imperfectives and perfectives (e.g., Comrie 1976).

(1) a. imperfective: ongoing, open-ended, viewed from inside
b. perfective: completed, closed, viewed from outside

An alternative to (G) that is arguably more faithful to (1) defines an interval I to be *inside* another interval I' that stretches to the left and right of I

$$I \sqsubset I' \iff (\exists x \in I')\ \{x\} \prec I \text{ and } (\exists y \in I')\ I \prec \{y\}.$$

We can then put the distinction between imperfectives and perfectives with event time E down to a viewpoint, analyzed as an interval R, with perfectives inside R, (2b), and R inside imperfectives, (2a).

(2) a. imperfective: R $\sqsubset$ E b. perfective: E $\sqsubset$ R

The contrast in (2) can be pictured as in (3), with an imperfective E segmented into three boxes, (3a), the middle of which contains R, and the perfective E left whole inside the middle box in (3b).

(3) a. E segmented: $\boxed{E_\circ}\boxed{E_\circ,R}\boxed{E_\circ}$ b. E whole: $\boxed{R_\circ}\boxed{E,R_\circ}\boxed{R_\circ}$

The strings of boxes in (3) are examples of the sequences mentioned by Prior above, which we will interpret model-theoretically, treating E and R as temporal propositions, not unlike Areces and Blackburn (2005), except that they are evaluated at an interval (which may exceed an instant), a snapshot of which is given by a box, arranged one after another, as in a comic strip (Fernando 2013).

With this in mind, let us fix a set Φ of temporal propositions, or *fluents* (for short), including E and R, and for every $\varphi \in \Phi$, the *φ-segment*, $\varphi_\circ$, satisfied by intervals I according to (4).

(4) a. $I \models \mathrm{I} \iff I = \mathrm{I}$ for $\mathrm{I} \in \{\mathrm{E}, \mathrm{R}\}$
b. $I \models \varphi_\circ \iff (\exists J \supseteq I)\ J \models \varphi$

(4a) treats E and R as names for themselves, while under (4b), φ-segments hold precisely at subintervals of φ-intervals. A commonly held view (shared by the avowedly Davidsonian Taylor 1977 and Montagovian Dowty 1979) is that a fluent φ representing a state holds at an interval I precisely if if holds at every instant in I—i.e., φ is pointwise in the sense defined in (5).

(5) φ is *pointwise* if for every interval I, $I \models \varphi \iff (\forall t \in I)\ \{t\} \models \varphi$

$E_\circ$ and $R_\circ$ are pointwise fluents, but neither E nor R is (unless they are singletons). For pointwise φ, we write $t \models \varphi$ for $\{t\} \models \varphi$ (as we do $t \prec t'$ for $\{t\} \prec \{t'\}$). Under this convention, (N) reduces to (S) for pointwise e. Furthermore, as made precise in Sect. 5, we can recast line (3) above using only pointwise fluents as (6), provided we analyze negation $\neg$ classically on points t

$$t \models \neg\psi \iff t \not\models \psi$$

before lifting satisfaction $\models$ to intervals I according to (5)

$$I \models \neg\psi \iff (\forall t \in I)\ t \models \neg\psi$$

(called *predicate negation* in Hamblin (1971), p. 131).

(6) a. $\boxed{E_\circ, \neg R_\circ}\boxed{E_\circ,R_\circ}\boxed{E_\circ, \neg R_\circ}$ b. $\boxed{R_\circ, \neg E_\circ}\boxed{E_\circ,R_\circ}\boxed{R_\circ, \neg E_\circ}$

3 Discourse, models and homogeneity

In a recent essay, Hans Kamp asserts

> when we interpret a piece of discourse—or a single sentence in the context in which it is being used—we build something like a *model* of the episode or situation described; and an important part of that model are its event structure, and the time structure that can be derived from that event structure by means of Russell's construction.

