An idea that changed web search: Google Page Rank

Value of webpage P_i = probability a web surfer is at P_i clicking links

- initially random
- move to P_j from P_i along link $P_i \rightarrow P_j$ probabilistically

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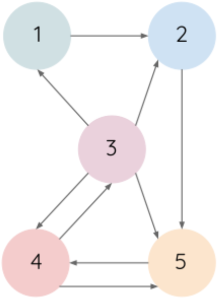
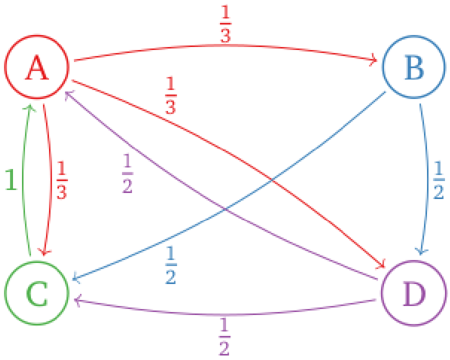


	Iteration 0	Iteration 1	Iteration 2	Final Rank
P_1	1/5	1/20	1/40	5
P_2	1/5	5/20	3/40	4
P_3	1/5	1/10	5/40	3
P_4	1/5	5/20	15/40	2
P_5	1/5	7/20	16/40	1

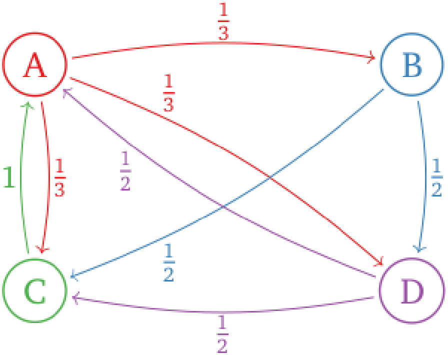
$$PR(P_5) = \tfrac{1}{5} + \tfrac{1}{5} * \tfrac{1}{4} + \tfrac{1}{5} * \tfrac{1}{2} = 7/20$$

From cs.brown.edu

From *Interactive Linear Algebra*, Margalit & Rabinoff



From *Interactive Linear Algebra*, Margalit & Rabinoff



$$\begin{pmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} c + \frac{1}{2}d \\ \frac{1}{3}a \\ \frac{1}{3}a + \frac{1}{2}b + \frac{1}{2}d \\ \frac{1}{3}a + \frac{1}{2}b \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}.$$

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$$\begin{pmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} c + \frac{1}{2}d \\ \frac{1}{3}a \\ \frac{1}{3}a + \frac{1}{2}b + \frac{1}{2}d \\ \frac{1}{3}a + \frac{1}{2}b \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}.$$

$$A\mathbf{x} = \mathbf{x}$$

is an eigenvalue-eigenvector problem that we can solve iteratively

$$\mathbf{x} = \lim_{n\to\infty} A^n \, \mathbf{x}_\circ$$

from uniform distribution $\mathbf{x}_\circ$ by repeated A-transitions

$$A^1 := A \qquad A^{n+1} := A^n A \ \text{(matrix multiplication)}$$

under certain assumptions about A.

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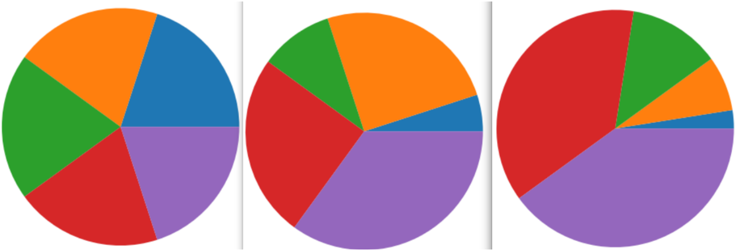

$p(0)$ $p(1)$ $p(2)$

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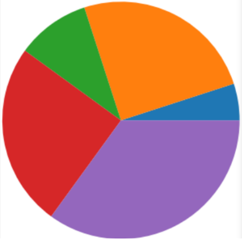
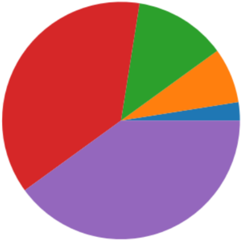
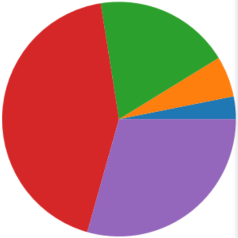
$p(0)$



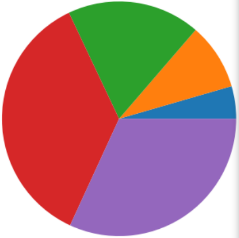
$p(1)$



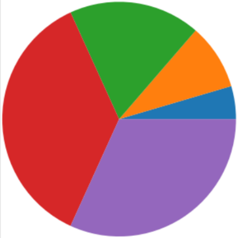
$p(2)$



$p(3)$



$p(10)$



$p(100)$

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https://www.codabrainy.com/en/python-compiler/

trinket Python3 Run Share

main.py

```python
import numpy as np
import matplotlib.pyplot as plt

A = np.array([[0, 0, 0.25, 0, 0], [1, 0, 0.25, 0, 0], [0, 0, 0, 0.5, 0],
              [0, 0, 0.25, 0, 1], [0, 1, 0.25, 0.5, 0]])

p0 = np.array([[0.2], [0.2], [0.2], [0.2], [0.2]])
p1 = np.matmul(A,p0)

def p(n):
    if n==0: return p0
    else: return np.matmul(A,p(n-1))    # problematic for n=1000

def q(n):
    temp = p0
    for i in range(n): temp = np.matmul(A,temp)
    return temp

def show(n):
    plt.pie(np.concatenate(q(n),axis=None))
    plt.show()

show(100)
```

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Adjustments for random exploration

with probability p, our surfer will surf to a completely random page; otherwise, he'll click a random link on the current page, unless the current page has no links, in which case he'll surf to a completely random page in either case.

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with probability p, our surfer will surf to a completely random page; otherwise, he'll click a random link on the current page, unless the current page has no links, in which case he'll surf to a completely random page in either case.

$$A = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{pmatrix} \xrightarrow{\text{modify}} A' = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{1}{4} & 0 \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{4} & 0 \end{pmatrix}$$

Choosing the damping factor $p = 0.15$, the Google Matrix is

$$M = 0.85 \cdot \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{1}{4} & 0 \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{4} & 0 \end{pmatrix} + 0.15 \cdot \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix}$$

$$\approx \begin{pmatrix} 0.0375 & 0.0375 & 0.2500 & 0.4625 \\ 0.3208 & 0.0375 & 0.2500 & 0.0375 \\ 0.3208 & 0.4625 & 0.2500 & 0.4625 \\ 0.3208 & 0.4625 & 0.2500 & 0.0375 \end{pmatrix}.$$

from *Interactive Linear Algebra*, Margalit & Rabinoff