(Kamp 2013, p. 11), adding that "discourse time" (as opposed to real time) is "made up by those comparatively few events that figure in this discourse" (p. 9). While "the notion of mental models as representations of discourse is uncontroversial" (Johnson-Laird 2004, p. 189), Kamp's invocation of events is potentially contentious, given that

> according to Prior events do not "exist" at all; strictly speaking, only things exist. "Events are just what things do and what happens to them", he said. . . . Points of time, instants and events seemed as mythical to him as matter did to Berkeley ... In his view time is not an object, and the earlier-later calculus is just "a convenient but indirect way of expressing truths that are not really about 'events' but about things".

(Øhrstrøm and Hasle 1993, p. 42). Steering clear of an ontology of events, Dowty 1979 embraces times as objects in "postulating a single homogeneous class of predicates—*stative predicates*—plus three or four sentential operators and connectives" (p. 71) for an account of various classes of events (going back to Aristotle, Ryle, Kenny and Vendler). We pursue a variant of this approach below, proceeding from a set X of pointwise fluents, and augmenting Fernando 2013 with a structural analysis of homogeneity.

An *X-strip* is a tuple $\langle T, \prec, \{[\![\varphi]\!]\}_{\varphi\in X}\rangle$ interpreting every $\varphi \in X$ as a unary predicate $[\![\varphi]\!] \subseteq T$ over a set T of times linearly ordered by $\prec$. Given X-strips $\mathfrak{A}_1 = \langle T_1, \prec_1, \{[\![\varphi]\!]_1\}_{\varphi\in X}\rangle$ and $\mathfrak{A}_2 = \langle T_2, \prec_2, \{[\![\varphi]\!]_2\}_{\varphi\in X}\rangle$, an *$X$-morphism from $\mathfrak{A}_1$ to $\mathfrak{A}_2$* is a function $h : T_1 \to T_2$ such that for all $t, t' \in T_1$,

$$t \preceq_1 t' \text{ implies } h(t) \preceq_2 h(t')$$

(where $\preceq_i$ is the union of $\prec_i$ with equality $=$ on T_i) and for all $\varphi \in X$,

$$t \in [\![\varphi]\!]_1 \iff h(t) \in [\![\varphi]\!]_2.$$

The *coimage of h* is the set

$$\mathrm{coim}(h) := \{\{t' \in T \mid h(t) = h(t')\} \mid t \in T\}$$

of equivalence classes induced by h. An interval I is *X-homogeneous* if for every $\varphi \in X$, I intersects with $[\![\varphi]\!]$ iff I is a subset of $[\![\varphi]\!]$

$$(\exists t \in I)\ t \in [\![\varphi]\!] \iff (\forall t \in I)\ t \in [\![\varphi]\!].$$

That is, for an X-homogeneous interval I and $\varphi \in X$, it's all or nothing, $I \subseteq [\![\varphi]\!]$ or $I \cap [\![\varphi]\!] = \emptyset$.

Fact 1 Given an X-strip $\langle T, \prec, \{[\![\varphi]\!]\}_{\varphi \in X}\rangle$, a partition P of T is the coimage of an X-morphism iff each member of P is an X-homogeneous interval.

Next, let $\sim_X$ be the relation on T that holds between t and t' whenever there is an X-homogeneous interval to which both t and t' belong—i.e.,

$$t \sim_X t' \iff \{t'' \in T \mid t \preceq t'' \preceq t' \text{ or } t' \preceq t'' \preceq t\} \text{ is } X\text{-homogeneous.}$$

Clearly, $\sim_X$ is an equivalence relation. Moreover,

Fact 2 Given an X-strip $\mathfrak{A} = \langle T, \prec, \{[\![\varphi]\!]\}_{\varphi \in X}\rangle$, the relation $\sim_X$ induces the coarsest partition of T into X-homogeneous intervals. In particular,

$$t \sim_X t' \iff h(t) = h(t') \textit{ for some } X\textit{-morphism } h \textit{ from } \mathfrak{A}$$

for all $t, t' \in T$.

For $t \in T$, let t_X be the $\sim_X$-equivalence class of t

$$t_X := \{t' \in T \mid t \sim_X t'\}.$$

The *X-collapse of* $\mathfrak{A}$ is the X-strip

$$\mathfrak{A}_X = \langle \{t_X \mid t \in T\}, \prec_X, \{[\![\varphi]\!]_X\}_{\varphi \in X}\rangle$$

ordering the $\sim_X$-equivalence classes I and I' by whole precedence

$$I \prec_X I' \iff (\forall t \in I)(\forall t' \in I')\ t \prec t'$$

and collecting in $[\![\varphi]\!]_X$ the $\sim_X$-equivalence classes contained in $[\![\varphi]\!]$

$$I \in [\![\varphi]\!]_X \iff I \subseteq [\![\varphi]\!].$$

$\mathfrak{A}$ is *X-reduced* if every X-morphism from $\mathfrak{A}$ is 1-1—i.e., for all $t \in T$, $t_X = \{t\}$. Clearly, $\mathfrak{A}$ is X-reduced iff $\mathfrak{A}$ is isomorphic to $\mathfrak{A}_X$.

Let us agree that an X-strip $\mathfrak{A}$ is finite if T is finite, in which case we may assume that for some integer $n > 0$, T is the set $[n] := \{1, \ldots, n\}$ of integers from 1 to n, and $\prec$ is the restriction $<_n$ of $<$ to $[n]$. Next, consider the question: when is $\mathfrak{A}_X$ finite? Obviously, fluents in X had better not alternate between true and false indefinitely. More precisely, let a *(φ, n)-alternation in* $\mathfrak{A}$ be a sequence $t_1 \cdots t_n \in T^n$ such that for each $i \in [n-1]$,

$$t_i \prec t_{i+1} \quad \text{and} \quad t_i \in [\![\varphi]\!] \iff t_{i+1} \notin [\![\varphi]\!].$$

Given a (φ, n)-alternation $t_1 \cdots t_n$ in $\mathfrak{A}$, observe that there is *no* $(\varphi, n+1)$-alternation in $\mathfrak{A}$ iff $\mathfrak{A}_{\{\varphi\}}$ is isomorphic to $\langle [n], <_n, \{[\![\varphi]\!]_{t_1 \cdots t_n}\}_{\varphi \in X} \rangle$, where each $\varphi \in X$ is interpreted as the set of i's with $t_i \in [\![\varphi]\!]$

$$[\![\varphi]\!]_{t_1 \cdots t_n} \ := \ \{i \in [n] \mid t_i \in [\![\varphi]\!]\}.$$

We say $\mathfrak{A}$ is *φ-alternation bounded* if there is an integer $n > 0$ for which *no* (φ, n)-alternation in $\mathfrak{A}$ exists.[1]

Fact 3 *Given an X-strip $\mathfrak{A} = \langle T, \prec, \{[\![\varphi]\!]\}_{\varphi \in X} \rangle$, if X is finite then*

$$\mathfrak{A}_X \text{ is finite} \quad \iff \quad (\forall \varphi \in X)\ \mathfrak{A} \text{ is } \varphi\text{-alternation bounded.}$$

4 Segmentations and strings

Let $\mathfrak{A} = \langle T, \prec, \{[\![\varphi]\!]\}_{\varphi \in X} \rangle$ be an X-strip. A finite partition P of T into intervals can always be put in $\prec$-order—i.e., for some integer $n > 0$, $P = \{I_1, \ldots, I_n\}$ with $I_i \prec I_{i+1}$ for $1 \leq i < n$. Let us call a sequence $I_1 \cdots I_n$ of intervals a *segmentation of T* if

$$T = \bigcup_{i=1}^{n} I_i \quad \text{and} \quad I_i \prec I_{i+1} \text{ for } 1 \leq i < n.$$

A formal occurrence (in the sense of Galton 1987, as described in Sect. 2 above) is just a segmentation of T into 2 or 3 intervals. A segmentation $I_1 \cdots I_n$ of T induces the X-strip $\langle [n], <_n, \{[\![\varphi]\!]_{I_1 \cdots I_n}\}_{\varphi \in X} \rangle$ where for each $\varphi \in X$, we put every $i \in [n]$ such that $[\![\varphi]\!]$ contains I_i in

$$[\![\varphi]\!]_{I_1 \cdots I_n} \ := \ \{i \in [n] \mid I_i \subseteq [\![\varphi]\!]\}.$$

Fact 4 *Given an X-strip $\mathfrak{A} = \langle T, \prec, \{[\![\varphi]\!]\}_{\varphi \in X} \rangle$ and a segmentation $I_1 \cdots I_n$ of T, the following are equivalent*

(i) *the function from T to $[n]$ mapping $t \in T$ to the unique $i \in [n]$ such that $t \in I_i$ is an X-morphism from $\mathfrak{A}$* to $\langle [n], <_n, \{[\![\varphi]\!]_{I_1 \cdots I_n}\}_{\varphi \in X} \rangle$
(ii) *I_i is X-homogeneous for each $i \in [n]$*
(iii) *for every $\varphi \in X$,*

$$[\![\varphi]\!] \ = \ \bigcup \{I_i \mid i \in [n] \text{ and } I_i \subseteq [\![\varphi]\!]\}.$$

[1] That is, $\mathfrak{A}$ is φ-alternation bounded iff the boundary of $[\![\varphi]\!]$ is finite (where the *boundary of* a subset A of T is the closure of A minus the interior of A). We assume here the order topology, given by unions of sets (t, t') of instants $\prec$-between t and t'.

Given an X-strip $\mathfrak{A} = \langle [n], <_n, \{[\![\varphi]\!]\}_{\varphi \in X}\rangle$, let us define the *X-diagram of* $\mathfrak{A}$ to be the string $\alpha_1 \cdots \alpha_n$ of subsets

$$\alpha_i \ := \ \{\varphi \in X \mid i \in [\![\varphi]\!]\}$$

of X consisting of fluents φ in X with i in its interpretation $[\![\varphi]\!]$ (for $i \in [n]$). We extend the notion of an X-diagram to an arbitrary finite X-strip, identifying that X-strip with its unique isomorphic X-strip of the form $\langle [n], <_n, \{[\![\varphi]\!]\}_{\varphi \in X}\rangle$. Conversely, given a string $s = \alpha_1 \cdots \alpha_n$ over the alphabet 2^X of subsets of X, let $\mathfrak{A}_s$ be the X-strip $\langle [n], <_n, \{[\![\varphi]\!]_s\}_{\varphi \in X}\rangle$ where each $\varphi \in X$ is interpreted as the set

$$[\![\varphi]\!]_s \ := \ \{i \in [n] \mid \varphi \in \alpha_i\}$$

of positions in s where φ occurs. In $\mathfrak{A}_s$, "what you see is all there is" (WYSIATI, Kahneman 2011). To characterize strings such that $\mathfrak{A}_s$ is X-reduced, we implement

McTaggart's dictum that 'there could be no time if nothing changed'

(Prior 1967, p. 85) through a string function bc that reduces all repeating blocks $\alpha\alpha^n$ in a string s to α for its *block compression* $bc(s)$

$$bc(s) \ := \ \begin{cases} bc(\alpha s') & \text{if } s = \alpha\alpha s' \\ \alpha \ bc(\beta s') & \text{if } s = \alpha\beta s' \text{ with } \alpha \neq \beta \\ s & \text{otherwise.} \end{cases}$$

For example, for all integers $n, m \geq 0$, $bc(\alpha^n \alpha\beta\beta^m\alpha) = \alpha\beta\alpha$ provided $\alpha \neq \beta$. In general, $bc(bc(s)) = bc(s)$, and $bc(s)$ is stutter-less in that if $bc(s) = \alpha_1 \cdots \alpha_n$ then $\alpha_i \neq \alpha_{i+1}$ for $1 \leq i < n$.

Fact 5 Given a string $s \in (2^X)^+$, the following are equivalent

(i) *$\mathfrak{A}_s$ is X-reduced*
(ii) $s = bc(s)$
(iii) *for some X-strip $\mathfrak{A}$ such that $\mathfrak{A}_X$ is finite, s is the X-diagram of $\mathfrak{A}_X$.*

Notice that $bc(s) = s$ for any string s of length 1. The set T of times is $\emptyset$-homogeneous (vacuously), and is a segmentation of itself. As we add pointwise fluents into a set X, and require the components of a segmentation to be X-homogeneous, segmentations may need to be lengthened. To keep us from a segmentation into too many pieces, block compression bc desegments, essentially merging pieces whose union is X-homogeneous. In the next section, we compose bc with a string function ρ_X that takes account of the set X of fluents under consideration, with a view to letting X vary over the finite subsets of some large set Φ of fluents. Before then, however, let us keep X fixed, and say a string s over the alphabet 2^X *X-tracks* an X-strip $\mathfrak{A}$ if there is an X-morphism from $\mathfrak{A}$ to $\mathfrak{A}_s$.

Fact 6 For all strings s and s' over the alphabet 2^X that X-track an X-strip $\mathfrak{A}$, $bc(s) = bc(s')$ and $bc(s)$ X-tracks $\mathfrak{A}$.

Fact 6 suggests (in combination with Facts 4 and 5) the following definition. An X-*representation of* $\mathfrak{A}$ is a string s equal to $\mathit{bc}(s)$ that X-tracks $\mathfrak{A}$. It exists iff for every $\varphi \in X$, $\mathfrak{A}$ is φ-alternation bounded, in which case it is the X-diagram of $\mathfrak{A}_X$. Returning to the opening paragraph above from Prior (1967), a limited field of discourse is analyzed in terms of a finite set X of fluents and strings $\mathit{bc}(s)$, for $s \in (2^X)^+$.

5 Strings as models and as formulae

Familiar examples of X-representations of $\mathfrak{A}$-strips are provided by a calendar year, which we can represent as the string

$$s_{mo} \;:=\; \boxed{\text{Jan}}\boxed{\text{Feb}}\boxed{\text{Mar}}\cdots\boxed{\text{Dec}}$$

of length 12, or, were we interested also in days d1,d2. . .,d31, the string

$$s_{mo,dy} \;:=\; \boxed{\text{Jan,d1}}\boxed{\text{Jan,d2}}\cdots\boxed{\text{Jan,d31}}\boxed{\text{Feb,d1}}\cdots\boxed{\text{Dec,d31}}$$

of length 366 for a leap year.[2] In contrast to the points in the real line $\mathbb{R}$, a box can split, as $\boxed{\text{Jan}}$ in s_{mo} does (30 times) in $s_{mo,dy}$, on introducing days d1, d2,. . ., d31 into the picture. Reversing direction and generalizing from

$$mo \;:=\; \{\text{Jan,Feb,}\ldots\text{Dec}\}$$

to any set X of fluents, we define the function ρ_X on strings (of sets) to componentwise intersect with X

$$\rho_X(\alpha_1 \cdots \alpha_n) \;:=\; (\alpha_1 \cap x) \cdots (\alpha_n \cap X)$$

(throwing out non-X's from each box) so that

$$\rho_{mo}(s_{mo,dy}) \;=\; \boxed{\text{Jan}}^{31}\boxed{\text{Feb}}^{28}\cdots\boxed{\text{Dec}}^{31}.$$

The idea is that ρ_X sees only X, just as bc sees time only through change, sending $\rho_{mo}(s_{mo,dy})$ to s_{mo}

$$\mathit{bc}\left(\boxed{\text{Jan}}^{31}\boxed{\text{Feb}}^{28}\cdots\boxed{\text{Dec}}^{31}\right) \;=\; \boxed{\text{Jan}}\boxed{\text{Feb}}\boxed{\text{Mar}}\cdots\boxed{\text{Dec}}.$$

Let bc_X be the sequential composition $\rho_X; \mathit{bc}$ mapping s to $\mathit{bc}(\rho_X(s))$. Then $\mathit{bc}_{mo}(s_{mo,dy}) = s_{mo}$ and

$$\mathit{bc}_{\{\text{Feb}\}}(s_{mo,dy}) = \boxed{}\boxed{\text{Feb}}\boxed{} \quad\text{and}\quad \mathit{bc}_{\{\text{d3}\}}(s_{mo,dy}) = \boxed{}\boxed{\text{d3}}\left(\boxed{}\boxed{\text{d3}}\right)^{11}\boxed{}.$$

[2] We draw boxes (instead of the usual curly braces { and }) around sets-as-symbols, reinforcing their cartoon/film strip reading.

To formalize the idea that in $s_{mo,dy}$, the fluent Feb is an interval but $\mathsf{d3}$ is not, let us use the vocabulary of predicate logic of which a string $s \in (2^X)^+$ is (as the X-strip $\mathfrak{A}_s$) a model. That vocabulary consists of

(i) a binary relation symbol $\dot{<}$, interpreted by s as $<_n$, where n is the length of s, and
(ii) for each $\varphi \in X$, a distinct unary relation symbol P_φ, interpreted by s as $[\![\varphi]\!]_s$.

With these symbols, we can say φ is an interval

$$\varphi\text{-interval} \;:=\; \exists x\, P_\varphi(x) \wedge \neg\exists y\ \mathrm{hole}_\varphi(y)$$

where $\mathrm{hole}_\varphi(y)$ says φ holds before and after y but not at y

$$\mathrm{hole}_\varphi(y) \;:=\; \exists u(P_\varphi(u) \wedge u \dot{<} y) \wedge \exists v(P_\varphi(v) \wedge y \dot{<} v) \wedge \neg P_\varphi(y).$$

Identifying s with the X-strip $\mathfrak{A}_s$, we have

(7) $$s \models \varphi\text{-interval} \quad\Longleftrightarrow\quad unpad(bc_{\{\varphi\}}(s)) = \boxed{\varphi}$$

where *unpad* strips off any initial or final empty boxes $\boxed{}$

$$unpad(s) \;:=\; \begin{cases} unpad(s') & \text{if } s = \boxed{}s' \text{ or else } s = s'\boxed{} \\ s & \text{otherwise} \end{cases}$$

so that $unpad(s)$ neither begins nor ends with $\boxed{}$. We can rewrite (7) as

(8) $$s \models \varphi\text{-interval} \quad\Longleftrightarrow\quad s \in \langle bc_{\{\varphi\}}\rangle\langle unpad\rangle\boxed{\varphi}$$

adopting the notation $\langle R\rangle L$ for the inverse image of a language L under a binary relation R (between strings)

$$\langle R\rangle L \;:=\; \{s \mid (\exists s' \in L)\ sRs'\}$$

and conflating a string s with the language $\{s\}$. The modal operator notation suggests that $\boxed{\varphi}$ is part of a formula (φ-interval), the relevant vocabulary of which is picked out by the subscript X in bc_X.

Beyond singletons $\{\varphi\}$, X may range over any finite set of fluents. For any such X, let us collect the strings in which every $\varphi \in X$ is an interval and apply bc_X and *unpad* for

$$Ivl(X) \;:=\; \left\{ unpad(bc_X(s)) \;\middle|\; s \in \bigcap_{\varphi \in X} \langle bc_{\{\varphi\}}\rangle\langle unpad\rangle\boxed{\varphi} \right\}.$$

For two distinct fluents e nd e', there are 13 strings in $Ivl(\{e, e'\})$, one per Allen interval relation (Allen 1983), refining the relations $\prec$ of full precedence and $\bigcirc$ of *overlap*

Table 1 From Russell-Wiener-Kamp (RWK) to Allen

RWK	Allen	$Ivl(\{e, e'\})$	Allen	$Ivl(\{e, e'\})$	Allen	$Ivl(\{e, e'\})$
$e \bigcirc e'$	$e \mathtt{=} e'$	$\boxed{e, e'}$	$e\ \mathtt{fi}\ e'$	$\boxed{e}\boxed{e, e'}$	$e\ \mathtt{f}\ e'$	$\boxed{e'}\boxed{e, e'}$
	$e\ \mathtt{si}\ e'$	$\boxed{e, e'}\boxed{e}$	$e\ \mathtt{di}\ e'$	$\boxed{e}\boxed{e, e'}\boxed{e}$	$e\ \mathtt{oi}\ e'$	$\boxed{e'}\boxed{e, e'}\boxed{e}$
	$e\ \mathtt{s}\ e'$	$\boxed{e, e'}\boxed{e'}$	$e\ \mathtt{o}\ e'$	$\boxed{e}\boxed{e, e'}\boxed{e'}$	$e\ \mathtt{d}\ e'$	$\boxed{e'}\boxed{e, e'}\boxed{e'}$
$e \prec e'$	$e\ \mathtt{m}\ e'$	$\boxed{e}\boxed{e'}$	$e < e'$	$\boxed{e}\boxed{\ }\boxed{e'}$		
$e' \prec e$	$e\ \mathtt{mi}\ e'$	$\boxed{e'}\boxed{e}$	$e > e'$	$\boxed{e'}\boxed{\ }\boxed{e}$		

used in the Russell–Wiener–Kamp construction of time from events (e.g. Kamp and Reyle 1993; Kamp 2013); see Table 1. We have

$$Ivl(\{e, e'\}) \quad = \quad \text{Allen}(e \bigcirc e') + \text{Allen}(e \prec e') + \text{Allen}(e' \prec e)$$

where Allen$(e \bigcirc e')$ consists of the nine strings in which e overlaps e'

$$\text{Allen}(e \bigcirc e') \quad := \quad \left(\boxed{e} + \boxed{e'} + \epsilon\right) \boxed{e, e'} \left(\boxed{e} + \boxed{e'} + \epsilon\right)$$

(with empty string ϵ), and Allen$(e \prec e')$ consists of the two strings in which e precedes e'

$$\text{Allen}(e \prec e') \quad := \quad \boxed{e}\boxed{e'} + \boxed{e}\boxed{\ }\boxed{e'}$$

and similarly for Allen$(e' \prec e)$. Rather than expressing $e \prec e'$ in the vocabulary of predicate logic (as we did with φ-interval), we can apply the modal operators $\langle bc_{\{e,e'\}}\rangle$ and $\langle unpad\rangle$ to strings for

$$s \models e \prec e' \quad \iff \quad s \in \langle bc_{\{e,e'\}}\rangle\langle unpad\rangle\left(\boxed{e}\boxed{e'} + \boxed{e}\boxed{\ }\boxed{e'}\right)$$

and do the same for the more refined Allen relations—e.g.

$$s \models e\ \mathtt{f}\ e' \quad \iff \quad s \in \langle bc_{\{e,e'\}}\rangle\langle unpad\rangle\boxed{e'}\boxed{e, e'}.$$

There are more strings in $Ivl(X)$ than Russell–Wiener–Kamp event structures $\langle X, \bigcirc, \prec\rangle$, because not all boxes in a string such as $\boxed{e}\boxed{\ }\boxed{e'}$ are $\subseteq$-maximal (as required of instants, under Russell–Wiener–Kamp). But this is easily rectified; for each $e \in X$, we add two forms of negations, $past(e)$ and $future(e)$, to turn, for example, $\boxed{e'}\boxed{e, e'}\boxed{e'}$ into $\boxed{past(e), e'}\boxed{e, e'}\boxed{future(e), e'}$, no two boxes in which are related by $\subseteq$.

6 Branching from history-laden instants

In Prior 1968, instants are, it is suggested, "tensed facts" and

> Philosophically the most interesting proposition which is true at a given instant only is the conjunction of all the propositions which are then true, but for formal purposes any proposition true at that instant only will do as its tense-logical "representative"

(p. 196). Rather than identifying an arbitrary box in a string such as $\boxed{a}\boxed{}\boxed{a'}$ with an instant, one can construe the entire string as an instant whose present is represented by the final box, $\boxed{a'}$, and whose past is given by the remaining substring, $\boxed{a}\boxed{}$. Put as a tensed fact, the instant $\boxed{a}\boxed{}\boxed{a'}$ is the temporal proposition

$$\underbrace{a' \wedge \neg a}_{\text{present}} \wedge \mathcal{Y}\underbrace{(\neg a \wedge \neg a' \ \wedge \ \mathcal{Y}(a \wedge \neg a' \ \wedge \ \neg\mathcal{Y}\top))}_{\text{past}}$$

where $\mathcal{Y}$ is the existential operator *Y*esterday (Prior 1967, p. 67; the converse of Tomorrow or Next). The idea is that an instant comes with its present and past, but not a future; thus, propositions true at that instant may pertain to the present or past, but not the future. A string s may branch any number of directions into the future, represented by the different strings $\hat{s}$ that can combine with s in the prefix relation

$$s \preceq_{\text{prefix}} s' \iff (\exists \hat{s})\ s\hat{s} = s'.$$

Note that an instant's present and past are finite only because we have limited the field of discourse to a finite set X of fluents (interpreted in an X-strip where each is alternation bounded).

For an infinite set Φ of fluents, we can build approximations indexed by the set $Fin(\Phi)$ of finite subsets of Φ. More precisely, strings over the various alphabets 2^X, for $X \in Fin(\Phi)$, are organized in the *inverse limit* $\mathfrak{IL}(\Phi)$ *of* $\{\mathit{bc}_X\}_{X \in Fin(\Phi)}$, understood as the set of functions $f : Fin(\Phi) \to (2^\Phi)^+$ such that

$$f(X) = \mathit{bc}_X(f(X')) \ \text{ whenever } X \subseteq X' \in Fin(\Phi).$$

The prefix ordering $\preceq_{\text{prefix}}$ is then lifted to $\mathfrak{IL}(\Phi)$ by universally quantifying over $X \in Fin(\Phi)$ for the irreflexive relation $\prec_\Phi$ given by

$$f \prec_\Phi f' \iff f \neq f' \text{ and } (\forall X \in Fin(\Phi))\ f(X) \preceq_{\text{prefix}} f'(X)$$

for all f and $f' \in \mathfrak{IL}(\Phi)$. Time branches under $\prec_\Phi$, which is tree-like—i.e., transitive and left linear: for every $f \in \mathfrak{IL}(\Phi)$, and all $f_1 \prec_\Phi f$ and $f_2 \prec_\Phi f$,

$$f_1 \prec_\Phi f_2 \ \text{ or } \ f_2 \prec_\Phi f_1 \ \text{ or } \ f_1 = f_2$$

(Fernando 2013). A copy of the real line $\mathbb{R}$ can, with Φ equal to the set $\mathbb{Q}$ of rational numbers, be obtained from $\prec_{\mathbb{Q}}$, restricted to suitable functions $f_r \in \mathfrak{IL}(\mathbb{Q})$, for $r \in \mathbb{R}$, encoding Dedekind cuts representing r.

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